Multiobjective Genetic Algorithms Applied to Solve Optimization Problems

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***Abstract*—In this paper, we discuss multiobjective optimization problems solved by evolutionary algorithms. We present the nondominated sorting genetic algorithm (NSGA) to solve this class of problems and its performance is analyzed in comparing its results with those obtained with four others algorithms. Finally, the NSGA is applied to solve the TEAM benchmark problem 22 without considering the quench physical condition to map the Pareto-optimum front. The results in both analytical and electromagnetic problems show its effectiveness.**

***Index Terms*—Electromagnetics, multiobjective evolutionary algorithms, nondominated sorting genetic algorithms.**

I. INTRODUCTION

MANY real-world electromagnetic problems involve simultaneous optimization of multiple objectives that often are competing. In a multiobjective optimization problem (MOP), there may not exist one solution that is best with respect to all objectives. Usually, the aim is to determine the tradeoff surface, which is a set of nondominated solution points, known as *Pareto-optimal* (PO) *or* noninferior solutions. In view of the fact that none of the solutions in the nondominated set is absolutely better then any other, any one of them is an acceptable solution. The choice of one solution over the other requires problem knowledge and a number of problem-related factors [1].

One way to solve multiobjective problems is to transform the original problem in a single-objective one, by weighting the objectives with a weight vector. This process allows the use of any single-objective optimization algorithm, but the obtained solution depends on the weight vector used in the weighting process. Genetic algorithms (GAs) work with a population of points, so we expect that they can find the Pareto-optimal front easily.

In this paper, we present the NSGA [2] and we analyze it regarding the solution of MOPs. Moreover, we compare its results with those obtained by the multiobjective evolutionary algorithms (MOEAs): VEGA [3], NPGA [4] and MOGA [5], and the classical method of objective weighting refereed as $P(\lambda)$ [1]. We compare its performances in the solution of two analytical test problems. Finally, we apply the NSGA to solve the TEAM problem 22 without taking into account the quench physical condition with the aim of find the (quasi) optimum tradeoff surface.

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II. MULTIOBJECTIVE OPTIMIZATION PROBLEM

Mathematically, we can write MOPs as

$$\begin{aligned}
\text{maximize} \quad & \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}),\, f_2(\boldsymbol{x}),\, \ldots,\, f_M(\boldsymbol{x})\} \\
\text{subject to} \quad & \boldsymbol{g}(\boldsymbol{x}) = \{g_1(\boldsymbol{x}),\, g_2(\boldsymbol{x}),\, \ldots,\, g_J(\boldsymbol{x})\} \le \boldsymbol{0} \\
& \boldsymbol{h}(\boldsymbol{x}) = \{h_1(\boldsymbol{x}),\, h_2(\boldsymbol{x}),\, \ldots,\, h_K(\boldsymbol{x})\} = \boldsymbol{0} \\
\text{where} \quad & \boldsymbol{x} = \{x_1,\, x_2,\, \ldots,\, x_N\} \in \boldsymbol{X} \\
& \boldsymbol{y} = \{y_1,\, y_2,\, \ldots,\, y_M\} \in \boldsymbol{Y}
\end{aligned} \tag{1}$$

and $\boldsymbol{x}$ is the vector of decision variables, $\boldsymbol{y}$ is the objective vector, $\boldsymbol{X}$ is the decision space, and $\boldsymbol{Y}$ is called the objective space. The solution of (1) is usually no unique, but a set of equally efficient, noninferior or nondominated solutions, known as Pareto-optimal set [1].

A noninferior solution is one that is not dominated by any other feasible solution. Mathematically, in the maximization case, we say that the solution $\boldsymbol{x}^1$ dominates $\boldsymbol{x}^2$, or $\boldsymbol{x}^1$ is superior to $\boldsymbol{x}^2$, i.e.,

$$\forall i \in \{1, \ldots, M\},\, \boldsymbol{y}(\boldsymbol{x}^1) \ge \boldsymbol{y}(\boldsymbol{x}^2) \quad \wedge \exists i \in \{1, \ldots, M\} | y_i(\boldsymbol{x}^1) > y_i(\boldsymbol{x}^2).$$

If any other $\boldsymbol{x}$ in the feasible space of design variables does not dominate $\boldsymbol{x}^1$, hence, $\boldsymbol{x}^1$ is a noninferior, nondominated or a Pareto-optimal point. Two Pareto-optimal points are indifferent to each other.

The optimization algorithm should be terminated if any one of the Pareto-optimal solutions is obtained. But in practice, since there could be a number of Pareto-optimal solutions and the suitability of one solution depends on a number of factors, including the designer's choice and problem environment, finding the entire set of Pareto-optimal solutions may be desired. In the following section, we describe in details the nondominated sorting genetic algorithm (NSGA).

III. NSGA DESCRIPTION

The basic idea behind NSGA is the ranking process executed before the selection operation, as shown in Fig. 1. This process identifies nondominated solutions in the population, at each generation, to form nondominated fronts [2], based on the concept of nondominance criterion as explained in Section II. After this, the selection, crossover, and mutation usual operators are performed.

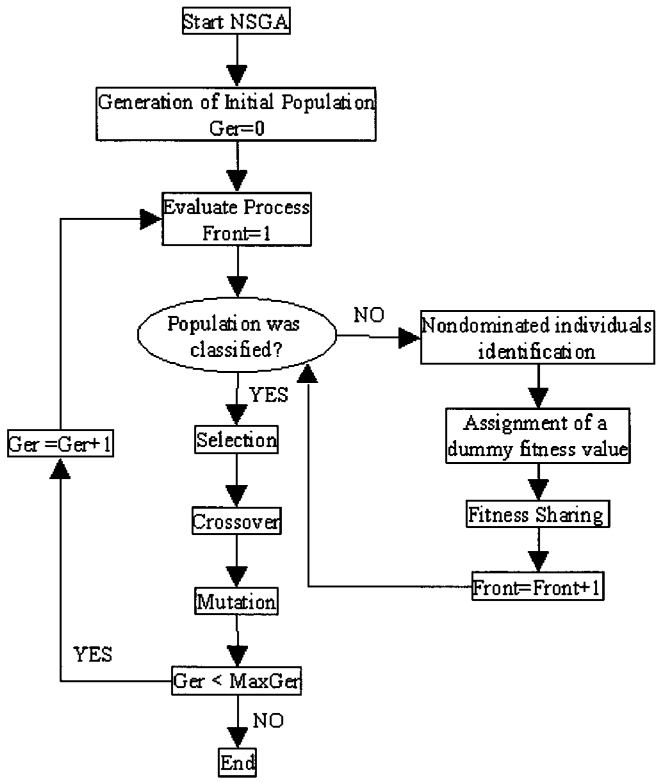

Fig. 1. Flow chart of NSGA.

In the ranking procedure, the nondominated individuals in the current population are first identified. Then, these individuals are assumed to constitute the first nondominated front with a large dummy fitness value [2]. The same fitness value is assigned to all of them. In order to maintain diversity in the population, a sharing method is then applied. Afterwards, the individuals of the first front are ignored temporarily and the rest of population is processed in the same way to identify individuals for the second nondominated front. A dummy fitness value that is kept smaller than the minimum shared dummy fitness of the previous front is assigned to all individuals belonging to the new front. This process continues until the whole population is classified into nondominated fronts. Since the nondominated fronts are defined, the population is then reproduced according to the dummy fitness values.

The NSGA was first proposed with a stochastic remainder proportional selection (SRS) procedure. However, it is possible to use any other selection technique as roulette wheel or tournament [6]. As the individuals in the first front have higher fitness value, they always get more copies than the rest of the population. This method was intended to search for nondominated regions, and sharing helps to distribute the individuals over this region. By emphasizing nondominated points, NSGA favors the schemata representing the Pareto-optimum regions [2].

NSGA implements both aspects of Goldberg's suggestion in the better way [6], i.e., the ranking procedure is performed according to the nondominance definition over the population and a uniform distribution of the nondominated is guaranteed using a niche formation technique. Both aspects produce distinct nondominated points to be found in the population.

Fitness Sharing: In genetic algorithms, sharing techniques aim at encouraging the formation and maintenance of stable subpopulations or niches [7]. This is achieved by degrading the fitness value of points belonging to a same niche in some space. Consequently, points that are very close to, with respect to some space (decision space $\boldsymbol{X}$ in this paper), will have its dummy fitness function value more degraded. The fitness value degradation of near individuals can be executed using (2) and (3), where the parameter d_{ij} is the variable distance (Euclidean norm) between two individuals i and j, and σ_{shared} is the maximum distance allowed between any two individuals to become members of a same niche. In addition, dfi is the dummy fitness value assigned to individual i in the current front and $df'i$ is its corresponding shared value. N_{pop} is the number of individuals in the population. For details about niching techniques, see [8]

$$Sh(d_{ij} = \begin{cases} 1 - \left(\dfrac{d_{ij}}{\sigma_{\text{shared}}}\right)^2, & \text{if } d_{ij} < \sigma_{\text{shared}} \\ 0, & \text{if } d_{ij} \geq \sigma_{\text{shared}} \end{cases} \tag{2}$$

$$df_i' = df_i \left[\sum_{j=1}^{N_{\text{pop}}} Sh(d_{ij}) \right]^{-1}. \tag{3}$$

IV. Description of $P(\lambda)$, VEGA, MOGA, and NPGA

The method of objective weighting, $P(\lambda)$, is probably the simplest of all classical techniques where multiple objectives are combined into one overall objective function. In this method, the optimal solution is controlled by a weight vector w and modifying the corresponding weight can change the preference of an objective. The only advantage of using this technique is that the emphasis of one objective over the other can be controlled and the obtained solution is usually a Pareto-optimum solution [1].

An early GA application on multiobjective optimization by Schaffer opened a new avenue of research in this field. The algorithm, called vector evaluated genetic algorithm (VEGA), performs the selection operation based on the objective switching rule, i.e., selection is done for each objective separately, filling equally portions of mating pool [3]. Afterwards, the matting pool is shuffled, and crossover and mutation are performed as usual.

Fonseca and Fleming [5] proposed a Pareto-based ranking procedure (MOGA), where the rank of an individual is equal the number of solutions found in the population where its corresponding decision vector is dominated. The fitness assignment is determined by interpolating the fitness value of the best individual (nondominated) and the worst one (most dominated). The MOGA algorithm also uses a niche-formation method to distribute the population over the Pareto-optimal region based on the objective space.

The niched Pareto genetic algorithm (NPGA) proposed by Horn, Nafpliotis, and Goldberg uses the concept of Pareto dominance and tournament selection in solving MOPs [4]. In this method, a comparison set of T_{dom} individuals is randomly picked from the current population before the selection procedure. In addition, we choose two candidates from the current population that will compete to survive to the selection operation. For selecting the winner, these two candidates are compared with those of T_{dom} set using a nondomination criterion as described in Section II.

V. Criterion for Performance Measurements

The performance measurement criterion used to evaluate the Pareto fronts produced by the EAs is the coverage relationship [7]. Given two sets of nondominated solutions, we compute for each set the fraction of the solutions that is not covered (not dominated) by the other. Since this comparison focus on finding the Pareto-optimal set, this criterion uses the off-line performance method. The nondominated solution set taken to perform the comparison between all EAs is the summation of nondominated solutions found by each algorithm at each run, after application of a nondominance criterion.

VI. Results

Analytical Problems

Two problems Γ_1 and Γ_2 were chosen in order to test the multiobjective genetic algorithms discussed in this paper. The Γ_1 problem has a convex Pareto-optimal front and is given by

$$f_1(x_1, x_2, \ldots, x_m) = x_1$$
$$f_2(x_1, x_2, \ldots, x_m) = g(\boldsymbol{x})^* \left(1 - \sqrt{f_1/g(\boldsymbol{x})}\right). \quad (4)$$

The second problem Γ_2 is the nonconvex counterpart to Γ_1

$$f_1(x_1, x_2, \ldots, x_m) = x_1$$
$$f_2(x_1, x_2, \ldots, x_m) = g(\boldsymbol{x})^* \left[1 - (f_1/g(\boldsymbol{x}))^2\right]. \quad (5)$$

In both cases, $m = 30$, $x_i \in [0, 1]$ and the Pareto-optimal front is formed with $g(\boldsymbol{x}) = 1$. The function $g(\boldsymbol{x})$ is defined by

$$g(x_2, \ldots, x_m) = 1 + 9 \sum_{i=2}^{m} x_i/(m-1). \quad (6)$$

The multiobjective EAs were executed 30 times for each problem with the same initial population. The results of each execution was stored in an auxiliary vector and at the end the nondominance criterion was applied to the points belonging to the auxiliary vector, resulting a nondominated set that was taken as outcome. The set of genetic parameters used are: $N_{\mathrm{ger}} = 250$, $N_{\mathrm{pop}} = 100$, $p_c = 0.80$, $p_m = 0.01$, $\sigma_{\mathrm{shared}} = 0.4886$ and $T_{\mathrm{dom}} = 10$ (for NPGA). The graphic results are shown in Figs. 2 and 3.

The direct comparison of the outcomes achieved by the different multiobjective EA is presented in Table I. Each cell gives the percentage of solutions evolved by method B that are nondominated by those achieved by method A for both problems Γ_1 and Γ_2. For example, the cell NPGA/MOGA signifies that 90% of solutions found by NPGA are nondominated by those found by MOGA for Γ_1 problem and 89% in the case of problem Γ_2. These results show that all methods give rise to similar solutions with a slight superiority for NSGA method, with exception of VEGA. The result for VEGA method is explained by the fact of its selection procedure does not use information of nondominated fronts.

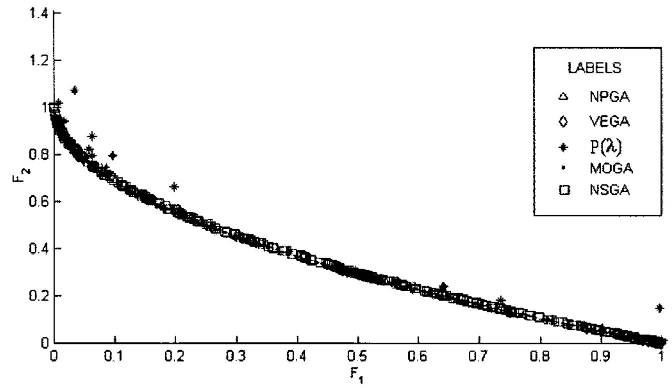

Fig. 2. Nondominated points for EAs—function Γ_1.

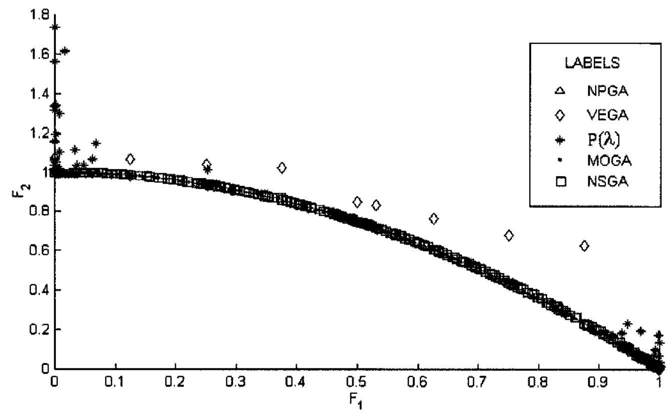

Fig. 3. Nondominated points for EAs—function Γ_2.

TABLE I
EA Performance Measurement

B/A	VEGA	MOGA	NSGA	NPGA	P(λ)
VEGA	-	0/0	0/0	0/0	0/0
MOGA	100/100	-	90/89	100/98	100/100
NSGA	100/100	100/100	-	100/100	100/100
NPGA	100/100	90/89	90/88	-	100/100
P(λ)	100/100	88/79	87/90	79/89	-

Optimization in Electromagnetics

The TEAM Benchmark Problem 22 was chosen to show the application of NSGA described in the previous study in solving a multiobjective electromagnetic optimization problem. In this paper, we search to find the (quasi) Pareto-optimal front of a superconducting magnetic energy storage device (SMES) [9]. The aim is to find the multiple Pareto-optimal points considering two objective functions: 1) the first objective considers the stray field and 2) the second one takes into account the stored energy related to a prescribed value. The constraint conditions are the bounds in the design variables. The quench physical condition that guarantees superconductivity was neglected in this simulation [10].

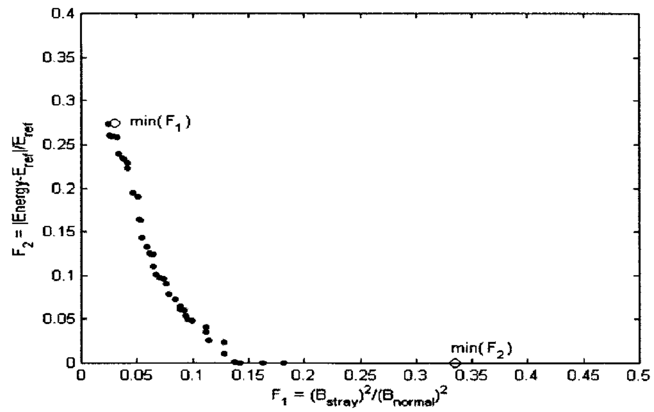

Fig. 4. Pareto-optimal points ($F_2 \times F_1$).

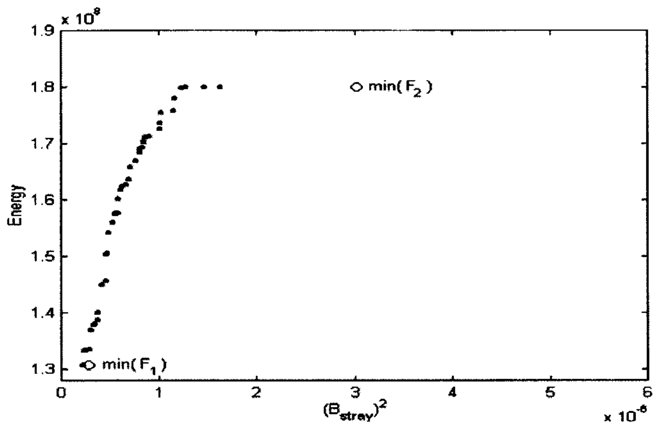

Fig. 5. Pareto-optimal points ($\text{Energy} \times B_{\text{stray}}^2$).

Mathematically, the multiobjective optimization problem for the SMES problem was stated as

$$\text{minimize} \quad \boldsymbol{f} = \{F_1, F_2\} = \left\{ \left(\frac{B_{\text{stray}}}{B_{\text{normal}}}\right)^2, \frac{|E_{\text{nergy}} - E_{\text{ref}}|}{E_{\text{ref}}} \right\} \quad (7)$$

where $B_{\text{normal}} = 3^*10^{-3}(T)$ and $E_{\text{ref}} = 1.8^*10^8(J)$.

The problem was solved considering three design variables in continuous case with a fixed current density equal to 22.5 A/mm^2. The nondominated points have been found using NSGA method (with roulette wheel selection, $p_c = 0.9$, $p_m = 0.05$, $N_{\text{pop}} = 30$ and $N_{\text{ger}} = 50$) coupled with a finite element code for energy and field calculations. The domain was subdivided in triangular elements of first order. The results are presented in Figs. 4 and 5.

VII. Conclusion

In this paper, a nondominated sorting genetic algorithm, proposed by K. Deb, is described and compared with four others algorithms using two test problems. In this comparison, the NSGA performs better than the others do, showing that it can be successfully used to find multiple Pareto-optimal solutions. Its application to the SMES problem show that it is reliable to solve multiobjective optimization in electromagnetics and that the TEAM22 Pareto-optimal front must be convex.

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