Designing Free Software for Marketing: A Game Theoretic Approach

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Abstract

We develop a vertical differentiation game-theoretic model that addresses the issue of designing free software samples for attaining follow-on sales. When software samples are akin to durable goods, a Monopolist giving a free sample away is likely to engender the cannibalization of sales of its commercial product. We analyze the optimal design of free software according to two characteristics: the trial time allotted for sampling (potentially renewable) and the proportion of features included in the sample. We find that these two dimensions play different roles whenever the software product is innovative or standard. We draw implications regarding the effectiveness of marketing strategies depending on the type of software product offered by a Monopolist.

Keywords: Vertical Differentiation, Monopolist, Free sample, Software, Durable goods, Sales Cannibalization, Optimal Design.

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I. Introduction

There are several reasons why firms give away free samples of their products. Some firms offer free samples to increase customers' costs of switching to alternative products. Other firms are attempting to leverage possible network effects. Many companies may aim at selling upgrades and/or complementary products. However, going back to the roots of traditional marketing, one of the primary purposes of free samples is to enhance sales, by providing first-hand experience to users. When this experience is positive it usually results in increased sales.

In this paper, we consider the case of a company that gives free software samples in order to build product awareness and to attain follow-on sales. We consider a class of software products such as shareware, or computer games, mostly B-to-C products. A unique attribute of such products is that the value of the *intrinsic* features of the product becomes more evident via the sample. In contrast, free samples for products such as Adobe Acrobat reader and MS PowerPoint viewer are designed mainly to take advantage of the network effects stemming from a large installed customer base. These network effects can be regarded as *extrinsic* features of the product (Katz and Shapiro (1985)). Our focus, in this paper, is on the intrinsic features of software products for which network effects are not necessarily the dominant factor (example: non-networked computer games, photo/art/design or music software).

The nature of samples used in selling traditional physical goods differs from information goods such as software products. Typically a physical good sample provides a limited experience of the actual product. Even in the case of a durable good like an automobile, a sample in the form of a test drive gives an experience of the majority of features that the customer needs, but only for a limited time. However, in the case of software products, the consumer is often able to continue using the sample instead of the product, when the sample provides a good proportion of the actual desired product features. In that instance, the distribution of free samples leads to cannibalization of actual product sales. For example, individuals may be able to reinstall the same sample on a repeated basis. Therefore, the sample trial time, which controls the frequency of reinstallation, and the proportion of features included in the sample are critical characteristics of focus for the design of free software.

Meanwhile, it is true that when a software product benefits from network effects, these effects may play a role in mitigating the cannibalization of sales (Haruvy and Prasad (2001)). Nevertheless, network effects may have an adverse impact on sales especially when a large existing base of free software users could entice newcomers to keep using the free sample rather than buying the actual product.

We examine the problem of a monopolist designing free software samples, which addresses the issue of sales cannibalization while focusing on the intrinsic features of the software product.[1] Our model is relevant to the case of B2C markets where the sample is distributed to a broad segment of consumers and where the final purchase of the product is not binding even though the customer may continue to use the sample.

The remainder of the paper is organized as follows, section II introduces the model, section III presents the main results regarding the subgame perfect equilibria of the game, and the conditions on trial time and proportion of features that induce these results. Section IV presents the comparative statics results, and section V the concluding comments.

[1] For the traditional analysis of the standard durable good monopolist case see Coase (1972) and Stokey (1981). Dhebar (1994) studies how the speed of improvement of the intrinsic quality of software (or other durable good products) affects the sales dynamics.

II. The Model

We use a vertical differentiation game-theoretic model (Tirole (1988)). Vertical differentiation is a business strategy that focuses on choosing an optimal combination of a product's quality and price in order to gain market share. In that framework, consumers differ according to their reservation price, but have a unanimous preference ranking over a product's attributes and/or performance, such as its user-friendliness and operating speed.[2] For example, given two similar products with identical prices, it is typically true that consumers will prefer the product with more features, and greater execution speed. This particular framework applies well in the context of software samples. In practice, deciding the optimal attributes of a software sample is a nontrivial task, and may impact the market share by directly influencing consumers' ultimate purchase decision. We posit that the proportion of features included and the duration of the trial period together constitute measures of sample attributes and by extension a surrogate of the quality of the actual product attributes.

Generally, the software industry is considered to be relatively concentrated with, for example, Microsoft and Oracle together accounting for about 68% of the industry's profits in 2000.[3] This is because the up-front cost of developing software products is prohibitively high. Presumably, sample and product quality improvements can be mostly achieved through R&D related expenditures (fixed costs) with a small increase in marginal cost. Without loss of generality, we consider a monopolistic market structure.

The model consists of a two-stage sequential game with complete and perfect information. There are two players: a Consumer and a Monopolist. A standard game period, normalized to 1,

[2] For a good introduction on vertical product differentiation see Sutton (1986).
[3] Source: Computerworld, August 2000.

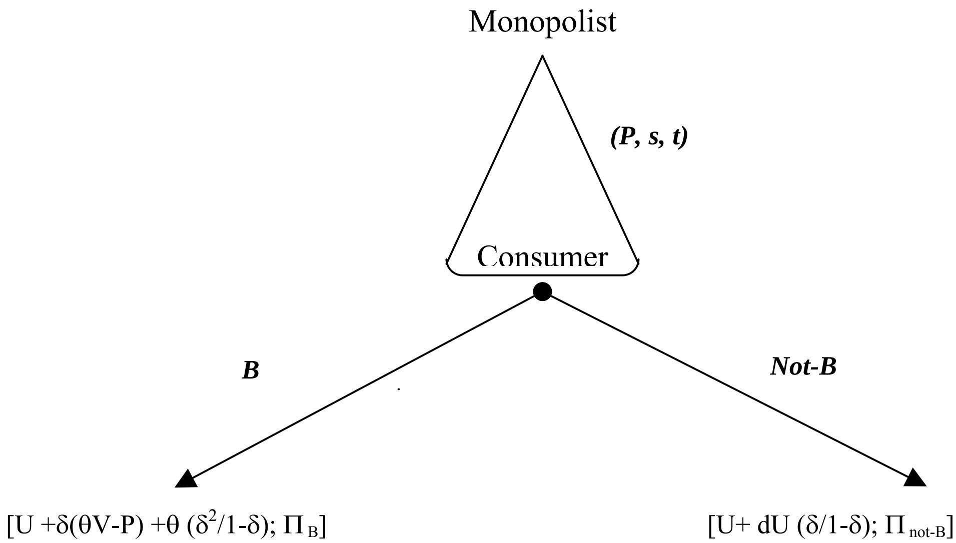
is interpreted as the industry standard for the *maximum* duration of time a Consumer may use a free sample. The first stage of the game has only one period. In that stage, the Monopolist chooses the price P at which she sells her product, and also decides whether to offer a free sample or not. It is assumed that the Monopolist has already developed the commercial or full version of the software product. If a free sample is offered, the Consumer will evaluate it at this stage.

The free sample is a version of the commercial product that may be adjusted along two attributes: the proportion s of features available in the free sample and the duration of the trial period t. The variable s takes values between 0 and 1. The sample trial period t is a fraction of the game period. We also assume that there is a minimum proportion of features s_0 and a minimum trial time t_0 that are needed in order for the Consumer to meaningfully assess the intrinsic features of the product.

The second stage has an infinite number of game periods, but strategic decisions only occur in the first period of this stage. In the first period, the model considers two cases: the Consumer may decide to buy (B) or not to buy (*Not-B*) the commercial product. Buying the product will provide a discounted stream of utility payoffs to the Consumer for the remaining infinite future. On the other hand, the Consumer may decide not to buy the product and continue using the sample as a *substitute for the product itself*.[4] This also results in a stream of discounted utility payoffs that depends on how easy it is for the Consumer to reinstall the sample and the proportion of features included in the sample.

During Stage 2, we assume that the Consumer has the ability to re-use the sample. One possible way to re-use the sample is to reinstall the current copy. Another way is to obtain a new copy of the same sample from the Monopolist, who would then have complete control over the duration of the (repeated) sample. The game tree and final node payoffs for the Consumer and Monopolist are depicted in Figure 1.

FIGURE 1: The Game in Extensive Form



2.1- The Consumer

The one-period utility function for the Consumer from using the product (or the sample) is:

$$U = \begin{cases} 0 \text{ if } s<s_0 \text{ or } t<t_0 \\ \theta ts \text{ if } s_0 \leq s \leq 1 \text{ and } t_0 \leq t \leq 1 \end{cases}$$

The parameter $\theta \in [\theta_L;\theta_H]$ represents a preference parameter, and is distributed according to a uniform probability distribution function $F(\theta) = \frac{\theta - \theta_L}{\theta_H - \theta_L}$. The parameter θ also captures the consumer's reservation price for the product.

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[4] Stage 2 collapses all future periods payoffs by using a discount rate. Takeyama (2002) shows that a durable good monopolist may indeed choose to cannibalize its own high-end product by producing goods of lesser quality, which result in a consumer welfare improving equilibrium.

It is assumed that the utility U is directly proportional to the proportion of features *s* and the trial time *t* during which the free sample can be used. The utility U will be zero if the proportion of features in the sample is too small ($s<s_0$) or the trial time is too short ($t<t_0$). By extension, the commercial product is identical to the sample when $t = 1$ and $s = 1$ and no restrictions are placed on reinstallation.

We also assume that there is an opportunity cost borne by the Consumer when he does not buy the commercial product and continues to use the sample in stage 2. This cost, represented by parameter *d*, captures the intensity of the effort needed for reinstalling the sample. The larger the parameter *d*, the *smaller* the cost. The parameter *d* is assumed to be exogenous. In practice, one can imagine that the Monopolist has some control over this parameter, by making reinstallation procedures more cumbersome. On the other hand, as long as reinstallation is feasible, this parameter will depend to some extent on the Consumer's perception of the reinstallation effort. In the limit, when $d = 0$ or trial time $t = 0$, the utility per period reduces to zero since the Consumer is unable to utilize the sample. Conversely, when $d = 1$ and trial time $t = 1$, then the Consumer benefits from unlimited sample use.

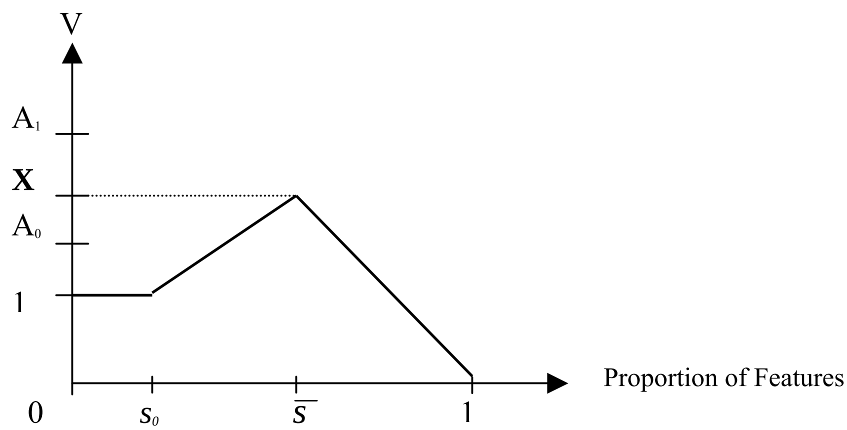
By using the sample, the Consumer acquires additional information about the product, its performance and its intrinsic quality. We posit that the impact of sample use is to raise the Consumer's reservation price[5], which is modeled as follows:

[5] We could interpret this assumption in terms of consumers discovering the product through sampling and updating their prior reservation price. This assertion should also be interpreted in terms of expected utility. That is, our conclusions would not be altered if we assumed a heterogeneous population with two types of individuals, some who will like the product after discovering the sample, and some who will not.

$$V = \begin{cases} 1 \text{ if } s<s_0 \text{ or } t<t_0 \\ \dfrac{(X-1)s+\bar{s}-Xs_0}{\bar{s}-s_0} \text{ if } s_0 \leq s \leq \bar{s} \text{ and } t_0 \leq t \leq 1 \\ \dfrac{X(1-s)}{1-\bar{s}} \text{ if } \bar{s} \leq s \leq 1 \text{ and } t_0 \leq t \leq 1 \end{cases}$$

Function V captures the relationship between the marginal propensity to buy and the proportion of features included in the sample. V is graphically shown in Figure 2:

FIGURE 2: Marginal Propensity to Buy



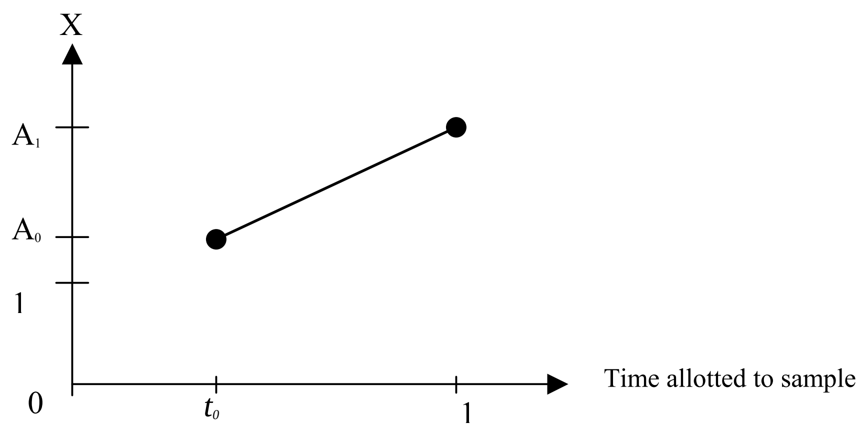
As can be seen from Figure 2, the function V indicates that when the proportion of features is lower than s_0 then the sample is deemed uninformative and the reservation price for the commercial product remains unchanged from the prior value θ. However, for proportions between s_0 and $\bar{s}$ the reservation price steadily increases, since the sample provides better information about the actual product's effectiveness and performance. Once the proportion of features crosses $\bar{s}$ the Consumer has less desire to buy the commercial product since the sample itself provides most of the functionality, resulting in a drop in the reservation price. This modeling approach is appropriate in the context of durable goods like software products, where free samples are offered.[6]

The function X represents the peak propensity to buy due to sample use. It is bounded between A_0 and A_1, with $A_1 > A_0 > 1$. We assume that X increases with the sample trial time t, and is given as:

$$X = \frac{(A_1 - A_0)t + A_0 - A_1 t_0}{1 - t_0}, \text{ so that } X = A_0 \text{ if } t = t_0\text{; and } X = A_1 \text{ if } t = 1.$$

The function X is graphically represented in Figure 3.

FIGURE 3: Behavior of the Peak Propensity to Buy



The larger the trial time t, the greater is the consumer's desirability for the product, and the larger the impact on the reservation price.[7] The discount rate $0<\delta<1$ captures the durability or shelf life of the good, or the degree of technological obsolescence. We assume that the discount rate is the same for the commercial product as well as the sample.

[6] We can easily imagine that $\bar{s}$ is a function of the parameter d. That is, for example as it becomes more difficult to re-use the sample (d close to 0), $\bar{s}$ may converge to 1. In fact, that would mean that the sample and the commercial products are less substitutable.

[7] Note that this effect is distinct from the greater incentive the Consumer has to be re-using the sample when more time is allotted between reinstallation dates.

2.2- The Monopolist

Similar to the vertical differentiation model presented in Tirole (1988), the Monopolist faces a demand given by $D(P,s,t) = N\,[1\text{-}F(\eta(P,s,t))]$, where N is the size of the total potential customer market. However, the segment of customers whose preference parameter is such that $\theta > \eta(P,s,t)$, constitutes the product demand. The function $\eta(P,s,t)$ represents a strategic response by the Consumer to the Monopolist's pricing and sample design decisions.

In order to realize the demand, the Monopolist must distribute samples to a set of potential customers larger than D, which is represented by $D_\gamma(P,s,t) = N\,[1\text{-}F(\eta(P,s,t)\text{-}\gamma/s^2)]$. The quantity γ/s^2 represents the incremental proportion of customers who need to receive free samples in order to realize the demand D, while parameter γ represents the lower limit of such proportion. In fact, this represents the reach and coverage of the marketing campaign. The coverage (function γ/s^2) is assumed decreasing with the proportion of features. The implication being that when more features are included, the Monopolist will better target potential customers to send the free samples to, since it is now more costly to produce such a sample (see below).

The Monopolist's profit is given by: $\Pi_\eta = D(P,s,t)\times[P\text{-}c]\text{- } D_\gamma(P,s,t)cs^2$, where c is the marginal cost of producing the commercial product. The expression cs^2 represents the marginal cost of producing a sample, which is increasing and convex in the proportion of features included in the sample. This assumption is justified when the fixed cost (re-programming the source code) per unit increases when more features are included in the sample.

Based on the above discussion, we can rewrite the Monopolist's profit as $\Pi_\eta = D(P,s,t) \times [P - c(1+s^2)] - N\gamma c/(\theta_H - \theta_L)$. Thus the term $N\gamma c/(\theta_H - \theta_L)$ represents a fixed cost for the monopolist (for a fixed market size).

III. Equilibrium Analysis

The analysis focuses on equilibria wherein the Consumer eventually buys the product. Our analysis comprises three cases based on the proportion of features that is optimal to include in the sample. These cases that are presented as propositions, provide insights for both the Monopolist and the Consumer, regarding the effective design of software samples. The first case (Proposition 1) provides conditions under which the Monopolist's best strategy is to offer no free sample.

Proposition 1: Given the conditions $\theta_H > (1-\delta)\, c$ and $K = (\delta + (1-\delta) A_1 - dt_0 s_0) < 1$, the Subgame Perfect Equilibrium (SPE) indicates that it is optimal for the Consumer to buy the product directly, when his preference parameter is $\theta > (1-\delta)\ P_1^*$. Furthermore it is also optimal for the Monopolist not to offer any free samples. The Monopolist's price is $P_1^* = \dfrac{\theta_H + (1-\delta)c}{2(1-\delta)}$ and the Monopolist's profit is $\Pi_1^* = \dfrac{N}{\theta_H - \theta_L} \dfrac{1}{4(1-\delta)} \left[\theta_H - (1-\delta)c\right]^2$

The condition $\theta_H > (1-\delta)c$ is a minimal condition for the Monopolist to supply the product. It requires that the Consumer's maximum reservation price (in present value terms) be greater than the marginal cost of producing the good. The second condition provides a limit under which neither the Monopolist nor the Consumer derives value from the free sample. This condition suggests that offering free samples is not always an effective marketing strategy. In this case, the variable K measures the relative attractiveness of the commercial product to the Consumer, after having used the free sample with the minimum proportion of features s_0 and trial time t_0.

For the condition K < 1 to hold, the effort involved in reinstalling the sample has to be low (parameter d is large), while the minimum time and minimum proportion of features necessary for the Consumer to learn the software have to be high (t_0 and s_0 are close to one). The product has to be fairly durable (δ is close to one) and the effect of the sample duration on the Consumer's willingness to pay should be negligible (A_1 is small). In general terms, this condition covers the scenario wherein one player (Consumer) obtains all the benefits from the sample while the other player (Monopolist) only incurs losses.

The next proposition identifies the conditions under which the Monopolist offers a free sample with a non-trivial proportion of features $s_0 < s^* < \bar{s}$ and the optimal trial time is also determined.

Proposition 2: Given the conditions $\theta_H > (1-\delta)\, c$, $K^* = (\delta + (1-\delta)V^* - dt^* s^*) > 2$ and γ small, the SPE indicates that it is optimal for the Consumer to buy the product, after using the sample, provided the preference parameter is $\theta > (1-\delta)\; P_2^* / K^*$. It is also optimal for the Monopolist to offer a free sample with a unique proportion of features s^* such that $s_0 < s^* < \bar{s}$.
The optimal proportion of features is $s^* = \dfrac{W^* + \Delta^{1/2}}{6c}$. The Monopolist's price is
$P_2^* = \dfrac{K^*\theta_H + (1-\delta)c(1+s^{*2})}{2(1-\delta)}$, and the Monopolist's profit is

$$\Pi_2^* = \frac{N}{\theta_H - \theta_L}\left(\frac{K^*}{4(1-\delta)}\left[\theta_H - \frac{(1-\delta)c(1+s^{*2})}{K^*}\right]^2 - \gamma c\right)$$

Where $W^* = \theta_H E^* - 4\dfrac{c}{E^*}\left(G + \delta/_{1-\delta}\right)$ with W* < 0; $\Delta = W^{*2} + 12c\left(c + \theta_H\left(G + \delta/_{1-\delta}\right)\right)$;
$E^* = \dfrac{\partial V^*}{\partial s} - \dfrac{dt^*}{1-\delta} > 0$; with $\dfrac{\partial V^*}{\partial s} = \dfrac{X-1}{\bar{s} - s_0}$ and $G = \dfrac{\bar{s} - Xs_0}{\bar{s} - s_0}$. In the case where
$\dfrac{\partial E}{\partial t} = \dfrac{A_1 - A_0}{(1-t_0)(\bar{s} - s_0)} - \dfrac{d}{1-\delta} < 0$, then the optimal trial time is $t^* = t_0$. On the other hand, if
$\dfrac{\partial E}{\partial t} > 0$, and $s^* - \dfrac{s_0(A_1 - A_0)}{\left[(A_1 - A_0) - \dfrac{d(1-t_0)(\bar{s} - s_0)}{1-\delta}\right]} < 0\ (> 0)$ then $t^* = t_0$ ($t^* = 1$).

The above results differ from those of Proposition 1 in several respects. Given that the software sample is being offered, the variable K* reflects the degree of attractiveness of the product, however, after adjusting for a proportion of features $s^* > s_0$ and trial time t^*. In this case the marginal propensity to buy V* is strong enough for the Consumer to purchase the product after having used the sample. As K* increases it engenders two opposing effects. The first effect is to allow the Monopolist to raise her price because of the increased attractiveness of the product, which tends to lower sales. The second effect leads to a market expansion (condition $\theta > (1-\delta) P_2^* / K^*$) because the sample use raises the reservation price of some of the customers whose prior reservation prices were below the threshold. The condition K*>2 ensures a low reservation price threshold (relative to the no-free sample case of Proposition 1) thereby allowing for a larger customer base, even though the Monopolist's price P_2^* is now higher. This condition guarantees that the expansion of market share more than offsets the adverse impact on sales due to a higher price. The purpose of a small γ is to prevent the fixed cost of offering free samples from becoming prohibitively large.

The function E* refers to the marginal effect on K* due to an increase in the proportion of features. In the design of free software samples it is possible to view trial time and proportion of features as substitutes or complements. In Proposition 2, the condition $\frac{\partial \mathrm{E}}{\partial t} < 0$ corresponds to the case where trial time t and the proportion of features s are substitutes. Therefore, more trial time tends to lower the desirability of the marginal feature and hence the proposition indicates that optimal trial time should be $t^* = t_0$.

On the other hand, in the case of complements ($\frac{\partial \mathrm{E}}{\partial t} > 0$), the proposition reveals that the optimal proportion of features s* is greater with $t^*=1$ than s* with $t^* = t_0$. This implies that it is optimal to provide longer trial time and greater proportion of features in tandem. Note that for trial time and proportion of features to be strong complements the marginal propensity to buy must be large, the cost of reinstallation must be high and the gap between the minimum and maximum proportion of features must be small. From these observations and the threshold expression for s*, it is apparent that as trial time and proportion of features become more complementary the likelihood is greater that the optimal trial time $t^*=1$.

The variable W* enters into the computation of the optimal proportion of features s*. The condition W*<0 entails that E* be small. A small positive E* implies that an increase in the reservation price (due to additional features in the sample) slightly dominates the benefit of re-using the sample (due to the ease of reinstallation). This assumption is necessary for the results of Proposition 2 to hold, and allows for characterizing different software products for which free samples may potentially cannibalize sales. In contrast, if E* is negative, then the Monopolist does not offer any free sample, since the Consumer would continue to reinstall the sample forever.

A third possible case is when E* is positive and large, which is not covered by Propositions 1 and 2. A plausible scenario is when a consumer is bedazzled by the mere offer of a sample and willing to purchase the commercial product. This may occur when the product is very innovative. So in this case, the Monopolist offers a sample with a proportion of features slightly greater than s_0, so as to generate a dramatic increase in the reservation price. There is a continuum of equilibrium strategies depending on how close the chosen proportion of features is to s_0. In the limit, if the Monopolist offers the proportion s_0, the sample would lose its appeal to the consumer, and thus the best strategy would collapse to offering no free sample. This reverts to Proposition 1. Nevertheless, we will see in Proposition 3 that the solution is to offer the maximum number of features $\bar{s}$ whenever $\bar{s}$ is close to s_0 which is true in the case of innovative software products.

<u>Proposition 3</u>: Assume that the Monopolist offers the same optimal trial time as the optimal trial time in Proposition 2. Further assume that $\theta_H > (1-\delta)\, c$, $\overline{K}^* = (\delta + (1-\delta)X - d\bar{t}^*\bar{s}) > 2$ and γ small. Then, the SPE indicates that it is optimal for a Consumer with reservation price $\theta > (1-\delta)\, P_3^* / K^*$ to buy the product after having used the free sample. For the Monopolist it is optimal to offer a free sample with the proportion of features $\bar{s}$, whenever $\bar{s} \le s_{\lim} < 1$.

The monopoly price is $P_3^* = \dfrac{\overline{K}^* \theta_H + (1-\delta)c(1+\bar{s}^2)}{2(1-\delta)}$, and the Monopolist's profit is

$$\Pi_3^* = \frac{N}{\theta_H - \theta_L}\left(\frac{\overline{K}^*}{4(1-\delta)}\left[\theta_H - \frac{(1-\delta)c(1+\bar{s}^2)}{\overline{K}^*}\right]^2 - \gamma c\right) \text{ and}$$

$$s_{\lim} = \frac{-\left[G + \delta/_{1-\delta}\right] + \left[\left(G + \delta/_{1-\delta}\right)^2 + \overline{E}^{*2}\right]^{0.5}}{\overline{E}^*} \text{ where } \overline{E}^* = \frac{X-1}{\bar{s}-s_0} - \frac{d\bar{t}^*}{1-\delta} > 0;\ G = \frac{\bar{s} - Xs_0}{\bar{s}-s_0}.$$

When $\dfrac{A_1 - A_0}{(1-t_0)\bar{s}} - \dfrac{d}{1-\delta} < 0\ (> 0)$, then the optimal trial time is $\bar{t}^* = t_0$ ($\bar{t}^* = 1$).

Proposition 3 outlines conditions for the Monopolist to offer a free sample with a maximum proportion of features $\bar{s}$. Note that this proposition is an extension of Proposition 2. Here $\bar{s}$ becomes the optimal proportion of features whenever it is smaller than the limit $s_{\lim}$. The limit can be interpreted as a threshold that separates the software product into two classes. When $\bar{s}$ is small, this implies that E* is large and hence a small increase in the proportion of features steeply raises the reservation price for the software product. In which case, we are dealing with a software product where a small proportion of features in the sample is enough to trigger a large purchase effect. For the case when $\bar{s} > s_{\lim}$, we revert to the result of Proposition 2 with an optimal interior solution for the proportion of features.

To summarize, Propositions 2 and 3 together lay the foundation for understanding the interplay between the design parameters for free software samples and the ensuing sales of commercial software products. They provide conditions under which the benefits of market expansion due to free samples more than offset the potential negative impact of sales cannibalization due to re-use of the free samples.

IV. Comparative Statics

In this section, we present comparative statics results, in the context of the Subgame Perfect Equilibrium wherein the Consumer buys the commercial product and the Monopolist offers a free sample with a proportion of features s* (Proposition 2). We consider several key parameters that influence the Monopolist's decision whether or not to offer the free software sample. The results are summarized in Tables 1 and 2 below.[8] These results are obtained under the necessary condition $4c-\theta_H E > 0$ which requires that the marginal cost of production be greater than a fraction of the largest value of the marginal propensity to buy across the population.[9] If this condition were violated, the Monopolist could easily provide close to the maximum proportion of features in the free sample, thereby rendering the trade-off between trial time and proportion of features meaningless. In contrast, when the condition is satisfied, it generates an upper bound on the attractiveness of the commercial product, and a lower bound on the marginal cost.

TABLE 1

Effect of Basic Parameters on the Optimal Proportion of Features

Parameter	**Effect on s***
Marginal cost of production *c*	(-)
Minimum proportion of features s_0	(+)
Maximum desirable proportion of features $\bar{s}$	(-)
Reinstallation utility cost parameter *d*	(-)
Discount rate δ	(-) (not as large as *d*)

From Table 1 we observe the following: as the marginal cost increases, the optimal proportion of features in the sample decreases. A small increase in marginal cost raises the price and lowers the quantity demanded. In order to avoid a decline in profit, the Monopolist has to mitigate the cost, which requires lowering the proportion of features in the sample. In doing so, the Monopolist does not stand to lose any significant market share, since the variable E is small (from the condition $4c\text{-}\theta_H E > 0$). With respect to other parameters, the effects on the optimal proportion of features are as expected.

For the purpose of gaining further insight, it is useful to consider a commercial software product and the free sample being offered along two dimensions. First, a software product could be referred to as Innovative or Standard. It is Innovative when it has a short life cycle (δ is small) and strong customer appeal (giving a few additional sample features induces a high level of

[8] All the calculations are available from the authors upon request.

demand for the product or a large value for E). In contrast it is considered a Standard product when it has a longer life cycle with a limited appeal. Another useful dimension is to consider the degree of substitutability between trial time and proportion of features in the free software sample.[10] As noted in section III, the trial time t and the proportion of features s are substitutes when $\frac{\partial E}{\partial t} < 0$, and are complements when $\frac{\partial E}{\partial t} > 0$. The comparative statics analysis in Table 2 is conducted along these dimensions.

TABLE 2

Effect on Optimal Proportion of Features

Parameter \ Type of Product	Standard (δ large, E small)	Innovative (δ small, E large)
Trial time t_0 ($\frac{\partial E}{\partial t} > 0$: t and s complements)	(+)	(+)
Trial time t_0 ($\frac{\partial E}{\partial t} < 0$: t and s substitutes)	(-)	(?)

From Table 2, we observe that as the minimum trial time t_0 increases, the effect on the optimal proportion of features varies depending upon whether the product is Standard or Innovative. For a Standard product, when t and s are complements, as t_0 increases a Monopolist should increase the optimal proportion of features in the free sample. This is because any additional increase in trial time alone has a minor impact on the propensity to buy, and leaves no choice but to increase the proportion of features in the sample.

[9] Based on the functional forms used in this paper, the fraction is equal to 25%.
[10] Parker and Van Alstyne (2000) articulate a strategy space for information product design. They classify information goods as strategic complements or substitutes according to their ability to generate network externalities.

However, when t and s are substitutes, as t_0 increases the Monopolist will reduce the optimal proportion of features. Normally, a Standard product is easy to comprehend, navigate and does not require any special training. Accordingly, as long as the sample is a good representation of the product, the Consumer is able to gauge correctly the scope and effectiveness of the product. Therefore, increasing both the trial time and proportion of features in the sample for a Standard product may actually act as a disincentive for the Consumer to purchase the product.

On the other hand, an Innovative product typically has a short shelf life because it is a cutting edge product and can be supplanted by other technologically superior products or generations of products. To use such an Innovative product, the Consumer might need to acquire special skills (programming, querying etc…) and could also need larger trial time to comprehend and navigate the sample. Thus, for an Innovative product, as t_0 increases the Monopolist should increase the optimal proportion of features in the free sample unambiguously only when t and s are complements. Since the Innovative product has a short shelf life, the Monopolist should provide a sufficient proportion of features in the sample to entice the Consumer to buy the product. Furthermore, the rapid obsolescence provides no incentive for the Consumer to re-use the sample. Instead he may prefer to purchase the commercial product so as to benefit from the full complement of features available in the product.

In the case of an Innovative software product, it is not clear whether it is meaningful to view t and s as substitutes. Hence in Table 2 we report an ambiguous effect. The complexity of such a product requires the Monopolist to strike a judicious balance between the trial time and the proportion of features included in the sample. For example, the trial time and the proportion of features have to be closely matched such that the Consumer can explore the sample and appraise the value of the product in an efficient manner. If the trial time is inadequate the Consumer feels frustrated and may decide not to purchase. On the other hand, a large trial time with no corresponding increase in the proportion of features prevents the Monopolist from garnering the gains due to a large enhancement of the propensity to buy.[11]

V. Conclusions

Software products come in a variety of forms. Some are sold without offering any samples (for example, advanced technical software used in electric utilities for control purposes), since the software is used for a specific application and requires a significant set-up cost. In contrast, other software products, for example Adobe Acrobat Writer, are sold by offering an unlimited number of free downloads of Acrobat Reader (free samples) since the cost of the sample copy is negligible, and the software product has mass appeal and market. However, there is a large variety of software products that fall somewhere between these two cases. For these software products, the market is not fully known a-priori and some training is necessary for the consumer to benefit from the product. Typically, these products are sold by offering a free sample that has partial features and limited trial time.

In this paper, we have developed a simple model for a Monopolist to analyze the role of free software samples in the marketing of software products. For this purpose, we consider an environment where the software sample is offered as a durable good rather than as a one-time-use-and-discard item. The software sample is modeled using two parameters namely: the proportion of features and trial time. Our results highlight the importance of the trade-off between proportion of features and trial time in developing prescriptive marketing and service

[11] It is interesting to note that when t and s are complements, the effect of changing the minimum trial time is positive and independent of whether the product is Standard or Innovative, because the two parameters δ and E

strategies. Specifically, our results provide conditions under which the benefits of market expansion due to free samples more than offset the potential negative impact of sales cannibalization due to re-use of the free samples.

We also differentiate between Innovative and Standard software products. A product is Innovative when it has a short life cycle and strong customer appeal. In contrast, it is considered a Standard product when it has a longer life cycle with a limited appeal. Another useful dimension is to consider the degree of substitutability between trial time and proportion of features in the free software sample. Complementarity means that more features and more trial time given together tend to enhance sales. Our comparative statics result point to the best free sample design response depending on the category of products offered by a monopolist.

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have offsetting impacts on the optimal proportion of features as t_0 is increased.

Appendix

Proof of Proposition 1:

Assume that the Monopolist strategy space is limited to picking a proportion of features so that $s<s_0$. By backward induction, we show that some consumers will buy (B) if:

$U+\delta[\theta V-P]+\delta^2\theta/(1-\delta) > U+ dU\delta/(1-\delta)$

As U =0, and V=1, we can show easily that the above inequality entails: $\theta/(1-\delta)-P>0$. Thus $D(P)=N[1-F((1-\delta)P)]$ is independent of *s* and *t*. Therefore the Monopolist's profits are declining in *s* as $\Pi = D(P)\times[P-c(1+s^2)] -N\gamma c/(\theta_H-\theta_L)$. Thus, the optimal response for the monopolist is to set $s^*=t^*= 0$. Moreover the standard first order condition for selecting the optimal price P^*, leads to $P^* = \dfrac{\theta_H+(1-\delta)c}{2(1-\delta)}$, and the Monopolist's profit is $\Pi_1^* = \dfrac{N}{\theta_H-\theta_L}\dfrac{1}{4(1-\delta)}\left[\theta_H-(1-\delta)c\right]^2$.

For the demand to exist we need $\theta_H > (1-\delta)c$. In order to show that the Monopolist's best response is $s^*= 0$ in the unrestricted strategy space we need to show that the profit Π_1^* is greater than the monopolist profit in the cases where the Monopolist would choose $s^*> s_0$. It is equivalent to showing that $\Pi_1^*>\Pi_2^*$ and $\Pi_1^*>\Pi_3^*$, where Π_2^* and Π_3^* are the maximum profits generated respectively in Propositions 2 and 3. We know that:

$$\Pi_1^* = \frac{N}{\theta_H-\theta_L}\frac{1}{4(1-\delta)}\left[\theta_H-(1-\delta)c\right]^2 \text{ and}$$

$$\Pi_2^* = \frac{N}{\theta_H-\theta_L}\left(\frac{K^*}{4(1-\delta)}\left[\theta_H-\frac{(1-\delta)c(1+s^{*2})}{K^*}\right]^2-\gamma c\right); \text{with } K^* = \delta+(1-\delta)V^* - dt^*s^*.$$

Where s^* and t^* are optimal strategies for the Monopolist in Case 2.and,

$$\Pi_3^* = \frac{N}{\theta_H-\theta_L}\left(\frac{\overline{K}^*}{4(1-\delta)}\left[\theta_H-\frac{(1-\delta)c(1+\bar{s}^2)}{\overline{K}^*}\right]^2-\gamma c\right); \text{with } \overline{K}^* = \delta+(1-\delta)X - d\bar{t}^*\bar{s}.$$ Where $\bar{s}$ and $\bar{t}^*$ are optimal strategies for the Monopolist in Case 3.

A sufficient condition for having $\Pi_1^* > \Pi_2^*$ is K^*<1, and a sufficient condition for $\Pi_1^* > \Pi_3^*$ is $\overline{K}^*$<1, and γ sufficiently small. Both of these inequality conditions will be true when $\delta + (1-\delta)A_1 - dt_0 s_0 < 1$. QED.

Proof of Proposition 2:

Assume that the Monopolist strategy space is limited to picking a proportion of features so that $s_0 \leq s < \bar{s}$. By backward induction, we show that some consumers will use the sample and then buy (B) if: $U+\delta[\theta V-P]+\delta^2\theta/(1-\delta) > U+ dU\delta/(1-\delta)$

In this case the first inequality above entails that $\theta > (1-\delta)P/K$, where $K = \delta + (1-\delta)V - dts > 0$ by assumption. Thus $D(P, t, s)=N[1-F((1-\delta)P/K)]$. Therefore the Monopolist's profit is $\Pi = D(P, t, s)\times[P-c(1+s^2)] - N\gamma c/(\theta_H-\theta_L)$. The optimal solution for P_2^* is found by using the following first order condition:

$$\frac{\partial \Pi}{\partial P} = \frac{\partial D}{\partial P}\left[P - c(1+s^2)\right] + D = 0 \; (1)$$

Solving equation (1) leads to $P_2^* = \dfrac{K\theta_H + (1-\delta)c(1+s^{*2})}{2(1-\delta)}$ (2)

We need to check that $\theta_H > (1-\delta) P_2^*/K$ which is implied by K>2 and $\theta_H > (1-\delta)c$. On the other hand, the optimal strategy s^* is given by the following first order condition:

$$\frac{\partial \Pi}{\partial s} = \frac{\partial D}{\partial s}\left[P - c(1+s^2)\right] - cD = 0 \; (3)$$

with $\dfrac{\partial D}{\partial s} = NP\dfrac{P}{K^2}\left[\dfrac{\partial V}{\partial s} - \dfrac{dt}{1-\delta}\right]\times F'(\cdot)$. Let $E = \dfrac{\partial V}{\partial s} - \dfrac{dt}{1-\delta}$. Noticing that K can be expressed as $K = (1-\delta)[Es+G]+\delta$ with $G = \dfrac{\bar{s} - Xs_0}{\bar{s} - s_0}$, K>2 implies E>0, this also implies that an interior solution exists. In order to solve for s^*, note that combining equations (1) and (3) leads to:

$$\frac{\partial D}{\partial s} + c\frac{\partial D}{\partial P} = 0 \; (4)$$

Solving for P_2^* again leads to a different expression: $P_2^* = \dfrac{cK}{(1-\delta)E}$ (5)

Combining equations (2) and (5), and expressing K as a function of E and G, we can therefore get to two possible solutions for $s^* = \frac{W^* \pm \Delta^{1/2}}{6c}$. Where $W^* = \theta_H E^* - 4\frac{c}{E}\left(G + \delta/_{1-\delta}\right)$, and $\Delta = W^2 + 12c\left(c + \theta_H\left(G + \delta/_{1-\delta}\right)\right)$. It is fairly easy to show that for s^* to be a maximum there is a unique solution, which is $s^* = \frac{W^* + \Delta^{1/2}}{6c}$. The optimal profit for the Monopolist is

$$\Pi_2^* = \frac{N}{\theta_H - \theta_L}\left(\frac{K^*}{4(1-\delta)}\left[\theta_H - \frac{(1-\delta)c(1+s^{*2})}{K^*}\right]^2 - \gamma c\right).$$

As for the optimal fraction of time t^* it is determined using the following condition: $\frac{\partial \Pi}{\partial t} = \frac{\partial D}{\partial t}\left[P - c(1+s^2)\right] > \text{or} < 0$. Thus the sign of $\frac{\partial \Pi}{\partial t}$ depends on the sign of $\frac{\partial D}{\partial t}$. But

$$\frac{\partial D}{\partial t} = NP\frac{P}{K^2}\left[\frac{\partial V}{\partial t} - \frac{ds}{1-\delta}\right] \times F'(\cdot)$$

Using the definition of the function V, we have that: $\frac{\partial V}{\partial t} - \frac{ds}{1-\delta} = \frac{\partial E}{\partial t}s^* - \frac{s_0(A_1 - A_0)}{(1-t_0)(\bar{s} - s_0)}$ where $\frac{\partial E}{\partial t} = \frac{A_1 - A_0}{1-t_0} - \frac{d}{1-\delta}$. In the case where $\frac{\partial E}{\partial t} < 0$, then the optimal solution is $t^* = t_0$. On the other hand, if $\frac{\partial E}{\partial t} > 0$, then the optimal solution is still $t^* = t_0$ whenever $s^* < \frac{s_0(A_1 - A_0)}{\left[(A_1 - A_0) - \frac{d(1-t_0)(\bar{s} - s_0)}{1-\delta}\right]}$; and $t^* = 1$ whenever

$$s^* > \frac{s_0(A_1 - A_0)}{\left[(A_1 - A_0) - \frac{d(1-t_0)(\bar{s} - s_0)}{1-\delta}\right]}.$$

As for the Monopolist we need to check that selecting $s_0 < s^* < \bar{s}$, is optimal over the global strategy space. This will be the case if $\Pi_2^* > \Pi_1^*$ and $\Pi_2^* > \Pi_3^*$, where Π_1^* and Π_3^* are the maximum profits generated respectively in Propositions 1 and 3. It is easy to check that

$\Pi_2^* > \Pi_1^*$ if K*>2 and γ is small enough. On the other hand, $\Pi_2^* > \Pi_3^*$ whenever W*<0 as this implies that $\frac{\partial \Pi}{\partial s} > 0$ for s> $\bar{s}$. QED.

Proof of Proposition 3:

Assume that the Monopolist strategy space is limited to picking a proportion of features so that $\bar{s} \leq s$<1. It is evident that in this case the same basic conditions for the Consumer: $\theta_H > (1-\delta)c$; and $\overline{\mathrm{K}}^*$>2 should be used. The same first order conditions will also hold as in Proposition 2. The optimal strategy s^* is given by the following first order condition:

$$\frac{\partial \Pi}{\partial s} = \frac{\partial \mathrm{D}}{\partial s}\left[P - c(1+s^2)\right] - c\mathrm{D} = 0 \; (6)$$

with $\frac{\partial \mathrm{D}}{\partial s} = \mathrm{N}P\frac{P}{\mathrm{K}^2}\left[\frac{\partial \mathrm{V}}{\partial s} - \frac{\mathrm{d}t}{1-\delta}\right] \times \mathrm{F}'(\cdot)$. On the other hand, we have $\frac{\partial \mathrm{V}}{\partial s} = \frac{-\mathrm{X}}{(1-\bar{s})} < 0$. Thus the solution for s^* is equal to $\bar{s}$. Solving the first order conditions for the Monopolist price leads to $P_3^* = \frac{\overline{\mathrm{K}}^*\theta_H + (1-\delta)c(1+\bar{s}^2)}{2(1-\delta)}$, and the Monopolist's profit is:

$$\Pi_3^* = \frac{\mathrm{N}}{\theta_H - \theta_L}\left(\frac{\overline{\mathrm{K}}^*}{4(1-\delta)}\left[\theta_H - \frac{(1-\delta)c(1+\bar{s}^2)}{\overline{\mathrm{K}}^*}\right]^2 - \gamma c\right).$$ Where $\overline{\mathrm{K}}^* = \delta + (1-\delta)\mathrm{X} - d\bar{t}^*\bar{s}$.

As for the optimal fraction of time t^* it is determined using the following condition:

$\frac{\partial \Pi}{\partial t} = \frac{\partial \mathrm{D}}{\partial t}\left[P - c(1+s^2)\right] >$ or < 0. Thus the sign of $\frac{\partial \Pi}{\partial t}$ depends on the sign of $\frac{\partial \mathrm{D}}{\partial t}$. But

$$\frac{\partial \mathrm{D}}{\partial t} = \mathrm{N}P\frac{P}{\mathrm{K}^2}\left[\frac{\partial \mathrm{V}}{\partial t} - \frac{\mathrm{d}s}{1-\delta}\right] \times \mathrm{F}'(\cdot)$$

Using the definition of the function V, we have that:

$\frac{\partial \mathrm{V}}{\partial t} - \frac{\mathrm{d}\bar{s}}{1-\delta} = \frac{\mathrm{A}_1 - \mathrm{A}_0}{1-t_0} - \frac{d\bar{s}}{(1-\delta)}$. In the case where $\frac{\mathrm{A}_1 - \mathrm{A}_0}{1-t_0} - \frac{\mathrm{d}\bar{s}}{1-\delta} < 0$, then the optimal trial time is $\bar{t}^* = t_0$. On the other hand, if $\frac{\mathrm{A}_1 - \mathrm{A}_0}{1-t_0} - \frac{\mathrm{d}\bar{s}}{1-\delta} > 0$, then the optimal solution is $\bar{t}^* = 1$.

1. As for the Monopolist we need to check that selecting the optimal proportion of features of $\bar{s}$ <1, is optimal over the unrestricted strategy space. This will be the case if $\Pi_3^* > \Pi_1^*$ and $\Pi_3^* > \Pi_2^*$, where Π_1^* and Π_2^* are the maximum profits generated respectively in Propositions 1 and 2. It is easy to check that $\Pi_3^* > \Pi_1^*$ if $\overline{\mathrm{K}}^*$ >2 and γ is small. On the other hand, $\Pi_3^* > \Pi_2^*$ will be true if $\frac{(1+\bar{s}^2)}{\overline{\mathrm{K}}} < \frac{(1+s^{*2})}{\mathrm{K}^*}$ as $\overline{\mathrm{K}}^* > \mathrm{K}^*$ when the Monopolist does not alter her choice of the optimal trial time. Where K^* ($\overline{\mathrm{K}}^*$) is the value of K corresponding to the optimal trial time t^* in Proposition 2 and an optimal proportion of features s^* ($\bar{s}$). The previous inequality is implied by $\frac{\partial}{\partial s}\frac{(1+s^2)}{\mathrm{K}} < 0$, or by $\bar{s} \le s_{\lim}$ <1, with $s_{\lim} = \frac{-\left[\mathrm{G} + \delta/_{1-\delta}\right] + \left[\left(\mathrm{G} + \delta/_{1-\delta}\right)^2 + \overline{\mathrm{E}}^{*2}\right]^{0.5}}{\overline{\mathrm{E}}^*}$ QED.

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