A Flatness Based Approach for the Thickness Control in Rolling Mills

Andreas Kugi*, Rainer Novak**, Kurt Schlacher** and Karl Aistleitner***
*Department of Automatic Control
Johannes Kepler University of Linz
**Christian Doppler Laboratory Automatic Control of Mechatronic Systems in Steel Industries
***VOEST-ALPINE Industrieanlagenbau GmbH & Co
Altenbergerstr. 69
4040-Linz, Austria, Europe
*kugi@mechatronik.uni-linz.ac.at

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Abstract

A multi-stand rolling mill is a highly complex coupled non-linear system. If the operating point is changing in a wider range the non-linearities can no longer be neglected in the controller design. For a control concept to be feasible in the rough industrial environment, it is absolutely necessary to consider the fact that not all quantities are directly measurable and that they are significantly corrupted by noise. The proposed control approach consists of a non-linear servo compensation in the inner loop and a flatness based thickness and interstand tension controller in the outer loop.

1 Introduction

In the last years, in rolling mills a strong tendency towards tighter thickness tolerances and higher efficiency can be observed. Apart from the mechanical equipment, the actuators and sensors, a big potential for improving the quality of the rolled product lies in the automation system and the used control techniques. The conventional control approach for a multi-stand rolling mill is based on several linear SISO- (single-input single-output) controllers, although the underlying physical structure is a highly complex non-linear coupled process. Since these coupling effects are not considered within the controller design, the performance of the overall closed loop system is not always satisfactory. However, in literature one can find successful applications of linear MIMO- (multi-input multi-output) controllers to multi-stand rolling mills, which, in fact, overcome the deficiencies of the classical single-loop control concepts (see, e.g., [17], [19], [8], [9]). Nevertheless, all these approaches assume that the process can be described by a linear nominal MIMO-model and the inherent non-linearities of the process are taken into account by means of uncertainty models, which are supposed to satisfy certain conditions depending on the used controller design strategy. This is why in literature the proposed controllers are based either on a linear robust approach, like e.g. the linear multivariable H_∞-design (see, e.g., [9] and the references cited therein) or on linear self-tuning concepts as it is presented e.g., in [4]. In many situations these model assumptions are no essential restriction, in particular, if the rolling mill is operating around a predefined pass schedule. But if the operating point is changing in a wider range, like it is the case for a flying gauge change in continuous rolling mills, then the non-linearities of the mathematical model can no longer be neglected. At this point it is worth mentioning that in connection with the development of continuous casting there is a strong trend to continuous hot rolling mills and also final strip thicknesses down to 0.7 mm. Here the demands on the thickness tracking control concept (flying gauge change) are very challenging and the presented control approach also seems to be a promising way to reach the desired goals.

The main objective of the thickness control concept presented in this paper is to minimize the off-gauge lengths for a flying gauge change in continuous cold rolling mills. This is only possible by making use of the full operating range of the actuating devices like the hydraulic adjustment system. In turn, this brings about that a linear approximation of the non-linear dynamics of the rolling process is no longer valid. We are aware of the fact that the non-linear models contain constitutive parameters which are only known rather inaccurate like e.g., the friction coefficient between the roll and the strip or the yield stress as a function of the strip reduction. On the other hand there are non-linear effects which stem from balance equations and hence can be covered fairly good by the model. The control task becomes even more difficult since in addition to these parameter uncertainties not all quantities are directly available through measurement and, in general, the measured signals are significantly corrupted by transducer and quantization noise. For instance, this fact makes it practically nearly impossible to determine the velocity in a higher dynamic range by an observer based on the position signal. A successful implementation of a non-linear con-

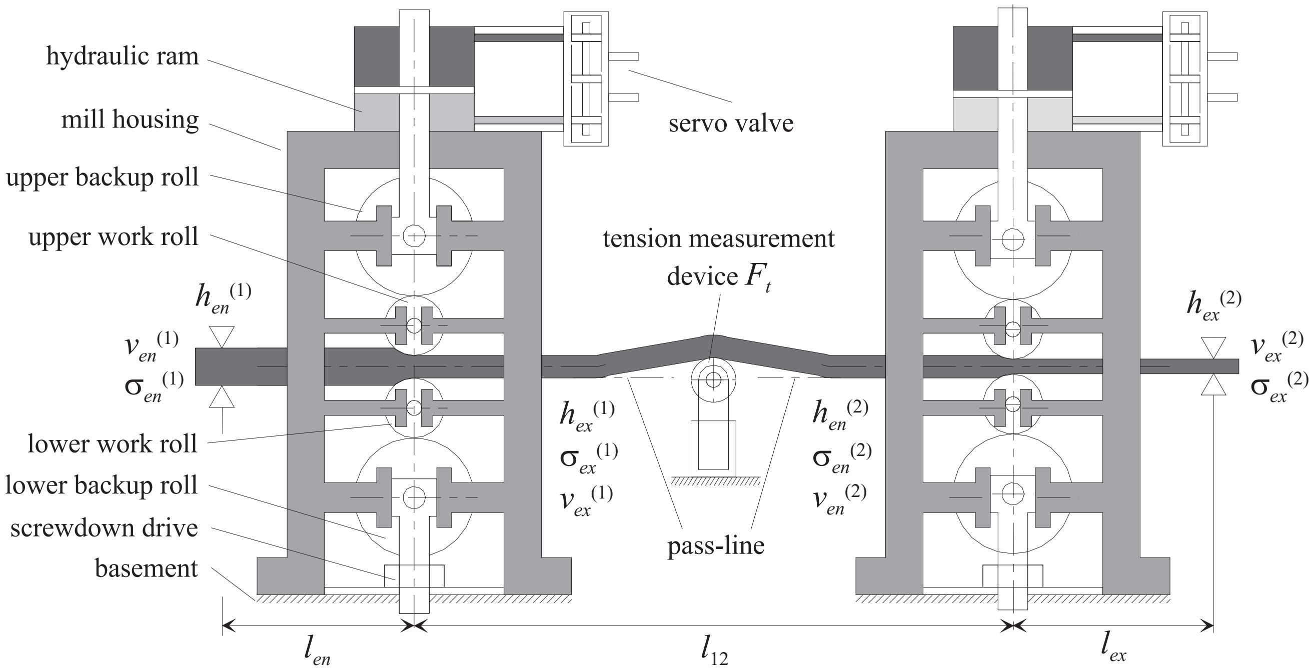

Figure 1: Schematic diagram of a two-mill stand configuration.

trol concept for the HGC (hydraulic gap control) in rolling mills can be found in [11], [18]. Maybe, these restrictions and features of the plant give a possible explanation of the fact that in the rough industrial environment the realization of non-linear control concepts is not so popular as the theory for non-linear control design would promise. Moreover, we have the experience from several practical applications that a straight-forward application of the available non-linear control design strategies to practical problems only succeeds in very rare cases. In this sense also in this paper the flatness (see, e.g., [5], [20], [3], [15]) based thickness control concept requires some additional considerations in order to be practically feasible. Throughout the paper we will try to elaborate the details of the controller design in such a way that the reader, who is not so familiar with rolling processes, will also understand the proposed methods.

Another important aspect, which has to be taken into account, is that the proposed control concept can easily be implemented and realized by the commissioning engineer and the start-up time of the mill can be kept to a minimum. This requires that the control concept is extensively tested in advance on a mill simulator, which contains a much more detailed model than the model which serves as a basis for the controller design. In the mill simulator all the ”dirty” effects, like the non-negligible dynamics of the sensors and actuators, the quantization, the transducer noise, the sampling process, stick-slip friction effects etc. have to be included and if possible, the constitutive parameters have to be adjusted by means of measurement results.

The paper is organized as follows. In Section 2 the various components of the mathematical model of a two-stand cold rolling mill with two four-high stands and a hydraulic adjustment system acting on the upper backup roll will be presented. This configuration has the advantage that on the one hand it covers all the essential non-linear coupling effects and on the other hand the complexity of this model still allows to understand the underlying physical structure. The description of the material deformation is based on the roll force model of Bland, Ford and Ellis for cold rolling with pure plastic deformation, see [6], [2] and [7]. Section 3 is devoted to the thickness control concept, which consists of an inner control loop for the hydraulic servo compensation and an outer control loop for the thickness and tension control. The thickness and the tension controller are designed on the basis of a flatness approach which automatically yields to a natural decoupling between an average specific interstand tension and the strip exit thickness. Some simulation results for the considered two-stand mill configuration on the mill simulator are presented in Section 4 and the last section, Section 5, contains some conclusions.

2 Mathematical Model

Typically, rolling mills consist of several mill stands with a pay off reel on the entry side and a tension reel on the exit side. In the case of continuous cold rolling mills in combination with a pickling line on the entry side the pay off reel is replaced by a bridle roll. For the sake of clearness we will subsequently restrict our considerations to two stands with the associated entry and exit section because this gives the smallest unit, which contains all the interconnection phenomena. However, without much effort the presented theory can be extended to a multi-stand rolling mill with an arbitrary number of mill stands. Figure 1 depicts the schematic diagram of a two-stand mill with two four-high mill stands and a hydraulic adjustment system acting on the upper backup roll. Thereby, the work rolls are effectively used for the strip deformation whereas the backup rolls serve to support the work rolls in order to prevent too excessive bending of the work rolls. The rolls are running in so called chocks which can move vertically in the mill housing and hence enable a change of the roll gap. Generally, the thickness of the rolled strip is determined by the gap between the two work rolls. The actual position control is then performed by the exact and fast-acting hydraulic adjustment system. For the entry and exit section of the two-mill stand configuration of Fig. 1 we assume that the specific entry strip tension $\sigma_{en}^{(1)}$, the strip entry speed $v_{en}^{(1)}$, the strip entry thickness $h_{en}^{(1)}$ and the specific exit strip tension $\sigma_{ex}^{(2)}$ are sufficiently smooth known functions of the time t. It will not be specified here, whether these quantities are directly available through measurement or indirectly by calculation on the basis of other measured quantities. Concerning the notation, let us arrange that an upper index (1) or (2) always refers to the first or second stand, respectively. In the following, we will give a detailed formulation of the different components of the two-mill stand configuration of Fig. 1 and we will always try to point out clearly the simplifications and neglects of the mathematical model.

2.1 Hydraulic Adjustment System

Let us assume without restriction of generality that the hydraulic adjustment system consists of a double-acting, double-ended hydraulic ram as shown in Fig. 2. This configuration covers all different constructions of single- and double-ended as well as single- and double-acting hydraulic actuators. The continuity equation written for the two chambers yields to a mathematical model of the form

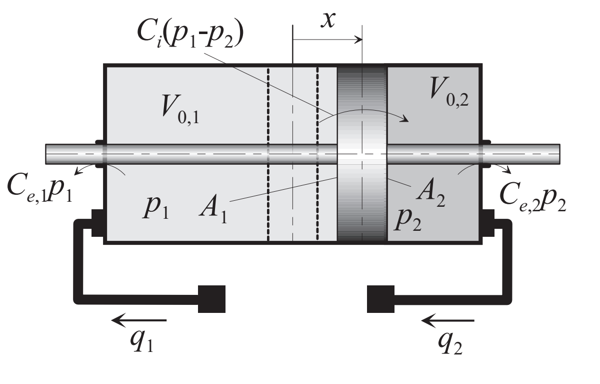

Figure 2: Double-acting double-ended hydraulic ram.

$$\frac{\mathrm{d}}{\mathrm{d}t}p_1 = \frac{E_{oil}\left(q_1 - A_1 v - C_i\left(p_1 - p_2\right) - C_{e,1}p_1\right)}{\left(V_{0,1} + A_1 x\right)}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}p_2 = \frac{E_{oil}\left(A_2 v - q_2 + C_i\left(p_1 - p_2\right) - C_{e,2}p_2\right)}{\left(V_{0,2} - A_2 x\right)} \tag{1}$$

with the displacement of the piston x, the piston velocity $v = \mathrm{d}x/\mathrm{d}t$, the pressures p_1, p_2 in the forward and return chamber and the flow from the valve to the forward chamber q_1 as well as the flow from the return chamber to the valve q_2. Furthermore, E_{oil} is the isothermal bulk modulus of oil, $V_{0,1}$ and $V_{0,2}$ denote the volumes of the forward and return chamber for $x = 0$, A_1 and A_2 are the effective piston areas and C_i, $C_{e,1}$ and $C_{e,2}$ are the leakage coefficients. Suppose that the servo valves are rigidly connected to the hydraulic ram and the supply pressure remains constant during all possible operations. Then the flows from and to the valve q_1 and q_2 can be calculated by

$$\begin{aligned} q_1 &= K_{v,1}\sqrt{p_S - p_1}\,\mathrm{sg}\left(x_v\right) - K_{v,2}\sqrt{p_1 - p_T}\,\mathrm{sg}\left(-x_v\right) \\ q_2 &= K_{v,2}\sqrt{p_2 - p_T}\,\mathrm{sg}\left(x_v\right) - K_{v,1}\sqrt{p_S - p_2}\,\mathrm{sg}\left(-x_v\right) \end{aligned} \tag{2}$$

with the supply and the tank pressure p_S and p_T, the valve displacement x_v, the function $\mathrm{sg}\left(x\right) = x$ for $x > 0$ and $\mathrm{sg}\left(x\right) = 0$ for $x \leq 0$ and the coefficients $K_{v,i} = C_d A_{v,i}\sqrt{2/\rho_{oil}}$, $i = 1, 2$, where $A_{v,i}$ is the orifice area, C_d is the discharge coefficient and ρ_{oil} is the density of oil (see, e.g., [14], [16]). Often the dynamics of the servo valve is much faster than the other components of the hydraulic adjustment system, and therefore, we will neglect the servo valve dynamics and consider the valve displacement x_v as the plant input to the system.

For the purpose of the controller design we may assume that the internal and external leakage flows are negligible compared to the flows from and to the valve. It is worth mentioning that this assumption is no substantial restriction of generality. In fact, the leakage flows themselves are neglected, but depending on the valve configuration (e.g., two valves in a cut-in circuit) they may cause pressures p_1 and p_2 in the forward and return chambers with a considerable offset value p_{off} from symmetrical pressure conditions and this effect is still contained in the model. In order to clarify this statement one can easily convince oneself that the same hydraulic force F_h can be obtained under totally different pressure conditions

$$F_h = A_1 p_1 - A_2 p_2 = A_1\left(p_1 + p_{off}\right) - A_2\left(p_2 + \frac{A_1}{A_2}p_{off}\right) \tag{3}$$

with p_{off} arbitrary, but restricted within certain boundaries. Now, especially this offset pressure has a dominating influence on the dynamic behavior of the hydraulic system, in particular, if the piston is near one of the two edges of the hydraulic ram. With these simplifications the continuity equations (1) can be formulated in terms of the pressure difference and hence the hydraulic force F_h satisfies the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}F_h = \frac{E_{oil}A_1\left(q_1 - A_1 v\right)}{\left(V_{0,1} + A_1 x\right)} - \frac{E_{oil}A_2\left(-q_2 + A_2 v\right)}{\left(V_{0,2} - A_2 x\right)} \tag{4}$$

with q_1 and q_2 from (2).

2.2 Mill Stand Behavior

Generally, the mathematical models of the mill stand used by design engineers of a rolling mill are highly complex and usually they are based on a finite element calculation. The problem is that these models are not useful for the purpose of a controller design. Depending on the application, in literature one can find various simpler dynamic models which are composed of discrete masses, springs and dampers. So, for instance, in [1] the stand model was particularly derived for the identification of the mill stretch coefficient and the deformation resistance and the stand model presented in [12] takes additionally into account the effect of roll eccentricities and the friction between the work and backup roll chocks and the mill housing. The drawback of all these models is the fact that the parameters like the damping coefficients or the spring constants are only known rather inaccurate. A detailed investigation shows that for the thickness control concept it suffices to model the upper rolls, the chocks and the hydraulic piston as a single rigid mass m and the mill stretch effect of the stand frame and the roll stack including bending is considered by means of a measured mill stretch calibration curve $f_{str}(F_h)$. Generally, the mill stretch $f_{str}(F_h)$ is a non-linear function of the hydraulic force F_h but mostly it can be fairly good approximated by an affine function in F_h. Figure 3 shows the scheme of the simple mill stand model

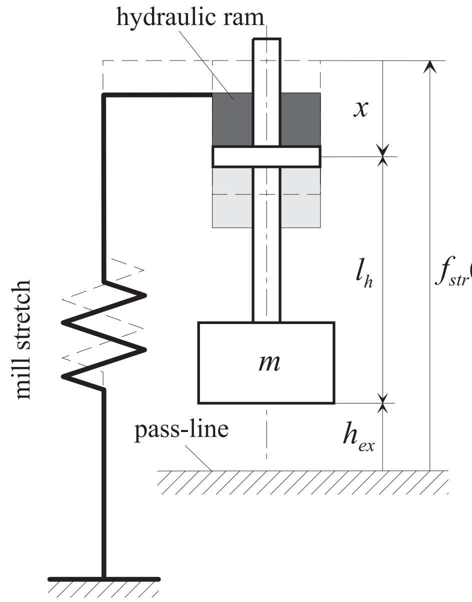

Figure 3: Simple mill-stand model with a schematic representation of the mill stretch.

and the associated equations of motion read as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} h_{ex} &= v_{h,ex} \\ m\frac{\mathrm{d}}{\mathrm{d}t} v_{h,ex} &= -F_h - d v_{h,ex} + F_r - mg + F_b \end{aligned} \tag{5}$$

with the strip exit thickness h_{ex}, the hydraulic force F_h due to (3), the damping coefficient d, the total mass of all moving parts m, the roll force F_r and the force due to the bending and balancing system F_b. Then the displacement of the hydraulic piston x is given by

$$x = h_{ex} + l_h - f_{str}(F_h) \tag{6}$$

with the constant length l_h. It is worth mentioning that for the thickness control concept we assume a uniform behavior of the system along the strip width and we do not distinguish between operator and drive side, although we are aware of the fact that in practice this will essentially influence the guiding of the strip. However, the differences between the two sides in the mill stand behavior as well as in the hydraulic adjustment system can be taken into account by means of an outer control loop, which adjusts the roll swivel.

2.3 Material Deformation Model

The mathematical model for the material deformation considered in this paper is based on the roll force model of Bland, Ford and Ellis for cold rolling without the approximation for the elastic contribution. In the following we just want to summarize the essential results of this deformation model. The interested reader is kindly referred to the original literature [6], [2] and [7]. Figure 4 presents the schematic diagram of the material deformation model and the associated quantities. The roll force model assumes a homogeneous,

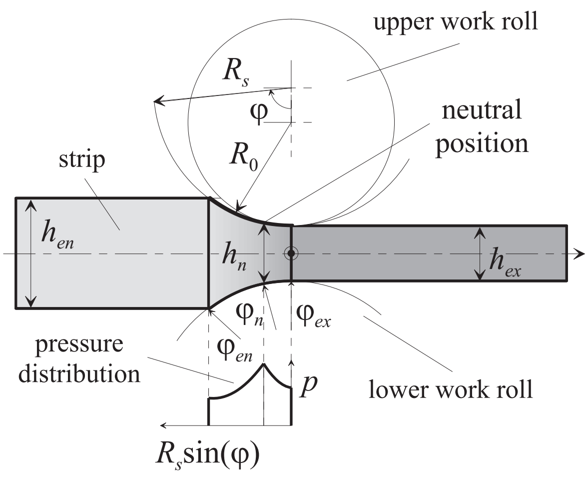

Figure 4: Schematic diagram for the material deformation model.

purely plastic deformation where the plane sections remain plane and Tresca's yield criterion is involved. The roll force F_r can be written as an integral over the pressure distribution p in the roll gap by

$$F_r = R_s b \left(\left(1 - \frac{\sigma_{ex}}{k_f(\varphi_{ex})} \right) \int_0^{\varphi_n} k_f(\varphi) \frac{h(\varphi)}{h_{ex}} e^{\mu \chi(\varphi)} \mathrm{d}\varphi + \right.$$

$$\left(1-\frac{\sigma_{en}}{k_f\left(\varphi_{en}\right)}\right)\int\limits_{\varphi_n}^{\varphi_{en}} k_f(\varphi)\frac{h(\varphi)}{h_{en}}e^{\mu(\chi(\varphi_{en})-\chi(\varphi))}\mathrm{d}\varphi\Bigg) \quad (7)$$

with the abbreviations

$$\begin{aligned}
\chi\left(\varphi\right) &= 2\sqrt{\frac{R_s}{h_{ex}}}\arctan\left(\sqrt{\frac{R_s}{h_{ex}}}\varphi\right) \\
h(\varphi) &= h_{ex}+R_s\varphi^2 \\
\chi_n &= \frac{\chi\left(\varphi_{en}\right)}{2}-\frac{1}{2\mu}\ln\left(\frac{h_{en}}{h_{ex}}\frac{1-\dfrac{\sigma_{ex}}{k_f\left(\varphi_{ex}\right)}}{1-\dfrac{\sigma_{en}}{k_f\left(\varphi_{en}\right)}}\right) \\
\varphi_n &= \sqrt{\frac{h_{ex}}{R_s}}\tan\sqrt{\frac{h_{ex}}{R_s}}\frac{\chi_n}{2}
\end{aligned} \quad (8)$$

and the radius R_s of the deformed but still cylindrical roll as a function of the radius R_0 of the undeformed roll and the roll force F_r

$$R_s = R_0\left(1+\frac{16\left(1-\nu_{roll}^2\right)}{E_{roll}\left(h_{en}-h_{ex}\right)\pi b}F_r\right) \quad (9)$$

due to Hitchcock's formula. Here and subsequently the indices en, ex and n always refer to the corresponding quantity for the entry, exit and neutral zone position, respectively. The symbol h denotes the thickness, σ is the specific strip tension, μ is the friction coefficient, which is supposed to be constant, $k_f\left(\varepsilon\left(\varphi\right)\right)$ denotes the yield stress as a function of the strip reduction $\varepsilon\left(\varphi\right)=\left(h_{en}-h_{ex}+2R_s\cos\left(\varphi\right)\right)/h_{en}$, E_{roll} and ν_{roll} are Young's modulus and Poisson's ratio for the roll material, respectively and b denotes the strip width. Thus, we can formulate the material model as an implicit equation of the form

$$f\left(F_r, h_{en}, h_{ex}, \sigma_{en}, \sigma_{ex}\right)=0\,. \quad (10)$$

2.4 Stand Interconnection Model

The two stands of Fig. 1 are connected via the strip and the interstand tension force F_t is measured by means of a tension measurement device which is located between the two stands. Let us subsequently assume that the deformation process does not change the strip width b and that the density of the strip also remains constant. By neglecting the strip mass, we can fairly good approximate the longitudinal strip behavior by means of a Hookean spring with an effective spring constant c_{eff}. Thus the simple model for the tension force F_t reads as

$$\frac{\mathrm{d}}{\mathrm{d}t}F_t = c_{eff}\left(v_{en}^{(2)}-v_{ex}^{(1)}\right), \quad (11)$$

where $v_{en}^{(2)}$ and $v_{ex}^{(1)}$ denote the entry and exit strip speed of the second and first stand, respectively. Note that if the strip thickness h is constant in between the distance l_{12} of the two stands, i.e. $h_{ex}^{(1)}=h_{en}^{(2)}=h$, then the effective spring constant c_{eff} simplifies to $c_{eff}=E_{strip}hb/l_{12}$ with E_{strip} as the Young's modulus of the strip material. As one can see from (10) the roll force model requires the specific entry and exit strip tension $\sigma_{en}^{(2)}$ and $\sigma_{ex}^{(1)}$ and they are related to the tension force F_t in the form

$$\sigma_{en}^{(2)}=\frac{F_t}{bh_{en}^{(2)}} \quad \text{and} \quad \sigma_{ex}^{(1)}=\frac{F_t}{bh_{ex}^{(1)}}\,. \quad (12)$$

The continuity equation for the strip passing the two stands, together with the assumption that the strip width and the density of the strip material do remain constant along the whole strip length lead to the following relations for the strip speeds

$$v_{ex}^{(1)}=\frac{\omega_{roll}^{(1)}R_0h_n^{(1)}}{h_{ex}^{(1)}}=\frac{v_{en}^{(1)}h_{en}^{(1)}}{h_{ex}^{(1)}} \quad \text{and} \quad v_{en}^{(2)}=\frac{\omega_{roll}^{(2)}R_0h_n^{(2)}}{h_{en}^{(2)}} \quad (13)$$

with $\omega_{roll}^{(i)}$ as the circumference velocity of the work rolls and $h_n^{(i)}=h^{(i)}\left(\varphi_n\right)$ due to (8) as the strip thickness in the neutral position of the i^{th}-stand, $i=1,2$. Of course, here we implicitly assume that the neutral position point, indicated by the index n, exists or in other words, there is always a point in the roll gap where the strip speed and the circumference velocity of the roll are equal. We can immediately see from (13) that for given $h_{en}^{(1)}$ and $h_{ex}^{(1)}$ the circumference velocity of the work rolls of the first stand $\omega_{roll}^{(1)}$ determines the strip entry speed $v_{en}^{(1)}$ by the relation

$$\omega_{roll}^{(1)}=\frac{v_{en}^{(1)}h_{en}^{(1)}}{R_0h_n^{(1)}}\,. \quad (14)$$

Consequently, in our case $\omega_{roll}^{(1)}$ is mainly responsible for the mass flow speed through the rolling mill and this is why we will call the first stand the speed master. In hot rolling mills the last stand serves as the speed master, but this does not change anything in the proposed control concept. Substituting (13) into (11), we end up with

$$\frac{\mathrm{d}}{\mathrm{d}t}F_t = c_{eff}\left(\frac{\omega_{roll}^{(2)}R_0h_n^{(2)}}{h_{en}^{(2)}}-\frac{v_{en}^{(1)}h_{en}^{(1)}}{h_{ex}^{(1)}}\right)\,. \quad (15)$$

3 Controller Design

The *primary objective* of the controller design is to track a predefined thickness trajectory $h_{ex,d}^{(1)}$, $h_{ex,d}^{(2)}$ and a prescribed average specific interstand tension $\sigma_d^{(1)(2)}$ with $\sigma^{(1)(2)}=(\sigma_{ex}^{(1)}+\sigma_{en}^{(2)})/2$. Henceforth, the supplementary index d always indicates the corresponding desired quantity. This paper will not concern the problem of reference trajectory generation because this is mainly performed with a view to technological reasons, e.g., the specific interstand tension strongly influences the final material quality and also the question, how the total strip reduction is shared among the number of mill stands, is answered within the pass schedule program. Thus, we may assume that the desired trajectories already meet all these technological demands and the proposed control concept should ensure to track these reference trajectories as good as possible. But our control approach will only be practically feasible if we can show at least in simulation that the control concept can cope with non-negligible quantization and transducer noise, the sampling process with a predefined sampling time of approximately 3 ms, stick-slip friction effects between the mill stand and the chocks, parameter inaccuracies in the roll force model, in particular the friction coefficient μ and the yield stress k_f and last but not least idealizations of the dynamics of the actuating hydraulic devices and main mill drives. We will henceforth call these requirements the *secondary objectives*.

For each stand the pressures p_1 and p_2 of the hydraulic ram, the piston position x, the forces of the roll bend cylinder F_{be} and the balance cylinder F_{ba} and the interstand tension force F_t are directly available through measurement. Furthermore, we assume that a thickness measurement device is located at a distance l_{en} before the first stand to measure the strip entry thickness $h_{en}^{(1)}$ and in addition the specific entry tension $\sigma_{en}^{(1)}$ as well as the specific exit tension $\sigma_{ex}^{(2)}$ are also supposed to be measurable at least indirectly via the tension force and the strip thickness. The strip entry speed of the first stand $v_{en}^{(1)}$ is either measured directly or can be obtained by means of the circumference velocity of the work rolls due to (14). Hence on the basis of this information we are able to calculate the hydraulic force $F_h = A_1 p_1 - A_2 p_2$, the strip exit thickness $h_{ex} = l_h - f_{str}(F_h) - x$ due to (6) and the specific tensions $\sigma_{en}^{(2)}$ and $\sigma_{ex}^{(1)}$ following (12). The valve displacement x_v (see (2)) for both mill stands and the circumference velocity of the second mill stand $\omega_{roll}^{(2)}$ serve as the plant inputs.

A closer investigation shows that $(h_{ex}^{(1)}, h_{ex}^{(2)}, F_t)$ is a set of flat outputs of the overall system with $(x_v^{(1)}, x_v^{(2)}, \omega_{roll}^{(2)})$ as the plant inputs. However, a controller can be directly obtained by a straight forward application of the theory of flat systems (see, e.g., [5], [20], [3], [15]). Unfortunately, extensive simulation studies show that this controller does not meet the secondary objectives as mentioned above. Therefore, we decided to go an alternative way where we use the full power of the flat systems approach but we do not need any of the measured quantities to be directly differentiated. There is only one part in the control concept where an approximate differentiation is used, but it turns out that this point is rather uncrucial. It is worth mentioning that in [11] also the approach of not explicitly using the velocity of the piston in the non-linear control concept was one of the cornerstones for the successful implementation of the non-linear HGC-concept.

3.1 Servo Compensation (Inner Loop)

In a first step, let us rewrite (4) in the form (see also [13])

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}F_h &= \frac{E_{oil}A_1 q_1}{(V_{0,1}+A_1 x)} + \frac{E_{oil}A_2 q_2}{(V_{0,2}-A_2 x)} - \\ &\quad \frac{\mathrm{d}}{\mathrm{d}t}\left(E_{oil}\ln\frac{(V_{0,1}+A_1 x)^{A_1}}{(V_{0,2}-A_2 x)^{A_2}}\right)\end{aligned} \tag{16}$$

with q_1 and q_2 from (2). Then the servo controller

$$x_v = \left(\frac{A_1 K_{v,1}\sqrt{p_S - p_1}}{V_{0,1}+A_1 x} + \frac{A_2 K_{v,2}\sqrt{p_2 - p_T}}{V_{0,2}-A_2 x}\right)^{-1} u \tag{17}$$

for $x_v > 0$ and

$$x_v = -\left(\frac{A_1 K_{v,2}\sqrt{p_1 - p_T}}{V_{0,1}+A_1 x} + \frac{A_2 K_{v,1}\sqrt{p_S - p_2}}{V_{0,2}-A_2 x}\right)^{-1} u \tag{18}$$

for $x_v < 0$ transforms (16) into

$$\frac{\mathrm{d}}{\mathrm{d}t}z = u \quad \text{with} \quad z = F_h + E_{oil}\ln\frac{(V_{0,1}+A_1 x)^{A_1}}{(V_{0,2}-A_2 x)^{A_2}} \tag{19}$$

and the new plant input u. Clearly, z is nothing else than the hydraulic force due to the pressures p_1 and p_2 extended by the deviation of the force due to the change of the chamber volumes. One can immediately see that z remains constant as long as the flows from and to the valve q_1 and q_2 are zero.

3.2 Flatness Based Control (Outer Loop)

Before starting with the derivation of the flatness based thickness and interstand tension controller, let us summarize the underlying mathematical model for the two stands of Fig. 1 in the form (see (5) and (15))

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}h_{ex}^{(i)} &= v_{h,ex}^{(i)} \quad , \quad i = 1,2 \\ \frac{\mathrm{d}}{\mathrm{d}t}v_{h,ex}^{(i)} &= \frac{-F_h^{(i)} - d^{(i)}v_{h,ex}^{(i)} + F_r^{(i)} - m^{(i)}g + F_b^{(i)}}{m^{(i)}} \\ \frac{\mathrm{d}}{\mathrm{d}t}F_t &= c_{eff}\left(\frac{\omega_{roll}^{(2)} R_0 h_n^{(2)}}{h_{ex}^{(1)}\left(t - T^{(1)(2)}\right)} - \frac{v_{en}^{(1)} h_{en}^{(1)}}{h_{ex}^{(1)}}\right)\end{aligned} \tag{20}$$

with the roll forces due to (10)

$$\begin{aligned}f^{(1)}\left(F_r^{(1)}, h_{en}^{(1)}\left(t - T^{(1)}\right), h_{ex}^{(1)}, \sigma_{en}^{(1)}, \sigma_{ex}^{(1)}\right) &= 0 \\ f^{(2)}\left(F_r^{(2)}, h_{ex}^{(1)}\left(t - T^{(1)(2)}\right), h_{ex}^{(2)}, \sigma_{en}^{(2)}, \sigma_{ex}^{(2)}\right) &= 0\end{aligned} \tag{21}$$

and $\sigma_{ex}^{(1)}, \sigma_{en}^{(2)}$ from (12). The strip exit thickness $h_{ex}^{(i)}$ and the average specific interstand tension $\sigma^{(1)(2)}$ written in terms of the measured quantities are given by (see, (6), (12))

$$h_{ex}^{(i)} = x^{(i)} + f_{str}^{(i)}\left(F_h^{(i)}\right) - l_h^{(i)} \tag{22}$$

and

$$\sigma^{(1)(2)} = \frac{F_t}{2b}\left(\frac{1}{h_{ex}^{(1)}\left(t - T^{(1)(2)}\right)} + \frac{1}{h_{ex}^{(1)}}\right). \tag{23}$$

The location of the strip entry thickness measurement device for $h_{en}^{(1)}$ at a distance l_{en} before the first stand and the distance l_{12} between the two mill stands cause velocity-dependent transport delays $T^{(1)}(t)$ and $T^{(1)(2)}(t)$, which can be calculated by means of the following integral equations

$$l_{en} = \int_{t-T^{(1)}(t)}^{t} v_{en}^{(1)} \mathrm{d}\tau, \, l_{12} = \int_{t-T^{(1)(2)}(t)}^{t} \frac{\omega_{roll}^{(1)} R_0 h_n^{(1)}}{h_{ex}^{(1)}} \mathrm{d}\tau \,. \tag{24}$$

Especially, in the case of speed-up and -down situations the time dependence of the transport delays cannot be neglected. An efficient way for an approximate evaluation of the time-dependent deadtime due to (24) can be found e.g., in [10]. The strip entry speed of the first stand $v_{en}^{(1)}$ and the circumference velocity of the work rolls of the first stand $\omega_{roll}^{(1)}$ are related due to (14) in the form

$$\omega_{roll}^{(1)} = \frac{v_{en}^{(1)} h_{en}^{(1)} \left(t - T^{(1)}\right)}{R_0 h_n^{(1)}} \,. \tag{25}$$

Note that in (20), (21), (23) and (25) it is explicitly indicated by the argument which quantities are delayed by which deadtime.

One can easily convince oneself that the system is flat with the outputs $y^T = [h_{ex}^{(1)}, \, h_{ex}^{(2)}, \, F_t]$, the control inputs $u_1^T = [F_h^{(1)}, \, F_h^{(2)}, \, \omega_{roll}^{(2)}]$ and the exogenous inputs $u_2^T = [F_b^{(1)}, \, F_b^{(2)}, \, h_{en}^{(1)}, \, v_{en}^{(1)}, \, \sigma_{en}^{(1)}, \, \sigma_{ex}^{(2)}]$ which are known sufficiently smooth functions of the time t and whose time evolution is fixed by the system environment and cannot be directly influenced within the considered framework. The thickness controller for tracking a desired thickness trajectory $h_{ex,d}^{(1)}$ and $h_{ex,d}^{(2)}$ includes an integrator in order to suppress stationary thickness errors, which may be caused e.g., by model uncertainties, and it takes the form

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} x_{I,h}^{(i)} &= h_{ex}^{(i)} - h_{ex,d}^{(i)} \\ F_h^{(i)} &= F_r^{(i)} - m^{(i)} g + F_b^{(i)} - m^{(i)} \frac{\mathrm{d}^2}{\mathrm{d}t^2} h_{ex,d}^{(i)} - \\ &\quad d^{(i)} \frac{\mathrm{d}}{\mathrm{d}t} h_{ex,d}^{(i)} - k_1^{(i)} \left(h_{ex}^{(i)} - h_{ex,d}^{(i)} \right) - k_2^{(i)} x_{I,h}^{(i)} \end{aligned} \tag{26}$$

with $F_r^{(i)}$ from (21) and suitable control coefficients $k_1^{(i)}$, $k_2^{(i)} > 0$ to adjust the closed loop dynamics. The associated error system reads as

$$m^{(i)} \frac{\mathrm{d}^3}{\mathrm{d}t^3} x_{I,h}^{(i)} + d^{(i)} \frac{\mathrm{d}^2}{\mathrm{d}t^2} x_{I,h}^{(i)} + k_1^{(i)} \frac{\mathrm{d}}{\mathrm{d}t} x_{I,h}^{(i)} + k_2^{(i)} x_{I,h}^{(i)} = 0 \,. \tag{27}$$

It is worth mentioning that in (26) no time derivative of a measured signal is required. The only situation where in our approach a time derivative would become necessary is if the natural damping of the overall system, described by $d^{(i)}$, is too restrictive for choosing an appropriate error dynamics of the closed loop (27). Analogous to the thickness controller the interstand tension controller is designed in order to track a prescribed trajectory of the average specific interstand tension $\sigma_d^{(1)(2)}$. Again by including an integral part, we get

$$\frac{\mathrm{d}}{\mathrm{d}t} x_{I,F} = F_t - F_{t,d}$$

$$\begin{aligned} \omega_{roll}^{(2)} = \frac{h_{ex}^{(1)} \left(t - T^{(1)(2)}\right)}{R_0 h_n^{(2)}} &\left(\frac{v_{en}^{(1)} h_{en}^{(1)}}{h_{ex}^{(1)}} + \frac{1}{c_{eff}} \frac{\mathrm{d}}{\mathrm{d}t} F_{t,d} - \right. \\ &\left. \frac{k_3}{c_{eff}} \left(F_t - F_{t,d}\right) - k_4 x_{I,F} \right) \end{aligned} \tag{28}$$

with suitable control coefficients k_3, $k_4 > 0$ and the desired tension force $F_{t,d}$ as a function of $\sigma_d^{(1)(2)}$ and $h_{ex,d}^{(1)}$

$$F_{t,d} = 2b\sigma_d^{(1)(2)} \left(\frac{1}{h_{ex,d}^{(1)} \left(t - T_d^{(1)(2)}\right)} + \frac{1}{h_{ex,d}^{(1)}} \right)^{-1} \,. \tag{29}$$

Thus, the closed loop error system takes the form

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} x_{I,F} + k_3 \frac{\mathrm{d}}{\mathrm{d}t} x_{I,F} + k_4 x_{I,F} = 0 \,. \tag{30}$$

The proposed control concept also brings about a decoupling between the thickness and the interstand tension control. This decoupling is indeed very desirable because the reference trajectory generation is always combined with an HMI (human machine interface) for the plant operator. Thus, e.g., if the plant operator decides to change the strip reduction distribution among the mill stands in order to relieve one mill stand concerning the required roll force, then he does not want the interstand tension to be altered. This can be, in practice at least approximately, ensured by a decoupling control approach. Finally, it should be pointed out that in general at a distance l_{ex} behind the last mill stand a thickness measurement device is located. Hence we have the additional information of the final strip exit thickness, in the case of Fig. 1 $h_{ex}^{(2)}$, with a very high accuracy but delayed with a velocity-dependent deadtime $T^{(2)}(t)$ due to the relation

$$l_{ex} = \int_{t-T^{(2)}(t)}^{t} \frac{\omega_{roll}^{(2)} R_0 h_n^{(2)}}{h_{ex}^{(2)}} \mathrm{d}\tau \,. \tag{31}$$

We will not take advantage of this possibility within our control approach but the reader can easily get an idea of how the presented control strategy can be extended, e.g., by means of a classical Smith predictor structure (see, e.g., [21]) or with the δ-flatness concept (see, e.g., [15]).

3.3 Control Concept

In the previous subsection we have designed the control laws for the hydraulic force $F_h^{(i)}$ (see (26)) and for the circumference velocity of the work rolls $\omega_{roll}^{(i)}$ (see (25), (28)). For the main mill drives we assume that they possess an ideal inner loop controller for the circumference velocity. The hydraulic adjustment system with the servo compensation (17) and (18) can be described due to (19) with the new plant input u. Now, with the control law for $F_h^{(i)}$ from (26) we get $z^{(i)}$ from (19) in the form

$$z^{(i)} = F_h^{(i)} + E_{oil} \ln \frac{\left(V_{0,1}^{(i)} + A_1^{(i)} x^{(i)}\right)^{A_1^{(i)}}}{\left(V_{0,2}^{(i)} - A_2^{(i)} x^{(i)}\right)^{A_2^{(i)}}} , \tag{32}$$

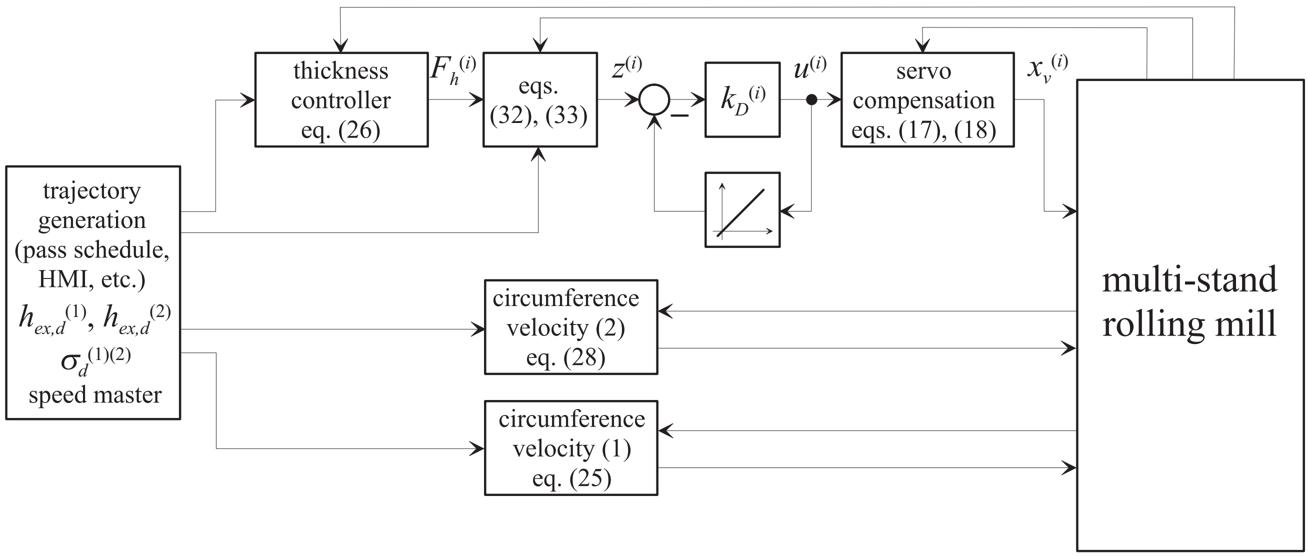

Figure 5: Block diagram of the control concept.

where $x^{(i)}$ is approximated by

$$x^{(i)} = h_{ex,d}^{(i)} + l_h^{(i)} - f_{str}^{(i)}\left(F_h^{(i)}\right). \qquad (33)$$

The plant inputs $u^{(i)}$ of the servo compensation (17) and (18) for the two mill stands $i = 1, 2$ are then obtained by

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} x_D^{(i)} &= -k_D^{(i)} x_D^{(i)} + z^{(i)} \\ u^{(i)} &= k_D^{(i)}\left(-k_D^{(i)} x_D^{(i)} + z^{(i)}\right) \end{aligned} \qquad (34)$$

with a sufficiently large $k_D^{(i)} > 0$. Of course, (34) is nothing else than an approximate differentiation of (32) but various simulation studies under realistic conditions show that nevertheless the secondary objectives determined at the beginning of this section are satisfied. Figure 5 depicts the block diagram of the overall control concept.

4 Simulation Results

In this section, we want to demonstrate the feasibility of the proposed control approach for both, reference tracking and disturbance rejection. The control concept is implemented on a two-stand mill simulator as shown in Fig. 1 with the specific entry strip tension $\sigma_{en}^{(1)} = 100 \cdot 10^6$ Nm^{-2}, the strip entry speed $v_{en}^{(1)} = 7.2$ ms^{-1}, the strip entry thickness $h_{en}^{(1)} = 3 \cdot 10^{-3}$ m and the specific exit strip tension $\sigma_{ex}^{(2)} = 140 \cdot 10^6$ Nm^{-2}. The two mill stands are equipped with a hydraulic adjustment system acting on the upper backup roll and it is assumed that they are built up identically, i.e. they possess the same physical parameters. The hydraulic adjustment systems consist of a single-acting hydraulic rams with an effective piston area $A^{(i)} = 0.66$ m^2, a total chamber volume $V_0^{(i)} = 0.1$ m^3 and a servo spool valve with a rated flow of 75 l/min. The supply pressure $p_S = 320 \cdot 10^5$ Pa and the tank pressure $p_T = 0$ Pa. The total mass m (see Fig. 2) of the moving parts (piston + upper backup roll + upper work roll) has the value $m = 30 \cdot 10^3$ kg and the mill stretch calibration curve $f_{str}(F_h)$ is supposed to be affine with a slope of $c_{str} = 5 \cdot 10^9$ Nm^{-1}. Note that c_{str} is often called the mill stretch coefficient. The distance between the two stands $l_{12} = 5$ m and the entry thickness measurement device is located $l_{en} = 1$ m before the first stand. The Young's modulus of oil $E_{oil} = 1.6 \cdot 10^9$ Nm^{-2} and of the strip $E_{strip} = 2.1 \cdot 10^{11}$ Nm^{-2}, respectively. For the deformation model the friction coefficient is fixed as $\mu = 0.05$ and the average yield stress $\bar{k}_f = 567 \cdot 10^6$ Nm^{-2}, the strip width is given by $b = 1$ m and the undeformed work roll radius reads as $R_0 = 0.25$ m. In order to get realistic simulation results a quantization of the piston position of $5 \cdot 10^{-6}$ m is included in the simulator and the transducer noise for the pressures in the chambers is modeled as a band-limited white noise with a noise power of $5 \cdot 10^5$ (MATLAB/SIMULINK). Furthermore, the controller is implemented with a sampling time of $3 \cdot 10^{-3}$ s.

Figure 6 contains the simulation results of several disturbance and tracking situations which may occur in a rolling mill. The nominal pass schedule is supposed to contain a reduction of the nominal strip entry thickness $h_{en}^{(1)} = 3 \cdot 10^{-3}$ m of 10 % in the first and of 30 % in the second stand. After 1 s the desired average specific interstand tension $\sigma_d^{(1)(2)}$ is changed from $100 \cdot 10^6$ Nm^{-2} to $150 \cdot 10^6$ Nm^{-2}. One second later, a disturbance of $0.2 \cdot 10^{-3}$ m in the strip entry thickness $h_{en}^{(1)}$ is simulated by $h_{en}^{(1)} = (3 + 0.2\xi(t-2)) \cdot 10^{-3}$ m with $\xi(t)$ as the unit step. The situation of a flying gauge change with an additional strip reduction of $0.6 \cdot 10^{-3}$ m in each stand, i.e. an additional total strip reduction of $1.2 \cdot 10^{-3}$ after passing both stands, is performed after 4 s. The desired thickness trajectories $h_{ex,d}^{(1)}$, $h_{ex,d}^{(2)}$ are also depicted in Fig. 6 in the

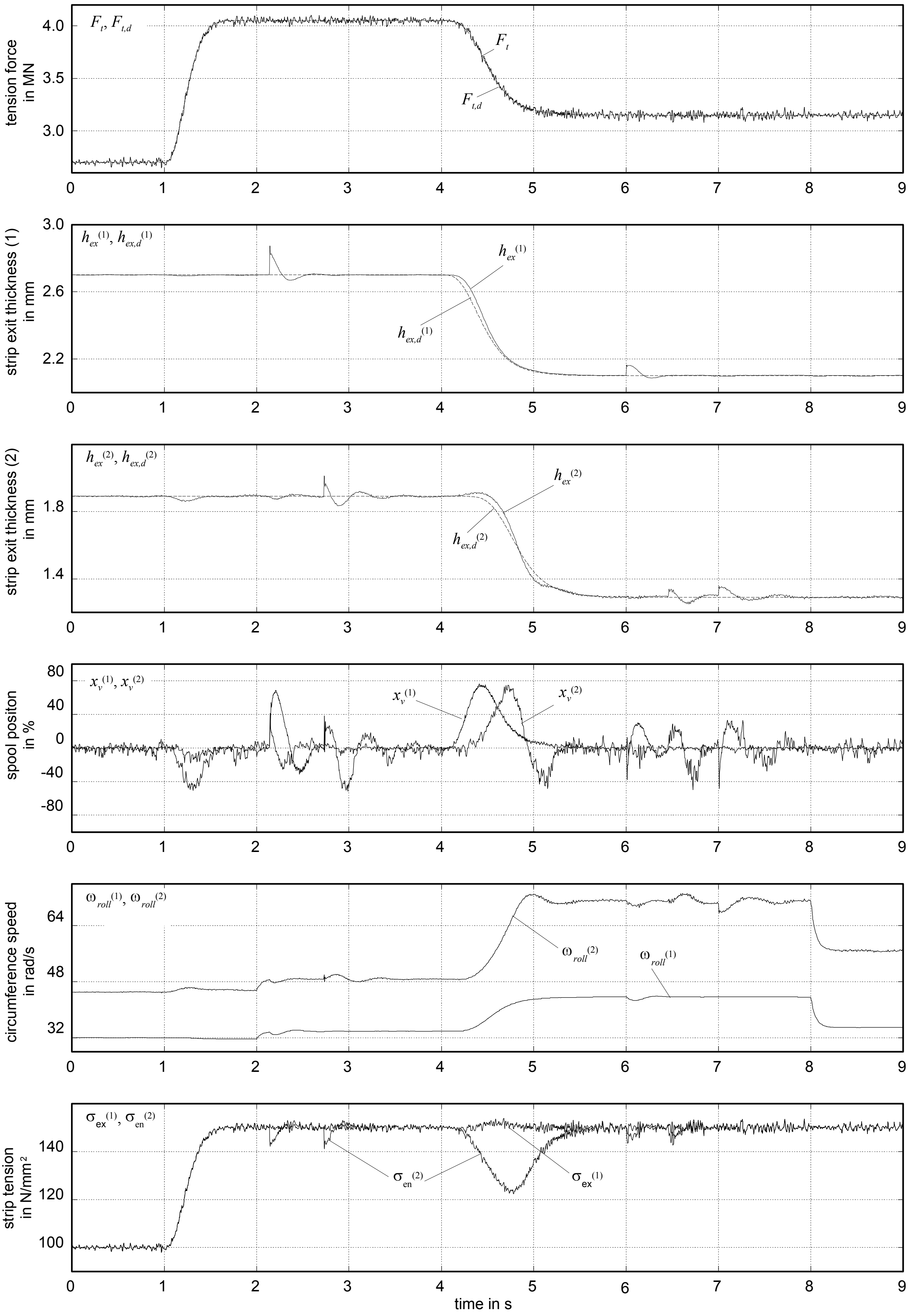

Figure 6: Simulation results for the two-mill stand configuration.

form of dashed lines. After 6 s and 7 s a step disturbance in the roll force of $400 \cdot 10^3$ N occurs in the first and second stand, respectively. These results should demonstrate how the proposed control concept handles changes in the strip material and errors in the material deformation model. Finally, after 8 s the speed master (first stand) changes the strip entry speed $v_{en}^{(1)}$ from 7.2 ms^{-1} to $v_{en}^{(1)} = 5.76$ ms^{-1}.

5 Conclusion

This paper is concerned with a new non-linear strip thickness and interstand tension control concept based on the theory of flat systems for multi-stand rolling mills. The proposed approach was extensively tested on a mill simulator under realistic conditions with all the "dirty" effects known from the rough industrial environment, like measurement quantization, transducer noise, parameter inaccuracies etc.. The results obtained so far are quite promising for a successful practical implementation.

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