# Part IV: State and Parameter Simultaneous Identification in Induction Motor Field Oriented Drives

## IV.2. General Solutions for State and Parameter Simultaneous Identification

This part deals with simultaneous identification of rotor flux and some of the parameters in flux equations. In this way an *adaptive observer* is obtained, and, if the parameters are also used for changing controller coefficients, we have an *adaptive control*. The solutions of state and parameter simultaneous identification problem are:

- solutions based on an *adaptation mechanism*. This compares the outputs of two models, outputs that are identical if the estimated parameters have right values. One of the models does not contain the estimated parameters and therefore it is called a *reference model*. The other, containing them is an *adaptive (adjustable) model*, because it is continuously corrected, according to the estimated parameters. These solutions are also called *model reference adaptive systems*. For our situations the two models can be rotor flux (or other quantities) observers, or the reference model is the motor itself and then the adaptive model is a stator current observer;
- solution that considers the parameters to be identified as states, with two components: one that is constant and other stochastic. In this case the same Kalman Filter gives both the system state and some of its parameters, considered, as we mentioned, to be states. It is called the Extended Kalman Filter *(EKF)* method;
- particular solutions, that, different to those presented above, are valid for a certain system and a specific parameter. They use for parameter identification relations deduced directly from system equations.

## IV.4. Solutions Based on an Adaptation Mechanism

The structure of an adaptive system of this type is presented in Fig. IV.1. The inputs in the two models may be not the same, but their outputs must be identical, if parameters are correctly estimated.

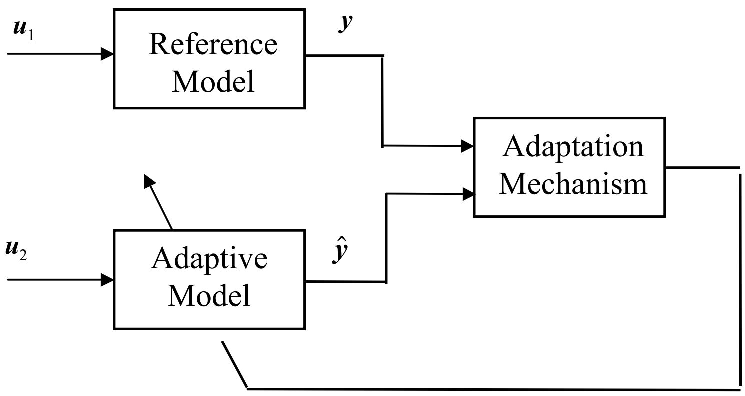


Fig. IV.1. Structure of an adaptive system based on an adaptation mechanism, for state and parameter identification

Both models are linear, thus a state space equation can be written for each of them. Let's suppose that only the $\boldsymbol{A}$ matrix in such an equation contains the parameters to be estimated. This is true for an induction motor, no matter if we want to identify the speed, the rotor resistance, or both of them.

We will consider the situation in which the reference model is the induction motor, while the adaptive one is a linear state observer (Luenberger). This is, as we will se, more general than all the others. The models are:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu} \\ \boldsymbol{y} = \boldsymbol{Cx} \end{cases} \qquad \begin{cases} \dot{\hat{\boldsymbol{x}}} = \tilde{\boldsymbol{A}}\hat{\boldsymbol{x}} + \boldsymbol{Bu} + \boldsymbol{L}(\hat{\boldsymbol{y}} - \boldsymbol{y}) \\ \hat{\boldsymbol{y}} = \boldsymbol{C}\hat{\boldsymbol{x}} \end{cases}$$

Matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are given by (I.1.), while the observer gain matrix, $\boldsymbol{L}$ is obtained as explained in II.1. For the adaptive model we denoted $\tilde{\boldsymbol{A}}$ instead of $\boldsymbol{A}$ because it contains the estimated parameters. This is the most general situation of all the adaptive schemes based on an adaptive mechanism, because of the observer structure, which is the most complex.
Denoting by $\boldsymbol{e}_x = \boldsymbol{x} - \hat{\boldsymbol{x}}$ the error between real and estimated state, we obtain the error dynamic equation:

$$\dot{\boldsymbol{e}}_x = \boldsymbol{Ax} - \tilde{\boldsymbol{A}}\hat{\boldsymbol{x}} + \boldsymbol{L}(\boldsymbol{Cx} - \boldsymbol{C}\hat{\boldsymbol{x}}) \qquad \dot{\boldsymbol{e}}_x = (\boldsymbol{A} + \boldsymbol{LC})\boldsymbol{e}_x + (\boldsymbol{A} - \tilde{\boldsymbol{A}})\hat{\boldsymbol{x}}$$

It describes a linear system ($\boldsymbol{A}$+$\boldsymbol{LC}$, $\boldsymbol{I}$, $\boldsymbol{C}$) connected by feedback with a nonlinear one, given by a function $\phi$ ($\boldsymbol{e}_y$). This nonlinear system has the output error $\boldsymbol{e}_y$ as input and gives $(\boldsymbol{A} - \tilde{\boldsymbol{A}})\hat{\boldsymbol{x}}$ at output. Considering the feedback to be a negative one, we have the situation shown in Fig. IV.2, where $\boldsymbol{\rho}$ denotes $-(\boldsymbol{A} - \tilde{\boldsymbol{A}})\hat{\boldsymbol{x}}$. This configuration is frequent in nonlinear system analyzes and control, being the configuration of Lur' e problem and of some of the problems considered by Popov [20]. Usually it is examined the situation of a single input, single output nonlinear system.

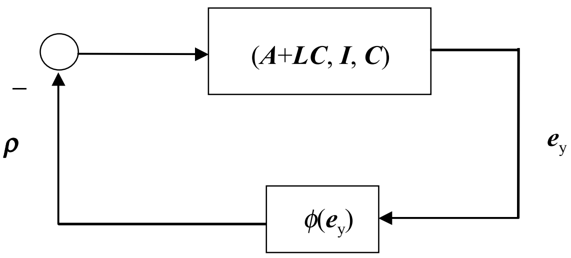


Fig. IV.2. Structure of the system describing the error dynamic equation for an adaptive system based on an adaptation mechanism

Considering, according to Popov terminology [20] the *nonlinear block* described by $\phi(\boldsymbol{e}_y)$, the associated input-output *integral index* is:

$$\eta(t_0, t_1) = Re\int_{t_0}^{t_1} \boldsymbol{e}_y^T(t)\boldsymbol{\rho}(t)dt\ ; \qquad \text{where: } \boldsymbol{e}_y^T \overset{not}{=} \begin{bmatrix} \boldsymbol{e}_y^T & 0 & 0 \end{bmatrix}$$

A necessary condition for the *hyperstabily* of this block is:

$$\eta(0, t_1) = \int_0^{t_1} \boldsymbol{e}_y^T(t)\boldsymbol{\rho}(t)dt \geq -\gamma^2(0)$$

for any input-output combination and a given positive constant $\gamma(0)$.
That means:

$$\eta(0,t_1) = -\int_0^{t_1} \boldsymbol{e}_y^T(\boldsymbol{A}-\widetilde{\boldsymbol{A}})\hat{\boldsymbol{x}}dt \geq -\gamma^2(0)$$

If the error $\boldsymbol{A}-\widetilde{\boldsymbol{A}}$ is given by only one parameter, we can write:

$$\boldsymbol{A}-\widetilde{\boldsymbol{A}} = (p-\widetilde{p})\boldsymbol{A}_{er}$$

with $p$ denoting that parameter (speed or rotor resistance), and $\boldsymbol{A}_{er}$ a constant matrix, depending on the position of $p$ among the coefficients of $\boldsymbol{A}$.

For any derivable function $f$ (we will consider it positive) it's easy to prove the inequality:

$$K\int_0^{t_1} \frac{df}{dt} f\,dt \geq -\frac{K}{2} f^2(0)$$

By writing:

$$\eta(0,t_1) = -\int_0^{t_1} \boldsymbol{e}_y^T \boldsymbol{A}_{er}\hat{\boldsymbol{x}}(p-\widetilde{p})dt$$

it can be seen that we will have a similar inequality for the considered integral index, if:

$$p-\widetilde{p} = f\,; \qquad -\boldsymbol{e}_y^T \boldsymbol{A}_{er}\hat{\boldsymbol{x}} = K\frac{df}{dt}$$

That means:

$$-K\boldsymbol{e}_y^T \boldsymbol{A}_{er}\hat{\boldsymbol{x}} = \frac{d}{dt}(p-\widetilde{p})$$

as $K$ is an arbitrary constant. If the variation of parameter $p$ is much more slower than that of the adaptive law, we obtain:

$$\hat{p} = \widetilde{p} = K\int \boldsymbol{e}_y^T \boldsymbol{A}_{er}\hat{\boldsymbol{x}}dt \tag{IV.1.}$$

This is the general relation for the adaptation mechanism in any model reference adaptive system.

When more than one parameter is to be identified, relation (IV.1.) is easily generalized. For example, in the case of two parameters, we have:

$$\boldsymbol{A}-\widetilde{\boldsymbol{A}} = (p_1-\widetilde{p}_1)\boldsymbol{A}_{er1} + (p_2-\widetilde{p}_2)\boldsymbol{A}_{er2}$$

and then two independent laws similar to IV.1. are obtained, one for each parameter.

The stability of the resulted adaptive observer can be proved either by Popov hyperstability theory or, by Liapunov theorem [8], [18]. We write the output error dynamic equation when only one parameter is estimated:

$$\dot{\boldsymbol{e}}_y = (\boldsymbol{A}+\boldsymbol{LC})\boldsymbol{e}_y + \boldsymbol{C}(\boldsymbol{A}-\widetilde{\boldsymbol{A}})\hat{\boldsymbol{x}}\,; \qquad \dot{\boldsymbol{e}}_y = (\boldsymbol{A}+\boldsymbol{LC})\boldsymbol{e}_y + (p-\widetilde{p})\boldsymbol{C}\boldsymbol{A}_{er}\hat{\boldsymbol{x}}$$

and we introduce the Liapunov function:

$$V = \boldsymbol{e}_y^T \boldsymbol{e}_y + c\,(p - \tilde{p})^2$$

with $c$ a positive constant. As one can prove, it verifies all the conditions of Liapunov functions. Its derivative is:

$$\dot{V} = \dot{\boldsymbol{e}}_y^T \boldsymbol{e}_y + \boldsymbol{e}_y^T \dot{\boldsymbol{e}}_y + 2c\,(p - \tilde{p})\frac{d}{dt}(p - \tilde{p}) = \dot{\boldsymbol{e}}_y^T \boldsymbol{e}_y + \boldsymbol{e}_y^T \dot{\boldsymbol{e}}_y + 2c\,(p - \tilde{p})(-K\boldsymbol{e}_y^T \boldsymbol{A}_{er}\hat{\boldsymbol{x}})$$

And, using the output error dynamic equation, it becomes:

$$\dot{V} = \boldsymbol{e}_y^T (\boldsymbol{A} + \boldsymbol{LC})^T \boldsymbol{e}_y + \boldsymbol{e}_y^T (\boldsymbol{A} + \boldsymbol{LC})\boldsymbol{e}_y + (p - \tilde{p})(\boldsymbol{CA}_{er}\hat{\boldsymbol{x}})^T \boldsymbol{e}_y + (p - \tilde{p})\boldsymbol{e}_y^T (\boldsymbol{CA}_{er}\hat{\boldsymbol{x}})$$
$$+ 2c\,(p - \tilde{p})(-K\boldsymbol{e}_y^T \boldsymbol{A}_{er}\hat{\boldsymbol{x}})$$

Choosing $c = \frac{1}{K}$ we obtain:

$$\dot{V} = \boldsymbol{e}_y^T (\boldsymbol{A} + \boldsymbol{LC})^T \boldsymbol{e}_y + \boldsymbol{e}_y^T (\boldsymbol{A} + \boldsymbol{LC})\boldsymbol{e}_y$$

Since all the eigen values of $\boldsymbol{A}+\boldsymbol{LC}$ have the real part strictly negative (because of the stability condition for this observer), the derivative of $V$ will be also strictly negative, for any non-zero $\boldsymbol{e}_y$. This means, that according to Liapunov stability theorem the system described by the output error dynamic equation, and thus the adaptive observer, will be globally asymptotically stable.

When more parameters are identified, the demonstration is similar, with the Liapunov function:

$$V = \boldsymbol{e}_y^T \boldsymbol{e}_y + c_1\,(p_1 - \tilde{p}_1)^2 + \ldots + c_i\,(p_i - \tilde{p}_i)^2$$

Remarks:

- if the system described by the output error dynamic equation is stable, also the one given by the state error dynamic equation will be stable, and this can be easily proved considering the direct, linear relation existent between these two errors;
- the stability of these systems does not imply the fact that the state and the parameters will be correctly estimated. For this to be true a so-called *persistent excitation* condition must be respected [18]. Normally this is respected if only one parameter is identified, but problems may appear for more parameters.

## IV.6. Rotor Flux and Speed Simultaneous Identification. Sensorless Drives

Respecting the most utilized terminology, the solutions are given by:

- methods based on an adaptation mechanism:

  i) ELO - (Extended Luenberger Observer) - the reference model is the induction motor, while the adjustable one is a Luenberger state observer;

  ii) MRAS - (Model Reference Adaptive System) - the reference model is a VI flux observer, while the adjustable one is an Iω flux observer;
- EKF - (Extended Kalman Filter);
- MEQ - methods computing the speed by solving motor equations.

The first two solutions are based on general methods, while the last is particular to this situation.

We will present the structure of each of these sensorless DFOC systems.

A. ELO Method

The adaptation mechanism is obtained from the general one (IV.1.), using the particular expression of each variable, that is:

$$\boldsymbol{e}_y = \begin{bmatrix} i_{qs} - \hat{i}_{qs} \\ i_{ds} - \hat{i}_{ds} \\ 0 \\ 0 \end{bmatrix}; \qquad \hat{\boldsymbol{x}} = \begin{bmatrix} \hat{i}_{qs} \\ \hat{i}_{ds} \\ \hat{\lambda}_{qr} \\ \hat{\lambda}_{dr} \end{bmatrix}; \qquad \boldsymbol{A}_{er} = \frac{1}{\omega_R - \tilde{\omega}_r}\left(\boldsymbol{A} - \tilde{\boldsymbol{A}}\right) = \begin{bmatrix} 0 & 0 & 0 & -\frac{L_m}{L_r L_s \sigma} \\ 0 & 0 & \frac{L_m}{L_r L_s \sigma} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

That means ($K$ >0 is an arbitrary constant):

$$\hat{\omega}_r = \tilde{\omega}_r = K\int \boldsymbol{e}_y^T \boldsymbol{A}_{er}\hat{\boldsymbol{x}}dt = K\int \begin{bmatrix} e_{iqs} & e_{ids} & 0 & 0 \end{bmatrix} \begin{bmatrix} -\frac{L_m}{L_r L_s \sigma}\hat{\lambda}_{dr} \\ \frac{L_m}{L_r L_s \sigma}\hat{\lambda}_{qr} \\ \hat{\lambda}_{dr} \\ -\hat{\lambda}_{qr} \end{bmatrix} dt = K\int \left[-e_{iqs}\hat{\lambda}_{dr} + e_{ids}\hat{\lambda}_{qr}\right]dt \tag{IV.2.}$$

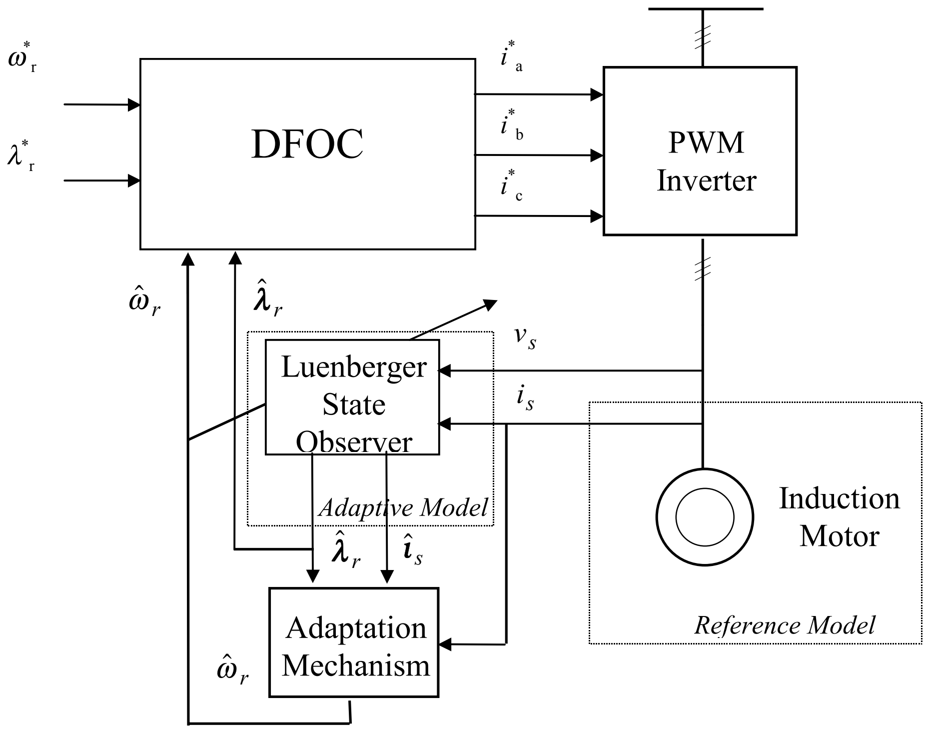


Fig. IV.3. Structure of a sensorless DFOC, based on ELO speed estimation method.

B. MRAS Method

Considering the particular structure of this scheme, we may write:

$$\boldsymbol{e}_y = \begin{bmatrix} \lambda_{qr} - \hat{\lambda}_{qr} \\ \lambda_{dr} - \hat{\lambda}_{dr} \end{bmatrix}; \qquad \hat{\boldsymbol{x}} = \begin{bmatrix} \hat{\lambda}_{qr} \\ \hat{\lambda}_{dr} \end{bmatrix}; \qquad \boldsymbol{A}_{er} = \frac{1}{\omega_R - \tilde{\omega}_r}\left(\boldsymbol{A} - \tilde{\boldsymbol{A}}\right) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

and the general adaptation mechanism becomes:

$$\hat{\omega}_r = \tilde{\omega}_r = K\int \boldsymbol{e}_y^T \boldsymbol{A}_{er}\hat{\boldsymbol{x}}dt = K\int \begin{bmatrix} e_{\lambda qr} & e_{\lambda dr} \end{bmatrix} \begin{bmatrix} \hat{\lambda}_{dr} \\ \hat{\lambda}_{qr} \end{bmatrix} dt = K\int \left[ e_{\lambda qr}\hat{\lambda}_{dr} - e_{\lambda dr}\hat{\lambda}_{qr} \right] dt$$

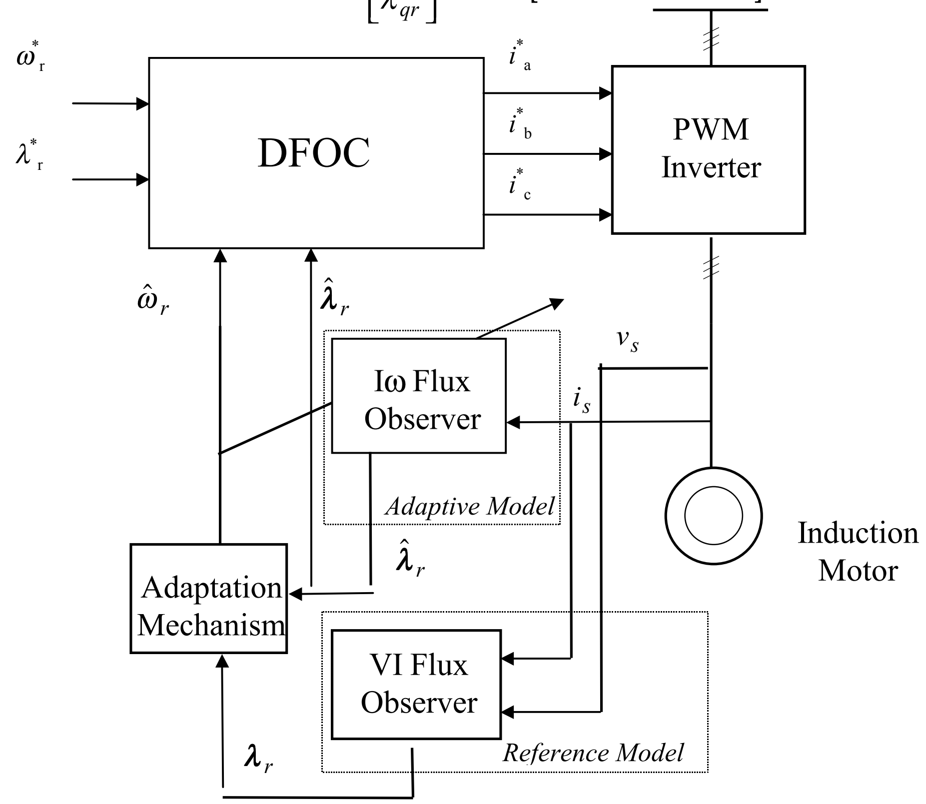


Fig. IV.4. Structure of a sensorless DFOC, based on MRAS speed estimation method.

C. EKF Method

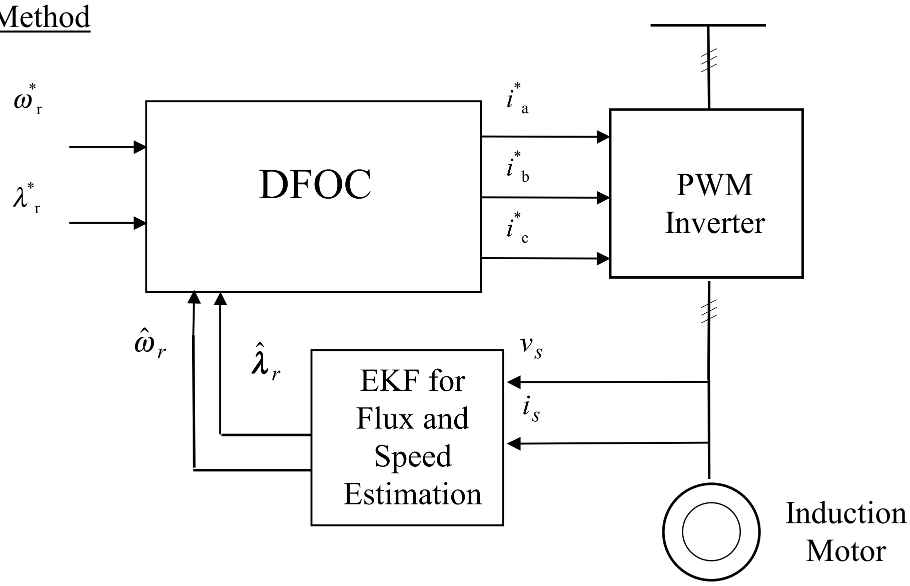


Fig. IV.5. Structure of a sensorless DFOC, based on EKF speed estimation method.

D. MEQ Method

Motor speed may be computed as the difference between the synchronous and the slip speed, both of these being expressed as functions of the estimated flux and stator current:

$$\hat{\omega}_r = \hat{\omega}_e - \hat{\omega}_s \qquad \hat{\omega}_e = \frac{\dot{\lambda}_{qs}\lambda_{ds} - \dot{\lambda}_{ds}\lambda_{qs}}{\lambda_{ds}^2 + \lambda_{qs}^2}; \qquad \hat{\omega}_s = \frac{L_m}{\tau_r}\frac{\lambda_{ds}i_{qs} - \lambda_{qs}i_{ds}}{\lambda_{ds}^2 + \lambda_{qs}^2}$$

The structure of a sensorless DFOC based on this speed estimation method is similar to the previous one, but instead of EKF block there is a flux observer plus the speed computation scheme, based on the above relations.