Computer generated models of intraorgan arterial beds: hydraulic conductivity and input admittance

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Abstract. Geometry of the arterial junctions is investigated on the plastic casts of the human intraorgan arterial vasculatures. Some statistical dependence between the lengths and diameters of the arterial segments are revealed. On the basis of the observed regularities for different inner organs the software for generation and visualization of the realistic models of the intraorgan arterial beds is elaborated. Pressure wave propagation in the generated branching systems of viscoelastic tubes is investigated.

1 Introduction

Geometry of the arterial intraorgan vasculatures defines blood supply to the organ and blood redistribution between the organs [1-2]. Structure-function relationships in the vascular beds are strongly influenced by their geometry [3]. The diameters and lengths of separate arterial segments and branching angles in junctions can be measured on the plastic casts of the arterial beds. The results of the measurements can be used for direct calculations of the hydraulic conductivity Z and wave input admittance of the vasculatures. Traditionally, the input impedance Z_{in} or input admittance $Y_{in} = (Z_{in})^{-1}$ has been used to characterize the dynamic properties of the arterial systems [4]. The input admittance is defined as the relationship of flow Q(t) and pressure P(t) at the entrance of the arterial bed and describes the ability of the arterial system for passive impeding the blood flow as well as active regulation of the wave propagation through the vasculature [5]. The data include a few thousand values that characterize geometry of the investigated vasculature only [6-8]. Arterial beds of different inner organs possess different topology and undergo significant individual variations. Physiological variations of the lengths ($\pm 30\,\%$) and diameters ($\pm 10\,\%$)

of arterial segments cause noticeable differences in the conductivity of the arterial systems of a given inner organ in different individuals.
The measurements on the casts have revealed the following regularities in geometry of the arterial bifurcations [9-11]:

- Murray law $\left(d_0^j\right)^\gamma = \left(d_1^j\right)^\gamma + \left(d_2^j\right)^\gamma$ (1)

- $$\cos(\alpha_{1,2}^j) = \frac{\left(\left(d_2^j\right)^\gamma + \left(d_1^j\right)^\gamma\right)^{4/\gamma} \pm \left(d_2^j\right)^4 \mp \left(d_1^j\right)^4}{2\left(d_2^j\right)^2\left(\left(d_2^j\right)^\gamma + \left(d_1^j\right)^\gamma\right)^{2/\gamma}} \quad (2)$$

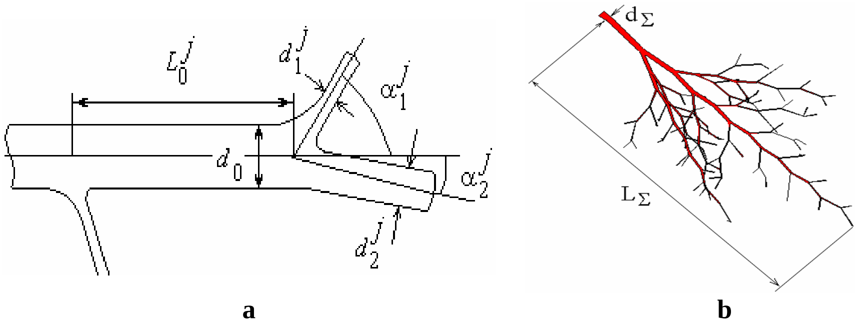

Fig. 1. Parameters of a separate bifurcation (a) and a subsystem of the arterial network.

where $d_{1,2}^j$, d_0^j are the diameters of the daughter and parent vessels in the j-th junction, $\alpha_{1,2}^j$ are branching angles (fig.1a,b). Taking into account the regularities the realistic models of arterial vasculatures can be constructed on the basis of a given set of the parameters $\{d^1, a, b, \gamma, \xi\}$ where d^1 is the diameter of the feeding artery of the vasculature, $\xi = \min\{d_{1,2}\}/\max\{d_{1,2}\}$ is an asymmetry coefficient. When the parameters a, b, γ, ξ are constant for all the junctions the model of the arterial vasculature as a self-similar tree-like system of compliant tubes can be constructed and used for calculations the hydraulic properties and the pressure wave propagation in the system taking into consideration wave reflection at bifurcations [12-14]. The models have been found to be very useful for biomechanical interpretation of the pulse wave curves P(t), Q(t) as an important tool for medical diagnostics [14-16]. The differences in geometry of the arterial beds of different inner organs have not been embedded into the fractal models yet.