LINEAR FILTRATION OF RANDOM ELECTRIC POWER PROCESSES: UNIFICATION OF CALCULATIONS

Eduard G. Kourennyi, George Isber, Lidiya V. Chernikova

The Donetsk State Technical University, 58 Artyoma Street, Donetsk 340000, Ukraine
Tishreen University, Lattakia, Syria

Problem description. Linear filters are component of mathematical models in electric power engineering. Filter models reaction $Y(t)$ of considered object to an input process $X(t)$. It is described by linear differential *n*-degree equation (in flickermeter $n = 11$ [1]).

For short we will consider random process $X(t)$ as stationary. It has following characteristics: average value $\overline{X}$, dispersion D_X, correlative function $K_X(\tau)$ or spectral density $G_X(\omega)$, which don't depend on time *t*. Formulas for calculation of appropriate characteristics of process after filtration are wellknown (e.g. [2]):

$$\overline{Y}(t) = \overline{X} \cdot H(t), \tag{1}$$

$$K_Y(\tau, t) = \int_0^t \int_0^t H'(v) H'(w) K_X(v - w + \tau) dv dw, \tag{2}$$

where $H(t)$ is response of filter to unit step function, *v* and *w* are variables of integration, τ is argument. Dispersion

$$D_Y(t) = K_Y(0, t). \tag{3}$$

Determination of correlative function entails cumbersome computations, since it is necessary to find response $H(t)$ by transfer function $W_n(s)$ and to fulfil double integration in each problem. This paper suggests the methods of calculations unification which allow to bring solution to determination of characteristic equation roots $s_1, s_2, \ldots, s_n$ without consequent integration.

Two types of unification. The idea of unification consists in the fact that initial model (Fig.1, a) is substituted for linear aperiodic links connected in parallel (bellow – simply "links"). Each link is characterized by transfer coefficient k and time-constant *J* or parameter $\gamma = 1/J$. Link has transfer function

$$W_J(s) = \frac{k}{Js + 1}$$

and response

$$h(t) = k(1 - e^{-\gamma t}). \tag{4}$$

Such substitution allows to represent process on filter output in form of sum of "partial" reactions $y(t)$ of each link to input process $X(t)$.

After feeding of stationary process on filter input transitional random process begins to proceed on filter output, and for $t \to \infty$ steady state set in. If according to problem conditions study of process $Y(t)$ on whole interval *t* from 0 to ∞ is required, then form of input process is retained without changes and filter is substituted for links in number *n* (Fig. 1, b). Such unification by type of process will be named partial unification.

If only steady state is considered, then total unification is possible (sing ~). In this case the process $X(t)$ is represented as result of passing of white noise $\xi(t)$ through hypothetical filter with transfer function

$W_X(s)$ and m-degree of appropriate characteristic equation (Fig.1, c). In consequence system with transfer function

$$\tilde{W}_N(s) = W_X(s) \cdot W_n(s) \tag{5}$$

has degree $N = n + m$ and it is represented by the same links number (Fig.1, d). Process $Y(t)$ is equal to sum of N partial reactions $\tilde{y}(t)$.

Partial unification. Let filter transfer function be equal ratio of two polynomials: $Q(s)$ with degree less n and $P_n(s)$ with n-degree. Denominator can be represented as product

$$P_n(s) = a_0 \prod_{r=1}^{n} (s - s_r),$$

where a_0 is coefficient to s^n.

For lack of zero and divisible roots of characteristic equation $P_n(s) = 0$ expansion

$$W_n(s) = \sum_{r=1}^{n} \frac{c_r}{s - s_r} \tag{6}$$

is correct. In expression (6)

$$c_r = \frac{Q(s_r)}{P_n^r(s_r)} = \left. \frac{Q(s_r)}{a_0 \prod_{r=1}^{n} (s - s_r)} (s - s_r) \right|_{S=S_r} . \tag{7}$$

Taking out value ($-s_r$) from denominator in (6), we will obtain following expression

$$W_n(s) = \sum_{r=1}^{n} \frac{c_r/(-s_r)}{s/(-s_r) + 1},$$

from which formulas (8), (9) for determination of parameters r-link result.

$$J_r = -1/s_r, \qquad \gamma_r = -s_r \tag{8}$$

$$k_r = -c_r/s_r = c_r \cdot J_r = c_r/\gamma_r \tag{9}$$

Usually filter consists of elementary links, that is why determination of roots doesn't meet with difficulties even for high n. For example denominator of transfer function of flickermeter filter is product of quadratic polynomial and two linear polynomial [1], that eliminates necessity in solution of biquadratic characteristic equation.

According to (1) and (4) average value of r-partial reaction

$$\bar{y}_r = \bar{X} k_r (1 - e^{-\gamma_r t}).$$

Average value of reactions sum

$$\overline{Y}(t) = \overline{X}\sum_{r=1}^{n} k_r (1 - e^{-\gamma_r t}) . \tag{10}$$

Partial dispersion of *r*-reaction is calculated with regard for (4) by formula (2) for $\tau = 0$:

$$D_r = \frac{k_r^2}{\gamma_r^2} \int_0^t \int_0^t e^{-\gamma_r u} e^{-\gamma_r v} K_X(u - v) du dv . \tag{11}$$

As one and the same process is fed on links inputs as partial reactions are correlative. Intercorrelative moment between *r* and *l* reactions is calculated by formula

$$b_{rl}(t) = \frac{k_r k_l}{\gamma_r \gamma_l} \int_0^t \int_0^t e^{-\gamma_r u} e^{-\gamma_r v} K_X(u - v) du dv , \tag{12}$$

which is analogous to formula (11).
Decision dispersion is

$$D_Y(t) = \sum_{r=1}^{n} D_r(t) + 2 \sum_{r \neq l} b_{rl}(t) . \tag{13}$$

Total number of items in formula (13) is equal

$$n + c_n^2 = n(n-1)/2 .$$

Integration gives solution in finite form for correlation functions which are met in practice. For instance in case of exponential function

$$K_X(\tau) = D_X e^{-\alpha|\tau|} \tag{14}$$

with parameter α we can obtain:

$$D_r(t) = D_X \frac{k_r^2}{\gamma_r(\alpha^2 - \gamma_r^2)} \left[\alpha - \gamma_r + (\alpha + \gamma_r) e^{-2\gamma_r t} + 2\gamma_r e^{-(\alpha + \gamma_r)t}\right], \tag{15}$$

$$b_{rl}(t) = D_X k_r k_l \gamma_r \gamma_l \left[\frac{1 - e^{-(\gamma_r + \gamma_l)t}}{\gamma_r + \gamma_l} \left(\frac{1}{\alpha - \gamma_r} + \frac{2}{\alpha - \gamma_l}\right) - \frac{1 - e^{-(\alpha + \gamma_r)t}}{(\alpha + \gamma_r)(\alpha - \gamma_l)} - \frac{1 - e^{-(\alpha + \gamma_l)t}}{(\alpha - \gamma_r)(\alpha + \gamma_l)}\right]. \tag{16}$$

In steady state

$$D_r = D_X \frac{k_r^2}{\gamma_r(\alpha + \gamma_r)} , \tag{17}$$

$$b_{rl} = D_X k_r k_l \gamma_r \gamma_l \frac{\gamma_r + \gamma_l + 2\alpha}{(\gamma_r + \gamma_l)(\alpha + \gamma_r)(\alpha + \gamma_l)} . \tag{18}$$

Parameters of links can be complex values, but as the result of summation in (10) and (13) the members with multiplier $j = \sqrt{-1}$ are reciprocally cancelled. In the same way solution for $\tau > 0$ is found. However for short it isn't given here.Thus the partial unification is brought to summation of algebraical expression without integration.

Total unification. For steady state characteristics of process $Y(t)$ can be calculated not only by formulas (1) and (3) for $t \to \infty$, but also through spectral density $G_X(\omega)$ of input process and through amplitude-frequency function (AFF) $A(\omega)$ of filter:

$$\overline{Y} = \overline{X} \cdot A(0), \qquad D_Y = \int_0^\infty G_X(\omega) \cdot A^2(\omega) d\omega, \tag{19}$$

where ω is angler frequency.

Proceeding from the condition of dispersion reproduction or spectral density $G_X(\omega)$, we will find type and parameters of linear filter which transforms white noise. White noise has constant spectral density which is equal value c_ξ and so for this condition realization AFF of hyposetical filter should have following form:

$$A_X(\omega) = \sqrt{G_X(\omega)/c_\xi}\,. \tag{20}$$

By form of this AFF filter with appropriate transfer function $W_X(s)$ and m-degree is selected. In following computations value c_ξ is cancelled therefore in calculations it is advisable to accept $c_\xi = 1$. However for totality we will keep this value.

As example we will consider process with exponential correlative function (14) which has spectral density

$$G_X(\omega) = D_X \frac{2\alpha}{\pi(\alpha^2 + \omega^2)}\,.$$

If we substitute this expression for $G_X(\omega)$ in formula (20), then we will obtain:

$$A_X(\omega) = \sqrt{\frac{2\alpha}{c_\xi \pi(\alpha^2 + \omega^2)}}\,.$$

Linear aperiodic link has AFF of such form:

$$A_X(\omega) = \frac{k_X}{\sqrt{1 + \omega^2 J_X^2}}\,,$$

where J_X and k_X are parameters of link. From comparison of those two expressions we will find

$$J_X = 1/\alpha, \qquad k_X = \sqrt{2\alpha D_X / c_\xi}\,. \tag{21}$$

In that way process with exponential correlative function is result of white noise passing through aperiodic link with transfer function

$$W_X(s) = \frac{k_X}{J_X s + 1}\,. \tag{22}$$

Total unification is fulfil in the same way as partial unification, but links number increases to N and formulas (8) and (9) are used for transfer function (5). As a result parameters $\tilde{k}$ and $\tilde{J}$ or $\tilde{\gamma}$ of links are determined.

For simplification of computations it is advisable to accept that the average value $\bar{\xi}$ of white noise is equal zero and to determine decision value $\bar{Y}$ by formula (10) for $t \to \infty$:

$$\bar{Y} = \bar{X} \sum_{r=1}^{n} k_r .$$

Otherwise we have to calculate the average value by formula

$$\bar{\xi} = \bar{X} \sum_{r=1}^{n} k_r \Big/ \sum_{r=1}^{N} \tilde{k}_r .$$

Correlative function of while noise is proportional to delta-function:

$$K_\xi(\tau) = \pi c_\xi \delta(t)$$

Substitution of this expression in (11) and (12) taking into account (4) for $\tau = 0$ and $t \to \infty$ gives partial dispersion

$$\tilde{D}_r = \frac{1}{2} \pi c_\xi \tilde{k}_r^2 \tilde{\gamma}_r \qquad (23)$$

and intercorrelative moment

$$\tilde{k}_{rl} = \pi c_\xi \tilde{k}_r \tilde{k}_l \frac{\tilde{\gamma}_r \tilde{\gamma}_l}{\tilde{\gamma}_r + \tilde{\gamma}_l} . \qquad (24)$$

According to (13) decision dispersion

$$D_Y = \pi c_\xi \left(\sum_{r=1}^{N} \frac{1}{2} k_r^2 \tilde{\gamma}_r + 2 \sum_{r \neq l} \tilde{k}_r \tilde{k}_l \frac{\tilde{\gamma}_r \tilde{\gamma}_l}{\tilde{\gamma}_r + \tilde{\gamma}_l} \right). \qquad (25)$$

Note should be taken that total unification can’t be used for nonstationary zones since transition to white noise is correct only for stationary random processes.

Particular cases. If polynomials $Q(s)$ and $P_n(s)$ have equal roots, then expressions (s-s_r) with equal roots are cancelled after expansion of $Q(s)$ on multipliers. For that reason the number of links is decreased by unification.

If characteristic equation $P_n(s)$ has divisible roots, then formula (7) for these roots gives infinity. In this connection we have to solve the problem in consecutive order: in the beginning only one of the same links is taken into account in transfer function, and solution for next link is found after determination of correlative function on output of such “truncated” filter etc.

In the case when there is zero root and there is ideal integrating link in denominator of transfer function in the beginning solution for filter without this link is obtained and then link with transfer function $1/s$ is added to.

Example. Let us assess possibility of capacitor banks (CB) feeding from bus bars of substation of metallurgical plant. Thyristor converters are connected up bus bars. Mains voltage contains the first harmonic and interference $u(t)$ which is nonsinusoidal component. Interference $u(t)$ is stationary random process with exponential correlative function and zero average value. Effective value of the first harmonic equal 100 % while interference has dispersion D_u = 120 (%)^2 and parameter α = 5000 s^{-1} [3]. According to [4] increase in current of CB from voltage nonsinusoidality hasn't to exceed 30%. Transfer function of CB has form [3]:

$$W_n(s) = g\frac{T_1 s + 1}{T_2^2 s^2 + T_3 s + 1},$$

in which $g = 3{,}72 \cdot 10^{-9}$ Sm; $T_1 = 1{,}57 \cdot 10^5$ s; $T_2 = 1{,}26 \cdot 10^{-5}$ s; $T_3 = 4{,}02 \cdot 10^{-6}$ s.

The average value of CB current component from interference equal zero, therefore it is sufficient to determine dispersion D_i of this component.

Let us solve this problem by conventional way characteristic equation has two complex conjugate roots.

$$s_{1,2} = -\lambda \pm j\beta,$$

where

$$\lambda = T_3 / 2T_2^2 = -1{,}27 \cdot 10^4 \text{ s}^{-1}; \qquad \beta = \sqrt{4T_2^2 - T_3^2} / 2T_2^2 = 7{,}86 \cdot 10^4 \text{ s}^{-1}$$

With regard for this transfer function

$$W_n(s) = g\frac{T_1 s + 1}{T_2^2 (s - s_1)(s - s_2)}.$$

We will have find response from Laplas inverse conversion tables by function $W_n(s)/s$:

$$H(t) = \frac{g}{T_2}\left(\frac{a_1}{s_1} e^{s_1 t} + \frac{a_2}{s_2} e^{s_2 t} + a_3\right),$$

in which

$$a_1 = \frac{T_1 s_1 + 1}{T_1 (s_1 - s_2)}, \qquad a_2 = \frac{T_1 s_2 + 1}{T_1 (s_2 - s_1)}, \qquad a_3 = \frac{1}{T_1 s_1 s_2}.$$

Taking into account absolute values of argument τ in formula (14) let us write formula (3) for dispersion of CB current from interference as

$$D_i(t) = \frac{g^2 D_u}{T_2^2} \int_0^t \left(a_1 e^{s_1 w} + a_2 e^{s_2 w}\right) dw + \int_0^w \left(a_1 e^{s_1 v} + a_2 e^{s_2 v}\right) \cdot e^{-\alpha(v-w)} dv + \int_w^t \left(a_1 e^{s_1 v} + a_2 e^{s_2 v}\right) \cdot e^{-\alpha(w-v)} dv$$

Integration by this formula doesn't meet with difficulties, but it is cumbersome. Results of integration are presented in Fig. 2, curve 1.

At steady state for $t \to \infty$ dispersion D_i = 52680 (%)^2. The same value is obtained by formula (19), for application of which in expression for $W_n(s)$ we will substituted s for $j\omega$ and find AFF. In consequence formula (19) have form

$$D_i = \frac{2}{\pi}\alpha g^2 D_u \int_0^\infty \frac{1 + \omega^2 T_1^2}{\left(\alpha^2 + \omega^2\right) \cdot \left[\left(1 - \omega^2 T_2^2 T_3^2\right)^2 + \omega^2 (T_2 + T_3)^2\right]} \cdot d\omega.$$

Integration of this expression by deduction theorem doesn't meet with difficulties, but it is cumbersome too.

Let us turn our attention to the methods of unification. For case of partial unification we will have found parameters of two links by formulas (7)-(9):

$$J_{1,2} = -\frac{1}{s_{1,2}} = \frac{1}{\lambda \mp j\beta} = \frac{\lambda \pm j\beta}{\lambda^2 + \beta^2} = \frac{1}{2}\left(T_3 \pm j\sqrt{4T_2^2 - T_3^2}\right);$$

$$k_1 = \frac{g(T_1 s_1 + 1)}{T_2^2 s_1 (s_2 - s_1)}; \qquad k_2 = \frac{g(T_1 s_2 + 1)}{T_2^2 s_2 (s_1 - s_2)}.$$

Substitution of correlative function parameters and parameters of two links in formulas (15), (16) and (13) without integration gives curve 1 at Fig. 2.

In case of total unification the third root $s_3 = -\alpha$ is added to two roots s_1 and s_2. For $c_\xi = 1$ formulas (7)-(9) give following parameters of three inertial links:

$$\tilde{J}_{1,2} = J_{1,2}, \qquad \tilde{J}_3 = 1/\alpha,$$

$$\tilde{k}_1 = \frac{-gz(T_1 s_1 + 1)}{T_\xi T_2^2 s_1 (s_1 - s_2)(s_1 - s_3)}, \quad \tilde{k}_2 = \frac{-gz(T_1 s_2 + 1)}{T_\xi T_2^2 s_2 (s_2 - s_1)(s_2 - s_3)}, \quad \tilde{k}_3 = \frac{-gz(T_1 s_3 + 1)}{T_\xi T_2^2 s_3 (s_3 - s_1)(s_3 - s_2)},$$

where $T_\xi = 1/\alpha$; $z = \sigma\sqrt{\frac{2}{\pi c_\xi \alpha}}$; σ is standard deviation.

Substitution of these values in formulas (17), (18) and (13) gives the same value of dispersion at steady state. If the erroneously use formulas (15) and (16) for finite value of t, then dispersion dependence on time (Fig.2, curve 2) will be substantially differ from actual dependence. Only for $t \to \infty$ curves 1 and 2 aim at one and the same value 52680 $(\%)^2$.

First harmonic of CB current, as one of voltage, is equal 100 %. Effective value of total current

$$I = \sqrt{100^2 + D_i} = 250\%$$

much exceeds permissible value 130 %, therefore feeding of CB from these bus bars is inadmissible.

Conclusion. Unified methods of calculation allow significantly to simplify the problem solution of linear filtration by reduction of them to algebraical computations instead of cumbersome integration.

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Information about authors

1. **Eduard G. Kourennyi**, professor, doctor of science, IAEE member. Professor of the chair of electric power supply of industrial plants and cities (EPG) of the Donetsk National Technical University (DonNTU) and consultation of the Koncern "Stirol".

2. **George Isber,** professor, ph-D, Tishreen University, Lattakia, Syria.

3. **Lidiya V. Chernikova**, ph-D, assistant professor of the chair EPG of DonGTU.

ANNOTATION

The unified analytic method for calculation of indices of random electric processes after linear filtration is described. The suggested method was named "partial reactions" method. It is based on considered linear system presentation in the form of connected in parallel linear inertial links. The "partial reactions" method application is shows on example of estimation of capacitor installation electromagnetic compatibility.