

A Note for Repeated Game (Discount Factor and Discount Rate)

1. Discount Factor

- Suppose that you will get \$100 in the next period.
- \$100 in the next period must be less worth than \$100 today.
- Suppose that \$100 in the next period is equivalent to today's \$100 to you (no difference between today and future)
 - This means your future is as important as today. In other words, your future is 100% as important as today.
- Suppose that \$100 in the next period is equivalent to today's \$80.
 - This means your future is 80% as important as today.
- Suppose that \$100 in the next period is equivalent to today's \$50
 - This means your future is 50% as important as today.
- If \$100 in the next period is equivalent to today's \$0.
 - This means your future is 0% as important as today.
- This % above is discount factor.
 - Discount factor=1 (100%) → no difference b/w today and future.
 - Discount factor=0 (0%) → only loves today. Any value from future is 0 value to you. Only today is important.
- Discount factor=Equivalent Current Value/Given Future Value.

2. Discount Rate

- Suppose that you currently have \$100. Let's find the amount of money in the next period that is as worth as current \$100 (It must be greater than \$100).
- Suppose that \$100 in the current period is equivalent to \$120 in the next period.
 - $120 = 1.2 * 100 = (1 + 0.2) * 100$. Here 0.2 is your discount rate.
- Suppose that \$100 in the current period is equivalent to \$150 in the next period.
 - $150 = 1.5 * 100 = (1 + 0.5) * 100$. Here 0.5 is your discount rate.
- Suppose that \$100 in the current period is equivalent to \$100 in the next period.
 - $100 = 1.0 * 100 = (1 + 0.0) * 100$. Here 0 is your discount rate. You don't discount future.
- Suppose that \$100 in the current period is equivalent to ∞ in the next period.
 - $\infty = \infty * 100 = (1 + \infty) * 100$. Here ∞ is your discount rate. The future value is just nothing. Only current period is important.
- Generally
 - Equivalent future value = (1 + discount rate) * given current value.

3. Discount Factor and Discount Rate

- Suppose that \$100 in the current period is equally as worth as \$120 in the next period.
 - Then we have
Future \$120 = (1 + 0.2) * given current \$100. Your discount rate is 0.2.

- Thinking this in the other way around, if \$ 120 in the next period is equally as worth as \$100 in the current period, your discount factor is $100/120$, which is same as $100/(1+0.2)*100=1/(1+0.2)$.
 - Therefore discount factor = $1/ (1+\text{discount rate})$.
- Suppose that \$100 in the current period is equally as worth as \$150 in the next period.
 - Then we have
Future \$150 = $(1+0.5) * \text{given current } \100 . Your discount rate is 0.5.
 - Thinking this in the other way around, if \$ 150 in the next period is equally as worth as \$100 in the current period, your discount factor is $100/150$, which is same as $100/(1+0.5)*100=1/(1+0.5)$.
 - Therefore discount factor = $1/ (1+\text{discount rate})$.
- Suppose that \$100 in the current period is equally as worth as \$100 in the next period.
 - Then we have
Future \$100 = $(1+0) * \text{given current } \100 . Your discount rate is 0.
 - Thinking this in the other way around, if \$ 100 in the next period is equally as worth as \$100 in the current period, your discount factor is $100/100$, which is same as $100/(1+0)*100=1/(1+0)$.
 - Therefore discount factor = $1/ (1+\text{discount rate})$.
- Suppose that \$100 in the current period is equally as worth as $\$ \infty$ in the next period.
 - Then we have
Future $\$ \infty = (1+\infty) * \text{given current } \100 . Your discount rate is ∞ .
 - Thinking this in the other way around, if \$ ∞ in the next period is equally as worth as \$100 in the current period, your discount factor is $100/\infty$, which is same as $100/(1+\infty)*100=1/(1+\infty)$.
 - Therefore discount factor = $1/ (1+\text{discount rate})$.