TENSOSENSITIVITY OF THE HOT-WIRE PROBE

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The principle of operation of a hot-wire anemometer is based on measurement of changes in the electrical resistance of some thin sensitive element. These changes, however, may be related not just to fluctuations of flow parameters, but also to probe deformations, mechanical vibrations, electromagnetic fields, etc. The false electric signals caused by various external factors may seriously distort measurement results. It is therefore of extreme importance to apply all possible measures to reduce the sensitivity of a hot-wire probe to such factors, vibrations for instance, that affect the probe resistance and bring about false signals. There are quite a number of methods that may be used for transforming vibrations into a reference electrical signal that bears no relation to fluctuations of flow quantities and therefore may be subsequently use to apply corrections for the vibrations. The strain gage effect is probably the most substantial source of false signals. The forces influencing a hot-wire probe were analyzed in [1-3], and estimations of the strains arising in a hot wire were given in these publications.

The contribution due to false electric signals may depend on the probe-overheating factor and on the manner in which the electric power is supplied to the probe, i.e., on the anemometer type. In a constant current anemometer, it is possible to measure and separate out the noise owing to the electric circuit of the anemometer; it is not the case with hot-wire anemometers of other types. To separate out the strain-induced component from the total signal is a very difficult task since this component arises alongside with the useful signal and directly at the hot wire. Therefore, in order to develop a way for diminishing the detrimental component owing to the strain gage effect or have a possibility to apply a proper correction for it, it is necessary to analyze the influence of different flow quantities, hot-wire probe properties and specific features of different types of hot-wire anemometers on probe tensosensitivity.

The equation relating the electric power supplied to a hot-wire probe to the thermal energy removed from it by the flow may be generally written as

$$ei = \pi l \lambda (T_w - T_e)\,\mathrm{Nu}, \tag{1}$$

where e, i – the electric voltage and current across the wire, l – typical size of probe, λ – coefficient of thermal conductivity of gas, T_w – temperature of the wire, T_e – recovery temperature of the wire, Nu – the Nusselt number.

This equation was analyzed in [4] when deriving relations for the probe sensitivity coefficients to flow parameters. To analyze the possibility of origination of a false signal owing to the strain gage effect, we assume the wire length, as well as all flow quantities, to be variable parameters. Unlike [4], we shall consider a general case in which no restrictions are imposed on the type of the anemometer used with the hot-wire probe.

As in [4], we use the following representation of the Nusselt number versus Reynolds number relation:

$$\mathrm{Nu} = (A + B\sqrt{\mathrm{Re}})(1 - ka_w)\,, \tag{2}$$

where A, B, and k are constants to be obtained in some calibration tests, the wire overheating factor is defined as

$$a_w = \frac{R_w - R_e}{R_e}\,, \tag{3}$$

and R_w and R_e are the electrical resistances of heated and cold wire, respectively.

In view of the aforesaid, Eq. 1 for supersonic flow velocities may be re-written as

$$ei = \pi l \lambda_0 (T_w - T_e)(A + B\sqrt{\mathrm{Re}})(1 - ka_w)\,. \tag{4}$$

The variables in this equation are the quantities dependent on the electrical parameters (e, i, T_w, a_w), the quantities determined by flow parameters and the quantities related to them (T_e, λ_0, Re), and also the wire length l. After non-dimensionalization, we obtain the following relation for small incremental changes of these parameters:

$$\frac{\Delta e}{e} + \frac{\Delta i}{i} = \frac{\Delta l}{l} + \frac{\Delta \lambda_0}{\lambda_0} + \frac{\Delta (T_w - T_e)}{T_w - T_e} + \frac{\Delta (A + B\sqrt{\mathrm{Re}})}{A + B\sqrt{\mathrm{Re}}} + \frac{\Delta (1 - ka_w)}{1 - ka_w}\,. \tag{5}$$

Consider the individual terms in this expression. For a power-law relation between the thermal conductivity and the dynamic viscosity in the expression for Reynolds number [4], we have

$$\frac{\Delta \lambda_0}{\lambda_0} = \frac{\Delta \mu_0}{\mu_0} = \omega \frac{\Delta T_0}{T_0}\,. \tag{6}$$

It follows from Eq. 3 that

$$\frac{\Delta a_w}{a_w} = \frac{a_w + 1}{a_w}\left(\frac{\Delta R_w}{R_w} - \frac{\Delta R_e}{R_e}\right). \tag{7}$$

For the Reynolds number with allowance for Eq. 6 we obtain

$$\frac{\Delta \mathrm{Re}}{\mathrm{Re}} = \frac{\Delta m}{m} - \omega \frac{\Delta T_0}{T_0} - \varepsilon \frac{\Delta l}{l}\,, \tag{8}$$

where ε is the Poisson's ratio,

$$\varepsilon = -\frac{\Delta d}{d} / \frac{\Delta l}{l}\,, \tag{9}$$

and d is the wire diameter.

The relation between the incremental changes in T_w and R_w may be found from the linear relation

$$R = R_*[1 + \alpha_*(T - T_*)]\,,$$

which gives

$$\Delta T_w = \frac{R_w}{R_* \alpha_*}\left(\frac{\Delta R_w}{R_w} - \frac{\Delta R_*}{R_*}\right), \tag{10}$$

$$\frac{\Delta (T_w - T_e)}{T_w - T_e} = \frac{a_w + 1}{a_w}\frac{\Delta R_w}{R_w} - \frac{R_* \alpha_* \eta T_0}{a_w R_e}\frac{\Delta T_0}{T_0} - \frac{a_w + 1}{a_w}\frac{\Delta R_*}{R_*}\,. \tag{11}$$

In view of Eq. 8, we have

$$\frac{\Delta (A + B\sqrt{\mathrm{Re}})}{A + B\sqrt{\mathrm{Re}}} = \frac{1}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}}\frac{\Delta \mathrm{Re}}{\mathrm{Re}} = \frac{1}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}}\left(\frac{\Delta m}{m} - \omega \frac{\Delta T_0}{T_0} - \varepsilon \frac{\Delta l}{l}\right). \tag{12}$$

The last term in Eq. 5 with allowance for Eq. 7 may be written as

$$\frac{\Delta (1 - ka_w)}{1 - ka_w} = -\frac{ka_w}{1 - ka_w}\frac{\Delta a_w}{a_w} = -\frac{k(a_w + 1)}{1 - ka_w}\left(\frac{\Delta R_w}{R_w} - \frac{\Delta R_e}{R_e}\right). \tag{13}$$

The relation between a change in the wire length and the related change in the electrical resistance of the wire owing to the strain gage effect has the form

$$\frac{\Delta R_*}{R_*} = \left(1 + 2\varepsilon\right)\frac{\Delta l}{l}, \tag{14}$$

from which for the change in the cold-state resistance we have

$$\frac{\Delta R_e}{R_e} = \frac{\Delta R_*}{R_*} + \frac{R_* \alpha_* \eta T_0}{R_e}\frac{\Delta T_0}{T_0} = \left(1 + 2\varepsilon\right)\frac{\Delta l}{l} + \frac{R_* \alpha_* \eta T_0}{R_e}\frac{\Delta T_0}{T_0}. \tag{15}$$

Substituting expressions Eqs. 6-15 into equation Eq. 5, we obtain the following relation between the changes in the electrical characteristics of the wire and mass flow pulsations, stagnation temperature fluctuations, and changes in the wire length:

$$\begin{aligned} \frac{\Delta e}{e} + \frac{\Delta i}{i} &= \frac{1}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}}\frac{\Delta m}{m} + \left(\frac{a_w + 1}{a_w} - \frac{k(a_w + 1)}{1 - k a_w}\right)\frac{\Delta R_w}{R_w} + \\ &+ \left(\omega - \frac{1}{a_w}\frac{R_* \alpha_* \eta T_0}{R_e} - \frac{\omega}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}} + \frac{k(a_w + 1)}{1 - k a_w}\frac{R_* \alpha_* \eta T_0}{R_e}\right)\frac{\Delta T_0}{T_0} + \\ &+ \left(1 - \frac{\varepsilon}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}} - \frac{a_w + 1}{a_w}\frac{1 - 2k a_w}{1 - k a_w}\left(1 + 2\varepsilon\right)\right)\frac{\Delta l}{l}; \end{aligned} \tag{16}$$

From the Ohm's law equation, we have

$$\frac{\Delta e}{e} = \frac{\Delta i}{i} + \frac{\Delta R_w}{R_w}, \tag{17}$$

which relation, together with Eq. 16, form the system that describes the relation between all variable quantities.

Up to now, no assumptions were made concerning the hot-wire anemometer type; hence, the system applies to all types of anemometers.

For finding sensitivity coefficients for a particular hot-wire anemometer, some quantities should be treated as constants.

1. For the constant temperature anemometer we have $\Delta R_w = 0$, and the above system yields

$$\begin{aligned} 2\frac{\Delta e}{e} &= \frac{1}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}}\frac{\Delta m}{m} + \left(1 - \frac{\varepsilon}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}} - \frac{a_w + 1}{a_w}\frac{1 - 2k a_w}{1 - k a_w}\left(1 + 2\varepsilon\right)\right)\frac{\Delta l}{l} + \\ &+ \left(\omega - \frac{\omega}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}} - \frac{1 - k a_w(a_w + 2)}{a_w(1 - k a_w)}\frac{R_* \alpha_* \eta T_0}{R_e}\right)\frac{\Delta T_0}{T_0} \end{aligned}, \tag{18}$$

or

$$\frac{\Delta e}{e} = F_{CTA}\frac{\Delta m}{m} - G_{CTA}\frac{\Delta T_0}{T_0} - H_{CTA}\frac{\Delta l}{l}, \tag{19}$$

where *F, G,* and *H* are the sensitivity coefficients with respect to the mass flux, stagnation temperature, and strain gage effect, respectively:

$$F_{CTA} = \frac{B\sqrt{\mathrm{Re}}}{4(A + B\sqrt{\mathrm{Re}})}, \tag{20}$$

$$G_{CTA} = \frac{\alpha_* R_*}{R_e} \frac{\eta T_0 [1 - ka_w(a_w + 2)]}{2a_w(1 - ka_w)} - \frac{\omega}{2}(1 - 2F_{CTA}), \tag{21}$$

$$H_{CTA} = \frac{a_w + 1}{2a_w} \frac{1 - 2ka_w}{1 - ka_w}(1 + 2\varepsilon) - \frac{1}{2}(1 - 2\varepsilon F_{CTA}). \tag{22}$$

2. For the constant current anemometer ($\Delta i = 0$), from the system of equations Eqs. 16 - 17 we obtain the following expression:

$$\frac{\Delta e}{e} - \left(1 + \frac{1 - ka_w(a_w + 2)}{a_w(1 - ka_w)}\right)\frac{\Delta e}{e} = \frac{1}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}}\frac{\Delta m}{m} +$$
$$+\left(1 - \frac{\varepsilon}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}} - \frac{a_w + 1}{a_w}\frac{1 - 2ka_w}{1 - ka_w}(1 + 2\varepsilon)\right)\frac{\Delta l}{l} + \tag{23}$$
$$+\left(\omega - \frac{\omega}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}} - \frac{1 - ka_w(a_w + 2)}{a_w(1 - ka_w)}\frac{R_*\alpha_*\eta T_0}{R_e}\right)\frac{\Delta T_0}{T_0};$$

or

$$\frac{\Delta e}{e} = F_{\mathrm{CCA}}\frac{\Delta m}{m} - G_{\mathrm{CCA}}\frac{\Delta T_0}{T_0} + H_{\mathrm{CCA}}\frac{\Delta l}{l}, \tag{24}$$

where

$$F_{CCA} = \frac{a_w(1 - ka_w)}{1 - ka_w(a_w + 2)}\frac{B\sqrt{\mathrm{Re}}}{2(A + B\sqrt{\mathrm{Re}})}, \tag{25}$$

$$G_{CCA} = \frac{\alpha_* R_* \eta T_0}{R_e} - \frac{\omega a_w(1 - ka_w)}{1 - ka_w(a_w + 2)}(1 - 2F_{CTA}), \tag{26}$$

$$H_{CCA} = \frac{(1 + a_w)(1 - 2ka_w)}{1 - ka_w(a_w + 2)}(1 + 2\varepsilon) - \frac{a_w(1 - ka_w)}{1 - ka_w(a_w + 2)}(1 - 2\varepsilon F_{CTA}). \tag{27}$$

3. In the constant voltage anemometer, the pulsations of flow parameters are judged from a voltage drop across a certain standard resistor, the electric current across this resistor being proportional to the electric current across the hot wire. Since the standard resistance here is constant, the voltage pulsations across the resistor are due to electric-current pulsations. Therefore, in this case the relation between the incremental changes of the electric current and those of flow quantities should be sought for. For constant voltage we assume $\Delta i = 0$, and then obtain from the above system

$$\frac{\Delta i}{i} + \left(1 + \frac{1 - ka_w(a_w + 2)}{a_w(1 - ka_w)}\right)\frac{\Delta i}{i} = \frac{1}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}}\frac{\Delta m}{m} +$$
$$+\left(1 - \frac{\varepsilon}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}} - \frac{a_w + 1}{a_w}\frac{1 - 2ka_w}{1 - ka_w}(1 + 2\varepsilon)\right)\frac{\Delta l}{l} + \tag{28}$$
$$+\left(\omega - \frac{\omega}{2}\frac{B\sqrt{\mathrm{Re}}}{A + B\sqrt{\mathrm{Re}}} - \frac{1 - ka_w(a_w + 2)}{a_w(1 - ka_w)}\frac{R_*\alpha_*\eta T_0}{R_e}\right)\frac{\Delta T_0}{T_0};$$

or

$$\frac{\Delta i}{i} = F_{CVA}\frac{\Delta m}{m} - G_{CVA}\frac{\Delta T_0}{T_0} - H_{CVA}\frac{\Delta l}{l}, \tag{29}$$

where

$$F_{CVA} = \frac{a_w(1-ka_w)}{1+2a_w-2ka_w-3ka_w{}^2}\frac{B\sqrt{\mathrm{Re}}}{2(A+B\sqrt{\mathrm{Re}})}, \tag{30}$$

$$G_{CVA} = \frac{1-ka_w(a_w+2)}{1+2a_w-2ka_w-3ka_w{}^2}\frac{\alpha_* R_* \eta T_0}{R_e} - \frac{\omega a_w(1-ka_w)}{1+2a_w-2ka_w-3ka_w{}^2}\left(1-2F_{CTA}\right), \tag{31}$$

$$H_{CVA} = \frac{(1+a_w)(1-2ka_w)}{1+2a_w-2ka_w-3ka_w{}^2}\left(1+2\varepsilon\right) - \frac{a_w(1-ka_w)}{1+2a_w-2ka_w-3ka_w{}^2}\left(1-2\varepsilon F_{CTA}\right), \tag{32}$$

Comparing formula Eqs. 20 - 22, Eqs. 25 - 27 and Eqs. 30 - 32, we find that

$$F_{CCA} = \varphi F_{CTA} \qquad G_{CCA} = \varphi G_{CTA} \qquad H_{CCA} = \varphi H_{CTA} \tag{34}$$

$$F_{CVA} = \psi F_{CTA} \qquad G_{CVA} = \psi G_{CTA} \qquad H_{CVA} = \psi H_{CTA} \tag{35}$$

where φ and ψ stand for combinations that appear in formulas Eqs. 25-27 and Eqs. 30 - 32:

$$\varphi = \frac{2a_w(1-ka_w)}{1-ka_w(a_w+2)}, \tag{36}$$

$$\psi = \frac{2a_w(1-ka_w)}{1+2a_w-2ka_w-3ka_w{}^2}. \tag{37}$$

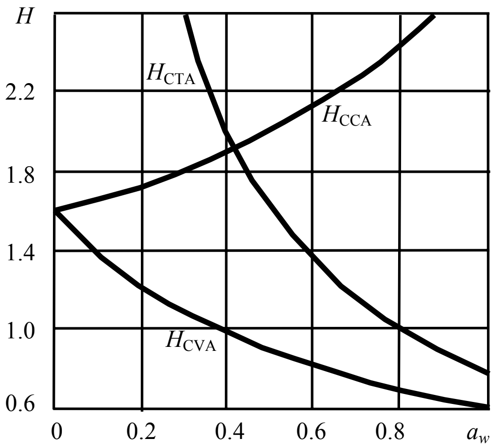

Fig. 1. Tensosensitivity coefficients as functions of the overheating factor of the wire

Figure 1 shows the tensosensitivity coefficients H_{CTA}, H_{CCA} and H_{CVA} as functions of the overheating factor of the wire. It is seen from the figure that the constant voltage anemometer is the least sensitive one to the parasitic strain gage effect for all overheating factors. In the range of small overheating factors, it is the constant temperature anemometer that exhibits a highest tensosensitivity, whereas at high overheating factors the constant current anemometer is most tensosensitive. As is seen from equations Eqs. 33-34, similar conclusions may be also drawn for the coefficients of device sensitivity to mass-flow rate and stagnation temperature fluctuations. In these cases, the relations between the sensitivity coefficients for any two types of hot-wire anemometers remain the same as above.

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