TECHNICAL NOTE:
INTEGRATING DESIGN AND PLANNING CONSIDERATIONS IN CELLULAR MANUFACTURING

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February 20, 1996
Revised: June 13, 1996

ABSTRACT

This paper presents a new model that integrates design and planning to prescribe a cost-effective cellular configuration that is responsive to real world considerations. The model incorporates practical engineering features such as the finite capacity of machines, use of alternative machines, multiple "copies" of a machine type, and limitations on cell size. It integrates design decisions, locating machines in each cell and identifying product families, with planning considerations, assuring that machine capacities are sufficient to produce required volumes and dealing with between cell movement to use alternative machines. Computational experience using a commercially available optimization package demonstrates that run time required to resolve problems of realistic size and scope can be quite reasonable.

ACKNOWLEDGEMENT

This material is based on work supported by the National Science Foundation on Grant numbers DDM-9114396 amd DMI-9500211. We also acknowledge an anonymous referee, whose helpful comments allowed us to strengthen an earlier version of this paper.

1.0 INTRODUCTION

The cell, which is becoming a widely used configuration in manufacturing, implements the concepts of group technology by producing each family of similar products on a group of dissimilar machines [Wemmerlov and Hyer (1987)]. Each cell specializes in processing a limited number of related products, thereby reducing setup requirements and offering the promise of improving material flow through reductions of work-in-process and cycle time.

A myriad of techniques have been proposed to design cells, but none seems to address the host of issues relevant to prescribing industrially relevant configurations. The process of designing cells is more an art relying upon ad hoc methods than a science based on comprehensive, quantitative methods.

The purpose of this paper is to introduce a model that integrates design and planning to prescribe a cost-effective cellular configuration that is responsive to real world considerations. The model incorporates practical engineering features such as the finite capacity of machines, use of alternative machines, multiple "copies" of a machine type, and limitations on cell size. It integrates design decisions, locating machines in each cell and identifying product families, with planning considerations, assuring that machine capacities are sufficient to produce required volumes and dealing with between cell movement to use alternative machines. While the focus of this paper is to introduce a new model, which may require philosophical changes in the way cells are conceptualized (i.e., the model does not advocate that each part family be produced in a single cell as do typical cell design approaches), it also demonstrates model application, reporting computational experience using a commercially available optimization package, showing that run time required to resolve problems of realistic size and scope can be quite reasonable.

Often, a robot is used to handle materials and tend machines in a cell. Since a robot - whether fixed position, track mounted, or gantry - has a work envelope of finite size, it can tend only a limited number of machines. This fact is not recognized by existing cell design procedures that advocate the completion - within *one* cell - of all operations required by a family of products. A more practical approach would restrict cell size and permit each product to visit more than one cell. Between-cell moves typically require longer, more expensive material handling transfers, and it may not be possible to maintain the level of flow control that is possible within a single cell. On the other hand, between-cell moves may promote the utilization of costly production machines and avoid the investment required to duplicate a certain type of machine by allowing the use of alternative (underutilized) machines.

Overviews of various aspects of cell design have been provided by Wemmerlov and Hyer (1986), Wemmerlov and Hyer (1987), Askin and Standridge (1993), Singh (1993), Offodile et al (1994), and Singh and Rajamani (1996). Research on cell design was initiated by a pragmatic analysis of manufacturing requirements [Burbidge (1971)]. Subsequently, a number of techniques using cluster analysis (i.e., similarity coefficients) [McAuley (1972), Witt (1980), Chan and Miller (1982), Waghodekar and Sahu (1984), Seifoddini and Wolfe (1986), Vakharia and Wemmerlov (1990)] or matrix manipulation (e.g., rank order clustering) [King and Nakornchai (1982), Chandrasekharan and Rajagopalan (1986), Vanelli and Kumer (1986)] have been proposed. The former requires the subjective use of somewhat arbitrary measures of similarity and the latter requires the designer to deal with bottleneck machines and exceptional parts using ad hoc methods [e.g., Srinivasan and Narendran (1991)].

More quantitative methods employing graph theory [Rajagopalan and Batra (1975)], mathematical programming [Askin (1986), Han and Ham (1986), Kusiak (1987), Choobineh (1988)], and heuristics [Askin and Subramamian (1986); Kumar, Kusiak and Vanelli (1986); Kusiak and Chow (1987)] have been proposed. Researchers have also have begun to explore practical issues related to cell design [e.g., Askin and Subramanian (1987), Ballakur and Steudel (1987), Heragu and Gupta (1994), Grznar et al (1994)] such as alternative routings [Kang and Wemmerlov (1993), Heragu and Chen (1995)]. To date, each model deals with a limited set of practical issues. In contrast, the model presented in this paper incorporates practical engineering features such as the finite capacity of machines, use of alternative machines, multiple "copies" of a machine type, and limitations on cell size.

The body of this paper is organized in three sections. The model is presented in section 2.0, and computational experience is described in section 3.0. Conclusions are given in section 4.0.

2.0 THE MODEL

The model, a mixed integer, linear program, is presented in this section. First, underlying assumptions and notation are presented.

2.1 Assumptions. We assume that the following information is known:

(1) Product Information

- (a) set of products to be produced
- (b) production volume of each product
- (c) sequence of operations for each product, and for each operation
  - (1) set of alternative machines
  - (2) processing time on each alternative machine
- (d) transfer batch size of each product

(2) Production Facility Information

- (a) number of machines of each type available
- (b) capacity (i.e., available time / period) of each machine

(3) Cell Configuration Information

- (a) number of cells to consider
- (b) upper bound on the size of (i.e., the number of machines in) each cell
- (c) location of each cell.

Furthermore, we assume that operations are deterministic (i.e., that production volumes and processing times are known with certainty).

The assumptions that operations are deterministic and that products to be produced over the planning period are known (1a) along with production volumes (1b), routings (1c), and the machines to be located (2a) are invoked by virtually all other papers that deal with the cell design problem. Similarly, it is expected that alternative machines capable of performing each operation (1c), transfer batch size (1d), and available machine capacity (2b) would be known when the cellular system is configured. If setup times are significant, machine capacities (2b) must be reduced to compensate, ensuring sufficient capacity.

It is expected that material handling devices to be used (e.g., robots) would be predetermined, for example, by considering product size and weight along with the positioning accuracy required. Furthermore, it is expected that good engineering practice would specify few different types of devices in the design, since use of a variety would increase maintenance, control, and training costs. These expectations underlie assumptions that an upper bound on the size of each cell (3b) and the number of cells (3a) are known. Finally, it is assumed that each cell is assigned a location in the layout (3c) so that inter-cell move costs can be based on estimates of the travel distance involved.

This problem arises in the context of devising a new, cellular configuration for an existing set of machines, perhaps installing new robotic handling equipment. Of course, the model could be applied in other contexts by repeated application (e.g., to evaluate alternative material handling equipments) or by extending the model (e.g., to prescribe the number and type of machines to be used in a new plant).

The model can deal with a number of products in reasonable run time, simultaneously prescribing cell composition and product families. Actually, problems of practical interest may not be too large; Drolet et al (1989) predict that future manufacturing systems may consist of no more than 20 machines.

2.2 Notation. Notation is summarized below for reader convenience:

Indices

k	= index for cells	= 1, 2, ..., $|K|$
o	= index for operations on product type p	= 1, 2, ..., $|O_p|$
m	= index for machines	= 1, 2, ..., $|M|$
p	= index for product types	= 1, 2, ..., $|P|$
q	= index for cells	= 1, 2, ..., $|K|$

Sets

A(po)	= set of alternative machines that can perform operation (po)
K	= set of cells

M = set of machines to be located
P = set of products to be produced
O_p = set of operations needed to produce product type p
$?_m$ = set of operations that can be processed by machine m

Known Parameters
c_{kq} = cost ($/foot) to move a transfer batch from cell k to cell q
D_{kq} = distance (feet) from cell k to cell q
L_p = transfer batch size (number of items) for product type p
S_m = processing cost ($/hour) on machine m
T_m = capacity (hours/period) of machine m
U_k = upper bound on the number of machines in cell k
V_p = production volume for product type p during the planning period (number of items)
$t_{po,m}$ = processing time (hours) per item for operation (po) on machine m

Derived Terms Known A Priori:
$t_{po,m}$ = processing time of operation (po) on machine m over the planning period
= $V_p\, t_{po,m}$
$C_{po,m}$ = processing cost of operation (po) on machine m
= $S_m\, t_{po,m}$
$B_{po,kq}$ = handling cost ($/period) moving from cell k to cell q after operation (po)
= $c_{kq}\, D_{kq}\, V_p / L_p$

Decision Variables:
x_{km} = 1 if machine m is assigned to cell k, 0 otherwise
$y_{po,km}$ = portion of the workload for operation (po) performed in cell k by machine m
$z_{po,kq}$ = portion of the workload for operation (po) that is performed in cell k and for which operation (p, o+1) is performed in cell q.

2.3 Formulation and Discussion. Three types of decision variables (x_{km}, $y_{po,km}$, and $z_{po,kq}$) are inter-related in the model. Binary integer variables, x_{km}, locate each machine in a cell, while the other two types deal with workload. Data that describe the products to be produced over the planning horizon and production volumes define the workload associated with each operation.

The portion of workload associated with operation (po) that is completed by each machine in the set of alternatives, A(po), is defined by decision variable $y_{po,km}$ ($0 \leq y_{po,km} \leq 1$), which is used to assure that machine capacities are observed. Multiple "copies" of a machine type can easily be handled by numbering each copy and defining the set of alternatives, A(po), appropriately. The model will prescribe the optimal locations for all copies, whether they are within one cell or among several cells.

If the entire workload for operation (po) is assigned to one machine, $m \in A(po)$, a processing cost of $C_{po,m}$ would be incurred. If only a portion, $y_{po,km}$, of the associated workload is assigned to machine m, a processing cost of $C_{po,m} \Sigma_{k \in K} y_{po,km}$ is incurred ($y_{po,km}$ is summed over k, since machine m can be located in any cell). Finally, if the total workload for operation (po) is completed in cell k, and the total for the subsequent operation (p, o+1) is completed in cell q ($q \neq k$), a between-cell material handling cost of $B_{po,kq}$ would be incurred. Decision variable $z_{po,kq}$ defines the portion of the workload that is to be produced in that manner and determines the between-cell material handling cost, $B_{po,kq}\, z_{po,kq}$, which is incurred by these two consecutive operations, assuming that handling cost is linearly proportional to the portion of workload $z_{po,kq}$. The cost of within-cell material handling is assumed, in comparison, to be negligible, although these costs could easily be included in the model if desired.

To highlight important trade-offs, we employ the objective of minimizing the costs of processing (using alternative machines) and of between-cell material handling movement. Model P_0 is now stated.

$$P_0\text{: Min } Z = \sum_{p \in P} \sum_{o \in \Omega_p} \sum_{m \in A(po)} 1\, C_{po,m} \sum_{k \in K} 2\, y_{po,km} + \sum_{p \in P} \sum_{o \in \Omega_p} \sum_{k \in K} \sum_{q \in K\_\{k\}} 3\, B_{po,kq}\, z_{po,kq} \qquad (1)$$

St.

$$\sum_{k \in K} 4\, x_{km} = 1 \qquad m \in M \qquad (2)$$

$$\sum_{m \in M} 5\, x_{km} \leq U_k \qquad k \in K \qquad (3)$$

$$\sum_{m \in A(po)} \sum_{k \in K} 6\, y_{po,km} = 1 \qquad p \in P,\ o \in O_p \qquad (4)$$

$$\sum_{(po) \in \Pi_m} 7\, t_{po,m}\, y_{po,km} \leq T_m\, x_{km} \qquad m \in M;\ k \in K \qquad (5)$$

$$\sum_{m \in A(po)} 8\, y_{po,km} - \sum_{q \in K\_\{k\}} 9\, z_{po,kq} + \sum_{q \in K\_\{k\}} 10\, z_{po,qk} - \sum_{m \in A(p,o+1)} 11\, y_{p,o+1,km} = 0$$
$$p \in P;\ o \in O_p;\ k \in K \qquad (6)$$

$$x_{km} = \{0, 1\} \qquad k \in K;\ m \in M \qquad (7)$$

$$y_{po,km} \geq 0 \qquad p \in P;\ o \in O_p;\ k \in K;\ m \in M \qquad (8)$$

$$z_{po,kq} \geq 0 \qquad p \in P;\ o \in O_p;\ k \in K;\ q \in K\backslash\{k\} \qquad (9)$$

The objective function in equation (1) incorporates the cost of performing each operation [each alternative machine may incur a different cost to complete operation (po)] as well as the cost of between-cell movement. Constraint (2) requires that each machine be located in some cell, and inequality (3) imposes the predetermined size of each cell.

The workload associated with operation (po) may be partitioned among applicable alternative machines, but equation (4) requires that the total workload be completed. Inequality (5) imposes machine capacity limitations. Binary and non-negativity restrictions are imposed by constraints (7) - (9).

Equation (6) defines the between-cell flow of materials for two successive operations, (po) and (p, o+1). If the portion of work completed in cell k for operation (po) [defined by the first summation in equation (6)] is the same as that for operation (p, o+1) [defined by the last summation in equation (6)], material flow between these operations can be accommodated entirely by within-cell movement, and both summations involving z variables will be zero. However, if the first summation is larger than the last, some workload [defined by the second summation in equation (6), which involves the $z_{po,kq}$ decision variables] must be moved between-cells (i.e., out of cell k and into cell q) for operation (p, o+1). Finally, if the first summation is less than the last, some workload [defined by the third summation in equation (6), which also involves the $z_{po,kq}$ decision variables] must be moved (between-cells) into cell q for operation (p, o+1). If between-cell movement is required for operation (p, o+1), it will entail movement either into or out of cell k, not in both directions, so either the second or third summation (or both) will equal zero, along with associated $z_{po,kq}$ variables. This logic that describes material flow is assured by the theory of linear programming; since columns associated with $z_{po,kq}$ and $z_{po,qk}$ are linearly dependent, at most one can be positive in any solution.

3.0 COMPUTATIONAL EXPERIENCE

In general, mixed integer programming problems are known to be NP-Hard [Nemhauser and Wolsey (1988)] so that, in the "worst case", even some relatively small problems may require prohibitive run times. Rather than addressing algorithmic issues, the purpose of this section is to provide some benchmark computational experience with the model using a commercially available optimization package. These tests show that the run time required to resolve problems of realistic size and scope can be quite reasonable and, thus, shed light on the "average case" performance that is typically of more interest in practical contexts.

This section describes the problems generated for testing purposes and discusses computational results. All tests were made on an IBM 3081-D using a "standard" implementation (i.e., using default settings) of the IBM Optimization Subroutine Library (OSL).

3.1 Test Problems. A set of twelve test problems, in which the number of products ($|P|$) was taken to be 5, 10, or 15 was generated. Drolet et al (1989) predicted that future manufacturing systems may not consist of more than 20 machines, so we used two levels, 15 and 20, for the number of machines ($|M|$). It seems practical to assume that cell size (U_k) be 3 to 7 machines, so two levels of the number of cells ($|K|$) were used: 4 and 5. Table 1 indicates the size of each test problem in terms of the numbers of products, machines, and cells; the resulting number of columns and rows in model P_0 are also noted.

The production volume for each product (V_p) was generated from a normal distribution with mean 700 and standard deviation 20, then rounded to the nearest integer. The number of operations for each product type ($|O_p|$) was generated from the discrete uniform distribution [2, 5]. The transfer batch size for all products was assumed to be 10 items ($L_p = 10$).

The processing cost per hour on each machine includes both labor and machine cost; we assumed S_m to be normally distributed with mean \$ 50/hour, standard deviation 10, and minimum \$ 20/hour. We determined machine capacity T_m assuming one shift operation (i.e., 40 hours/week) over 50 weeks/year.

The primary machine for each operation was selected at random from the 15 machines. The number of alternative machines that could perform each operation was generated from the discrete uniform distribution [0, 2], and specific alternatives were generated by random selection without replacement using the 15 machines. The processing time for operation (po), $t_{po,m}$, was generated randomly from the uniform distribution [0.2, 0.8]. Once generated, the processing time was increased by 10 percent for the first alternative machine (if any) and by 20 percent for the second alternative machine (if any).

Each machine must be located in one cell and each cell was assumed to require a 20 foot by 20 foot area. It was assumed that in problems involving four cells, the layout of cells would be a grid consisting of two rows and two columns; for problems involving five cells, the layout grid consisted of one row of three columns and a second row of two columns. The cost of moving a transfer batch between two cells was assumed to be c_{kq} = $1/foot. Parameters $B_{po,kq}$ were calculated using this value of c_{kq} and distances D_{kq} calculated assuming rectilinear travel between cell centers.

3.2 Test Results. Results are given in Table 2, which lists the solution value prescribed by OSL and the run time for each problem. These test cases do not represent any particular application, but they have been designed to reflect the size and scope of actual problems, and Table 2 shows that the proposed model can be solved within quite reasonable run times.

4.0 CONCLUSIONS

This paper presents a model for integrating design and planning considerations in cellular manufacturing. A number of practical engineering considerations that have been neglected by earlier approaches are incorporated, including machine capacity, multiple "copies" of a machine type, cell size limitations imposed by material handling equipment, the location of cells in a layout, the possible use of alternative machines for each operation, and prescription of material flow between and within cells.

Earlier approaches assume that all operations required by a product family should be completed within one cell. This principle may require extensive investment to duplicate machines. It also neglects the fact that within-cell material handling devices used to tend machines (e.g., robots) have a work envelope of finite size and typically cannot tend many machines. By considering material flow between successive operations, our model allows operations to be completed in the most cost effective manner, permitting use of machines in several cells to complete a product. Results establish an important economic trade-off between investing in additional machines or using alternative machines, perhaps located in different cells, at the cost of additional material handling. It is not necessary to identify product families *a priori* as in earlier approaches; products that are processed primarily in one cell form a family.

Computational tests using a commercially available optimization package to solve a set of problems devised to represent the size and scope of practical cases indicate that the model can be resolved within reasonable time. Therefore, it is expected that the model will be useful in actual industrial settings.

The model is relatively tractable; it involves numerous continuous variables but a relatively few binary variables. It is a prototype that could easily be embellished to incorporate broader issues such as prescribing the number of cells ($|K|$), the size of each cell (U_k), the layout of cells, and the types of material handling devices. In addition, other constraints could be incorporated, for example, to limit between-cell move time, the (footprint) area of machines located in a cell, the portion of workload done in more than one cell, or the utilization of machines (i.e., facilitating flow by affecting queueing). Queueing delays, cycle times, and scheduling operations are, however, most appropriately studied using a simulation model. In particular, the actual amount of time required to perform setups must be estimated using a model of time dependent operations. A model that deals with design and planning issues need only assure that sufficient capacity is provided to allow required setups as does our model.

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Table 1: Test Problem Sizes

Problem Number	Products (|P|)	Machines (|M|)	Cells (|K|)	Continuous Variables	Rows in (4)-(6)	Binary Variables	Rows in (2)-(3)
1	5	15	4	224	115	60	19
2	5	15	5	355	140	75	20
3	5	20	4	232	135	80	24
4	5	20	5	340	160	100	25
5	10	15	4	560	195	60	19
6	10	15	5	830	235	75	20
7	10	20	4	580	215	80	24
8	10	20	5	850	260	100	25
9	15	15	4	828	250	60	19
10	15	15	5	1210	300	75	20
11	15	20	4	884	270	80	24
12	15	20	5	1280	325	100	25

Table 2: Test Results

Problem Number	Solution	Time (CPU seconds)
1	259,448.0	84.3
2	259,440.2	193.3
3	248,773.1	112.3
4	248,798.2	279.4
5	575,126.7	91.5
6	573,780.6	315.1
7	555,068.5	274.4
8	555,039.0	1071.2
9	933,059.4	61.0
10	932,289.8	129.2
11	951,295.5	165.0
12	954,932.6	462.1