SIMULATION OF A BUSINESS FIRM

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Abstract

The focus of my PhD research is to build a model of a hypothetical business firm and to examine different growth trajectories using simulation. The model is dynamic and broadly speaking can be regarded as a system dynamics model: the business firm is viewed as a system for generating money and the simulation examines the behaviour of this system over time. The model of the firm is founded on the accounting identities and some additional behavioural equations; the model was not derived using a strict level-rate type analysis. The model uses the accounting identities as these are common to most firms and can be expressed as equations and consequently can be used as a basis for experimentation by means of simulation. The model is parsimonious in that as few equations as possible are used while still reasonably representing a business firm.

I will present the model and the results of the simulation to date. Also I will present insights that this has given me about the behaviour of real business firms. The paper will present ongoing rather than fully completed work and is ideally suited to a PhD stream in the Young OR conference if such exists.

Introduction

Simulation has long been used as a means of carrying out academic and commercial research. Scale models of buildings and mathematical models of traffic as a design aid, aircraft flight simulators as a teaching aid, and what-if type scenario analysis as a decision-making aid are well-known examples of the practical use of simulation in the commercial world (O'Sullivan and Kirwan, 2000; Kaye, 1994). In the academic world simulation has long been a teaching and research tool in the disciplines of operations research, system dynamics, economic, and organisation theory (Sterman, 1992; Homer, 1996; Fowler, 1999; Nelson and Winter, 1982; Cyert and March, 1992).

This research examines the behaviour of the business firm and adopts the view from the field of system dynamics that behaviour of a system depends on its structure. The firm is modeled as a set of equations and, following the practice set down by Naylor (1972:41), the equations are labeled either as accounting identities or behavioural equations. Defining the accounting identities is fairly straightforward as they are derived directly from the balance sheet and profit and loss statement. The behavioural equations are more problematic as some assumptions must be made in order to define the equation. For example, in this model of the firm production is assumed to change positively in proportion to the amount of fixed assets. While this assumption is reasonable it is an assumption nonetheless and raises questions of validity and verification.

Theoretical Foundations

According to the New Oxford Dictionary of English (1998:1188) a model is 'a system or thing used as an example to follow or imitate'; the same dictionary further refines this to: 'a simplified description, especially a mathematical one, of a system or process, to assist calculation and predictions'. Models of course can take many guises: physical, virtual, analogical, conceptual or mathematical. This research is primarily interested in mathematical models.

Simulation is defined by Bratley et al (1987:ix) as the process of 'driving a model of a system with suitable inputs and observing the corresponding outputs'. Other authors (Roberts et al, 1983:3; Ljung and Glad, 1994:15) suggest that the word simulate derives from the Latin word *simulare* meaning to pretend and that simulation therefore means to pretend to be or to imitate something.

These two sets of definitions illustrate the different emphasis of the two words. Definitions of model tend to emphasise noun or content words such as 'system' or 'thing' whereas definitions of simulation emphasise verb or process words such as 'driving' or 'imitate'. This demonstrates how the two concepts link together in the research process. Before undertaking a simulation generally speaking you must already have created some sort of model. Simulation is then the experimental process of using the model to gain some insight into its behaviour and hence into the behaviour of the reality that the model is meant to represent. It is worth noting however that simulation is only one way of using a mathematical model: an analytical approach giving an exact solution is sometimes an alternative for simple linear models. Law and Kelton (1990:4) give a useful taxonomy of models.

Models are particularly useful in the economic sciences as it is generally not possible to experiment with reality itself this being 'too expensive, too dangerous and too time-consuming' (Wood and Fildes, 1976:214). Researchers in this type of field therefore experiment with models of reality - conceptual models or mathematical models. Three different approaches are used when examining the behaviour of mathematical models: qualitative, analytical, and numerical (Blanchard et al, 1996). The qualitative approach evaluates the behaviour of simple systems of differential or difference equations by drawing and examining graphs; while the approach may give powerful insight for a relatively small amount of effort it is limited to examining the behaviour of simple linear models. The analytical approach derives formulae to determine exact solutions for the system of equations but again is limited to simple linear models. When dealing with complex or nonlinear models we must resort to numerical approaches of which simulation is one. However authors advise caution while using simulation as although it is 'a powerful tool, it is neither cheap nor easy to apply correctly and effectively. A simulation modeling effort should not be embarked upon lightly' (Bratley et al, 1987:3).

The model

This research uses the approach outlined above: firstly a mathematical model of the firm was built; this model represents the static or single period behaviour of the business firm and also represents the linking between one period and the next. The behaviour of the model over a number of periods of time was then simulated. This behaviour was then examined and insights gained into why such behaviour occurred. The author then attempted to generalise from this simulated behaviour to the behaviour of real firms.

The equations forming the model have been fully described in Brady (1999). The system of equations may be reduced to two fundamental functions: a supply function and a demand function. Unlike the neoclassical economic model the system does not seek to equate supply and demand and therefore the model does not operate at optimum conditions where marginal revenue equals marginal cost. Instead the model is based on the nonlinear function whereby the amount of goods sold is equal to either goods produced or goods demanded, whichever is the lower. This is a reasonable assumption for a single firm as in real life firms generally cannot match production exactly to demand and either sell all if demand is sufficiently high or else retain goods in inventory if it is not. The neoclassical economic model is an industry level model and assumes price setting and market clearing at an industry rather than at a firm level (Koutsoyiannis, 1979:160). The model on which this research is based is a firm level model and is not concerned with other firms in the industry, market clearing, or price setting.

The two fundamental functions in the model may be represented as:

demand = f (demand)
supply = g (net assets)

The remaining equations follow from these two fundamental functions. For example

revenue = max (supply, demand)
cost = h (supply)
profit = revenue - cost

Several nonlinearities exist in the model the most important of which is the maximisation function; this provides much of the interesting dynamical behaviour in the system being simulated. Other nonlinearities also exist. For example taxation is a nonlinear function of profits before tax - it is equal to either a proportion of profits before taxes if profit is positive or zero in the case of negative profits (losses). Demand is also modeled as a nonlinear function, in this case the logistic or sigmoid curve. This again is reasonable as demand often follows a lifecycle (ie. logistic or sigmoidal) curve. Additional nonlinearities can easily be incorporated into the model. For example, supply could be made a nonlinear function of net assets; cost could be made a nonlinear function of units produced. Again reasonable assumptions can be made to support these nonlinearities: the theory of diminishing returns implies that supply is a nonlinear function of productive assets; the theory of economies of scale implies that cost is a nonlinear function of units produced. For reasons of simplicity, these nonlinearities have not yet been introduced into this model.

The above reduced set of equations also illustrates the two fundamental types of equation in a mathematical model. The equation 'profit = revenue - cost' is clearly an identity based on the accounting profit and loss statement. The supply function is a behavioural equation and is based on an assumption that the level of net assets is the primary driver of production. The behavioural equations are clearly more difficult to define as they require the modeler to make assumptions about the behaviour of the system being modeled. In this model the assumption that revenues are in proportion to net assets is in accordance with financial theory where asset turnover or the ratio of assets to revenue is widely used in the analysis of financial performance. The behavioural equation whereby costs are assumed to be in proportion to units produced is in accordance with accounting theory which traditionally allocates labour and overhead costs to units of output. The demand function is also a behavioural equation and is based on the theory of product life cycles.

The model was simulated using two different approaches. The first approach used standard spreadsheet software. The model of the firm was set up as a series of equations in the cells in a row of the spreadsheet; each period of the simulation requiring a new row. Parameter values were stored in a separate area of the spreadsheet devoted to parameters. Initial conditions were entered directly into the cells of the spreadsheet devoted to period one. Once all values are in place the spreadsheet immediately calculates all values through time. Graphical output uses data in cells as source. The graphs are immediately updated as the value of cells changes. Macros have been built to allow the simulation to take place for a whole range of parameter values. The resulting output can be viewed as the graphs change dynamically.

The second approach used the C^{++} programming language. An identical model was coded up using C^{++} statements. The model used the same set of equations as did the spreadsheet model described above. One additional feature was included in the C++ model: the range of parameter values can be automatically varied incrementally from a start value to an end value and a resulting graphical image of results produced. This would be very difficult if not impossible to accomplish using a spreadsheet.

Results

The results using the spreadsheet software are described in Brady (1999). This paper demonstrated that the behaviour of the system is sensitive to initial conditions: when the parameter value representing the ratio of fixed assets to units produced varied only slightly the behaviour of the system varied greatly. The output of the spreadsheet model was viewed using a number of different graphical devices: time series showing variation of key variables (eg. net assets, profits) over time; phase diagrams plotting values of key variables in time periods t and t+1.

The results using the C^{++} software explored in more detail how changing parameter values caused the behaviour of the system to change. The result of the simulation for logistic growth in demand is shown in figure 1 and for compound growth in demand in figure 2. The shape of these images remains relatively constant as initial conditions or parameter values change. In this sense the images can be regarded as a signature of the system behaviour much as the island diagram is a signature or attractor for the Mandelbrot set (Mandelbrot, 1983) or the bifurcation diagram an attractor for the logistic function (May, 1976) or the butterfly diagram an attractor for the Lorenz equations (Lorenz, 1963; Crutchfield et al, 1986).

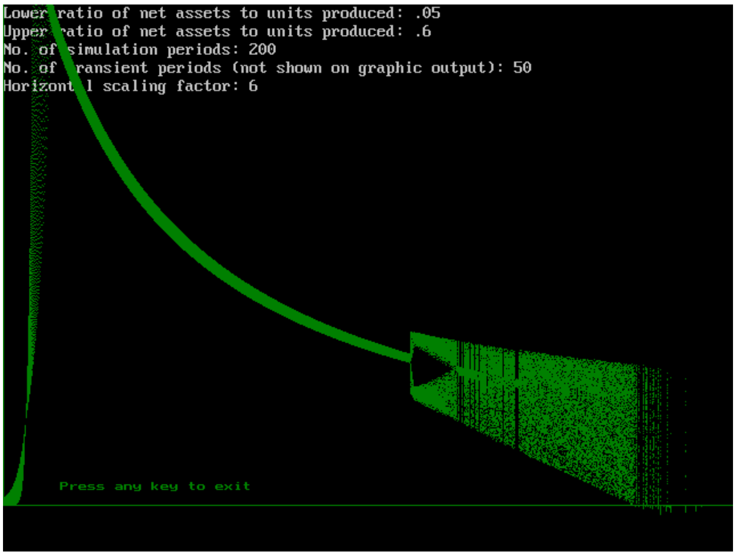

Figure 1: Simulation results (logistic growth in demand)

The two images while clearly different have some elements in common. Firstly, both diagrams show distinct bands of behaviour, although each set of bands is different. Secondly, some of the bands in each set are regular and some irregular (speckled), the irregular bands implying that level of net assets varies over time in some irregular fashion. Thirdly, the broad trend in each diagram is from top left to bottom right. This is somewhat counterintuitive as it implies that net asset levels decrease as the ratio of units produced per unit of net assets increases. One would have intuitively anticipated the opposite: that increasing the ratio of units produced for each unit of net assets would lead to increased levels of net assets.

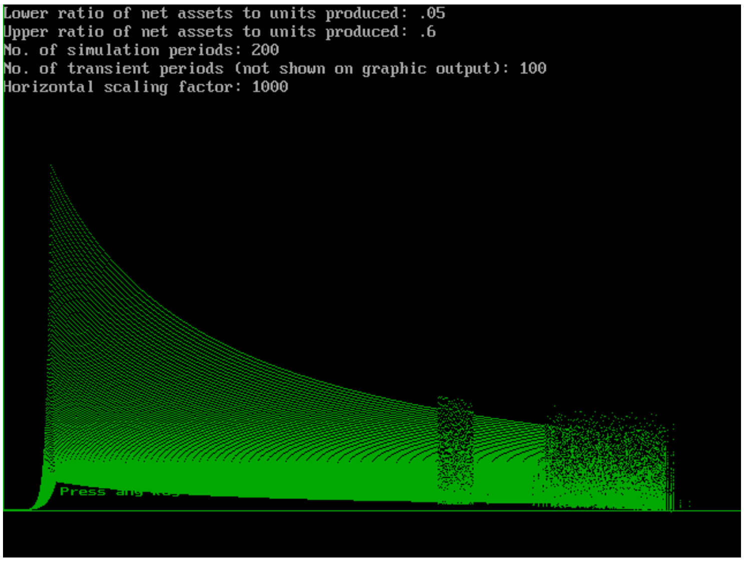

Figure 2: Simulation results (compound growth in demand)

Discussion

The advantage of simulation is clearly evident here. Because of the nonlinearities in the dynamical system it is not possible to 'solve' the set of equations analytically. A feasible way of examining the behaviour of the system is therefore to simulate the set of equations using a computer (hand simulation while theoretically possible would be extremely tedious and time consuming). Also, combining the simulated equations with graphical output gives a very clear picture of the results of the simulation. For example, the bifurcation diagram shown above very clearly identifies the bifurcation point; the researcher is thus led to examine in further detail the conditions that apply at or near the bifurcation point. The researcher can easily accomplish this by specifying the parameters so as to examine a particular area of the graph in more detail. For example, figure 3 shows the area around the bifurcation point in more detail.

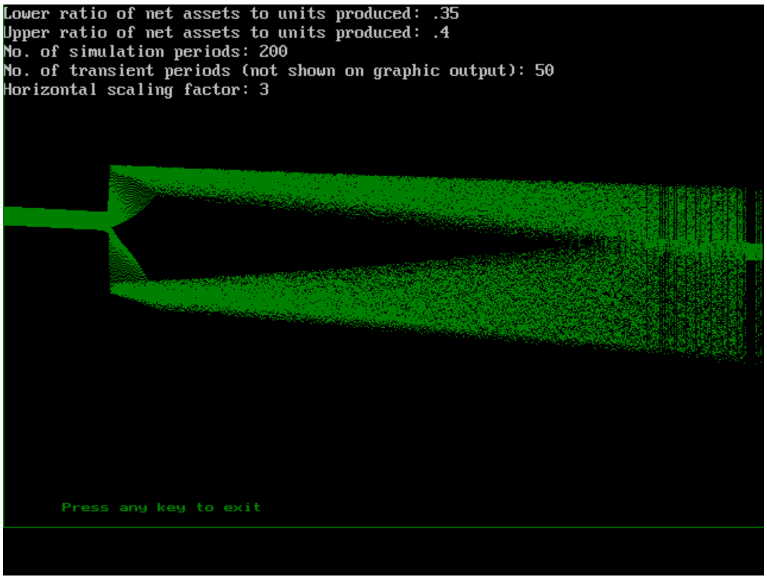

Figure 3: Blown up image of area around bifurcation point

The method is also clearly experimental in that a 'hypothesis' - namely a model of the business firm - is initially formed and the behaviour of this hypothetical firm then simulated and examined. Examination of results may lead the modeler to make further changes to the hypothesis and then to further examine the behaviour of the model.

It is worth spending some time comparing and contrasting the two approaches to simulation used. The spreadsheet approach has many advantages. It is very easy and straightforward to set up: spreadsheet software comes pre-loaded on many personal computers; even if not pre-loaded it is very straightforward to install. Entry of formulae into cells is very straightforward as is entry of parameter and initial value data. The graphical tools that accompany the spreadsheet software allow graphical output to be easily plotted. However some difficulties in simulating using spreadsheets were also evident. Extending simulated time required the formulae for one time period to be copied to a set of new rows. Also, the maximum number of periods that can be simulated is the number of rows that the spreadsheet is capable of handling (in practice however this is not a significant constraint: standard spreadsheets offer 2^{16} or 65536 rows which should be a sufficient number of time periods for most business simulations). In practice the main concern in using spreadsheets is the danger of creating 'spaghetti' code and the unfortunate necessity to create rows or columns containing temporary or intermediate data.

The second approach used in this research - creation of a simulation program using C^{++} - also has its advantages and disadvantages. The main advantage is that the modeler has full control over the specification of the model: exact formulae can be used; temporary or intermediate data is easy to arrange and manage; options - eg. the number of simulation periods - are easy to set at run time; iterations are easy to set up eg. simulation for a whole range of parameter values. There are also some disadvantages: a C^{++} development and production environment must be set up by the modeler - no trivial task; the software to handle all output including graphical output may need to be written by the developer; experience in computer programming is certainly useful and for complicated modeling may be vital; sufficient time must be allowed for specifying the model, writing the code, testing and debugging the software, and carrying out the simulation.

A third approach, not used in this research, is to used customised simulation software. Many such packages are available including several specific to the field of dynamical systems (eg. Dynamo, Stella). It is the intention of this author to use such a package at some future point in time.

It is worth pointing out that in the research process itself both approaches are used in tandem. The more general results from the model written in C^{++} often lead one to look at a specific area of results eg. the bifurcation point mentioned above. The spreadsheet program may then be set with parameter values and initial conditions to replicate this position at a detail level. The various spreadsheet outputs can then be examined in detail to determine underlying causes for the behaviour. For example, detailed examination showed that the bifurcation point in the case of logistic growth in demand occurred exactly when production so exceeded demand that losses were incurred by the hypothetical firm being simulated.

Future Research

The current model assumes that all profits are reinvested as productive assets in the firm. In reality firms do not do this; for example, Microsoft retains much of its profits as cash which it can then use for other purposes such as acquiring other companies or simply as an emergency reserve. Retaining a proportion of profits as cash rather than as new productive assets would significantly restrain the growth of the company, putting back the day when the firm will exceed its demand, and thus putting off the more complicated dynamic behaviour. This factor however could be incorporated relatively easily into the model.

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