Partial State Feedback Control of Induction Motors with Magnetic Saturation: Elimination of Flux Measurements*

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Abstract

In this paper, we present a singularity-free, rotor position tracking controller for the full order, nonlinear dynamic model of the induction motor that includes the effects of magnetic saturation. By utilizing the π-equivalent saturation model, we design a observer/controller strategy that achieves semi-global exponential rotor position tracking and only requires stator current, rotor velocity, and rotor position measurements. Experimental results are included to demonstrate the efficacy of the proposed algorithm.

1 Introduction

Because of the induction motor's relative inexpensive cost, rugged construction, and inherent operating reliability, many industrial manufacturers have opted to use the induction motor as the desired actuator for many constant speed drive applications. However, if one considers using the induction motor for high performance rotor position or velocity tracking applications, many of the previously designed velocity setpoint control methodologies may prove to be inadequate for the demands of position/velocity tracking applications. In an effort to use the induction motor for demanding tracking applications while still benefitting from its desirable features (*i.e.*, low-cost, low-maintenance, *etc.*), many nonlinear control researchers have endeavored to overcome the numerous difficulties associated with induction motor's complex, coupled, high-order electrical dynamics. Specifically, *early induction motor control algorithms targeted the following dilemmas:* i) lack of rotor flux measurements, ii) performance degradation due to noisy velocity measurements, iii) compensation for parametric uncertainties, and iv) singularity avoidance. To combat these potential drawbacks,

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nonlinear control design tools, such as adaptive controllers, observed integrator-backstepping, and feedback linearization, have been utilized to improve the performance and applicability of the induction motor. We refer the reader to [6], [8], [10], [15], [16], and [17] and the references therein for a *brief* literature survey of the various techniques utilized for induction motor control. As evidenced by the extensive literature, the shift of the induction motor's applicability from that of a constant speed source to that of a versatile, high-precision position/velocity actuator has required the assimilation of advanced, nonlinear control techniques.

With the recent intensity of nonlinear control research focused at the induction motor, a pinnacle appears to have been reached concerning the induction motor control problem; however, the current culmination of knowledge concerning the induction motor control problem has mainly relied on the assumption of a linear magnetic circuit. That is, the electrical subsystem model predicts linearly increasing flux levels with correspondingly increasing current values. However from experimental tests, a divergence of the actual rotor flux signal from that predicted by a linear flux model at higher current values is observed (*i.e.,* the rotor flux exhibits saturation characteristics). Hence, to further improve induction motor performance, it seems one must address the concern of incorporating magnetic saturation into the electrical subsystem dynamics. In response to this concern, Brown *et al.* [1] analyzed the effects of saturation in the main flux path on the generalized equations of alternating current machines. In addition, Kerkman *et al.* [9] presented a generalized induction motor model, incorporating the spatially dependent or cross-saturation effects, in an arbitrary reference frame. Unfortunately, many of the proposed saturation models do not conform to the Blondel-Park (BP) transformation [12] that converts the three phase representation of the induction motor into an equivalent two phase representation. With the inability to perform the BP transformation (the conditions for the BP transformation are outlined in [14]), it is not easy to see how the previously designed nonlinear control techniques based on the linear magnetic circuit assumption can be extended to include saturation effects. In an attempt to address the concern of modeling accuracy while simultaneously facilitating the ease of control analysis/development (*i.e.,* BP transformable), Sullivan *et al.* [18] developed a new electrical subsystem model, based on a nonlinear π-equivalent, magnetic circuit representation that includes magnetic saturation effects and still meets the requirement of the BP transformation. The saturation model proposed in [18] views the induction machine as being constructed of an infinite number of infinitesimal teeth. Under the assumption of a smooth air-gap and perfectly sinusoidally distributed windings, each tooth is modeled as a nonlinear π-equivalent circuit representation with nonlinear functions introduced to capture the saturation effects (Note that if these nonlinear functions were omitted, then linear circuit representation is recovered). The structure of the introduced nonlinear functions in the electrical subsystem model of [18] allows one to apply the BP transformation to develop a two phase equivalent model in terms of the rotor and stator flux. By using the nonlinear π-equivalent saturation model, Sullivan *et al.* [19], [20] discussed how one might modify standard induction motor control to account for saturation effects. In an effort to further the control design effort, Feemster *et al.* [7] utilized the nonlinear π-equivalent saturation model proposed by Sullivan *et al.* [18] to develop a global asymptotic rotor position tracking controller despite parametric uncertainty associated with the stator and rotor resistance; however, the controller exhibited a potential drawback in the sense that the stator and rotor flux signals are assumed to be able to be reconstructed from stator and rotor current measurements (Feemster *et al.* [7] illustrated the ability to implement the proposed controller on a wound rotor induction motor outfitted with slip rings which afford rotor current measurements).

In this paper, we extend upon the work presented in [7] to eliminate the need for the reproduction of the stator and rotor flux signals in the control algorithm. Specifically, we develop an observer/controller strategy that achieves semi-global exponential rotor position tracking despite saturation effects in the electrical subsystem and the lack of flux measurements. The observer/control design is developed under the assumptions that the introduced functions utilized to capture the magnetic effects are first order differentiable and satisfy some relatively simplistic properties (These properties are detailed in Section III). The remainder of the paper is organized as follows. In Section II, we present the electromechanical model of an induction motor actuating a mechanical subsystem with magnetic saturation effects. The control objectives in addition to the various error signals required to facilitate the control development are then presented in Section VI. In Section V, two closed-loop observers for the stator and rotor flux signals are proposed to accommodate for the lack of flux measurements in the control design. A preliminary Lyapunov analysis is then presented to illustrate the stability of the observation error system while providing motivation for the structure of the proposed observers. Section VI details the development of a desired torque signal to promote rotor position tracking. In the following subsections, the desired stator flux trajectory and the rotor flux trajectory are designed to ensure that: i) the desired torque is delivered to the mechanical subsystem, ii) rotor flux tracking is promoted, and iii) control singularities are avoided. The stator voltage control inputs are then designed to ensure stator flux tracking. In Section VII, a composite Lyapunov stability analysis is utilized to: i) analyze the closed-loop stability of the plant and control signals, and ii) illustrate the rotor position tracking result. Experimental results are provided in Section VIII to illustrate the performance of the proposed controller.

2 Electromechanical Model

The electromechanical model [18] for an induction motor actuating a mechanical subsystem in the presence of magnetic saturation effects expressed in the stator-fixed (a-b) reference frame is assumed to take the following form

$$M_m\ddot{q} + W_m(q,\dot{q})\theta_m = \alpha_L \psi_s^T \mathbf{J}_2 \psi_r \tag{1}$$

$$\dot{\psi}_s = -R_s I_s + V_s \tag{2}$$

$$\dot{\psi}_r = -R_r I_r + n_p \dot{q} \mathbf{J}_2 \psi_r \tag{3}$$

$$I_s = \left(\kappa_s(\|\psi_s\|^2) + \kappa_l\right)\psi_s - \kappa_l \psi_r \tag{4}$$

$$I_r = \left(\kappa_r(\|\psi_r\|^2) + \kappa_l\right)\psi_r - \kappa_l \psi_s \tag{5}$$

where $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$ represent the rotor position, rotor velocity, and rotor acceleration signals, respectively, M_m denotes the equivalent system inertia (including rotor inertia), $W_m(q,\dot{q}) \in \Re^{1\times p}$ represents a regression vector comprised of measurable quantities, $\theta_m \in \Re^{p\times 1}$ denotes a vector of constant system parameters, $\psi_s(t) = \begin{bmatrix} \psi_{sa}(t) & \psi_{sb}(t) \end{bmatrix}^T \in \Re^{2\times 1}$ and $\psi_r(t) = \begin{bmatrix} \psi_{ra}(t) & \psi_{rb}(t) \end{bmatrix}^T \in \Re^{2\times 1}$ are vectors representing the stator and rotor flux signals, respectively, R_s and R_r represent the stator and rotor phase resistances, respectively, n_p denotes the number of pole pairs of the induction motor, $V_s = \begin{bmatrix} V_{sa} & V_{sb} \end{bmatrix}^T \in \Re^{2\times 1}$ denotes the stator voltage control input vector, $I_s(t) = \begin{bmatrix} I_{sa}(t) & I_{sb}(t) \end{bmatrix}^T \in \Re^{2\times 1}$ and $I_r(t) = \begin{bmatrix} I_{ra}(t) & I_{rb}(t) \end{bmatrix}^T \in \Re^{2\times 1}$ represent the stator current and rotor current vectors, respectively, $\kappa_s(\|\psi_s\|^2)$ and $\kappa_r(\|\psi_r\|^2)$ are scalar saturation functions used to capture the flux saturation characteristics, L_l represents the leakage inductance, $\alpha_L = \dfrac{n_p}{L_l}$, $\kappa_l = \dfrac{1}{L_l}$ are constant model parameters related to the electric circuit parameters, $\|\cdot\|$ denotes the standard Euclidean norm of a vector, and the matrix $\mathbf{J}_2 \in \Re^{2\times 2}$ is a skewed-symmetric matrix defined as follows

$$\mathbf{J}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{6}$$

It is assumed that $W_m(q, \dot{q})$ is first order differentiable and that $W_m(q, \dot{q})$, $\dot{W}_m(q, \dot{q}) \in L_\infty$ if $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in L_\infty$.

3 Saturation Model

Typically, induction motor controllers are designed under the assumption of a linear magnetic circuit. That is, the electric subsystem model predicts linearly increasing flux levels with correspondingly increasing current values. For the case when the magnetic circuit is assumed to be linear, the saturation functions in (4) and (5), denoted by $\kappa_s\left(\|\psi_s\|^2\right)$ and $\kappa_r\left(\|\psi_r\|^2\right)$, are set to constant values. As one would expect, experimental data indicates a divergence in the flux levels at higher current levels of those predicted by the linear model. Hence, the functions of (10) have been introduced to more accurately reflect the phenomenon of saturated flux levels at higher current values. While there are many possible saturation models (*e.g.*, see [20] and [7]), the control design/stability analysis presented in this paper requires that the saturation functions be constructed to satisfy the following properties.

Property 1: The saturation functions must be first order differentiable and satisfy the following relationships

$$\kappa_i(x) > 0 \quad \forall x \in \Re^1 \tag{7}$$

$$\text{if } x > y, \text{ then } \kappa_i(x) > \kappa_i(y) \quad \forall x, y \in \Re^1 \tag{8}$$

where the subscript i is set equal to s or r.

Property 2: If (9) holds, then it easy to show that the following inequality holds

$$(\kappa_i(\|\mathbf{x}\|)\mathbf{x} - \kappa_i(\|\mathbf{y}\|)\mathbf{y})^T(\mathbf{x} - \mathbf{y}) \geqslant \kappa_i(0)(\mathbf{x} - \mathbf{y})^T(\mathbf{x} - \mathbf{y}) \qquad \forall \mathbf{x}, \mathbf{y} \in \Re^2 \tag{9}$$

where the subscript i is set equal to s or r. This property along with (7) is exploited in the subsequent stability analysis.

Property 3: The saturation model must be constructed in such a manner to ensure that $\kappa_i\left(\|\psi_i\|^2\right)$, $\dfrac{\partial \kappa_i\left(\|\psi_i\|^2\right)}{\partial \|\psi_i\|^2} \in L_\infty$ over all possible values of $\psi_i(t)$ where the subscript i is set equal to s or r. This property is utilized during the stability analysis of Section VII.

Remark 1 *While many complicated functions can be constructed to satisfy Property 1 and 2, one simple selection for the saturation function is given as follows*

$$\kappa_i\left(\|\psi_i\|^2\right) = \frac{\alpha_{i1}\sinh\left(\alpha_{i2}\sqrt{\|\psi_i\|^2}\right)}{\sqrt{\|\psi_i\|^2}} \tag{10}$$

where the subscript i is set equal to s or r, and α_{i1}, α_{i2} denote positive, scalar constants that must be experimentally determined. Since the selected saturated function is written in terms of flux