## Life Tables for Arkansas for 2000 By Race and Gender: Methodology and Construction



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## Overview of Life Tables

Life tables combine mortality rates of a population into a model that makes it possible to study longevity from a purely statistical viewpoint (Chiang, 1984). There are two types of life tables. Current life tables reflect the combined mortality experience of a single population at a particular time. Current tables are also referred to as period life tables. Life tables that record the mortality of a particular population over their lifetime are called cohort life table. Such life tables are based on the mortality experience of a cohort from the birth of the first cohort member to the death of the last cohort member. Life tables can either be complete providing data by every single years, or they can be abridged life tables providing data by age intervals (Anderson, 1999). The life tables produced by the Demographic Research Group at the Institute for Economic Advancement, University of Arkansas at Little Rock are abridged current life tables.

Life tables are similar in appearance because they have the same life table functions, and differ only in the way the functions they are calculated. Life table definitions and functions include:
x to $\mathrm{x}+\mathrm{n}$ : the period of life between two exact ages.
${ }_{\mathrm{n}} \mathrm{m}_{\mathrm{x}}$ : the age-specific death rate.
${ }_{n} q_{x}$ : the proportion of persons in the cohort alive at the beginning of the age interval ( $x$ ) and dying before reaching the end of the age interval $(x+n)$.
$l_{\mathrm{x}}$ : the number of persons living at the beginning of the indicated age interval (x) out of a beginning cohort of 100,000 live births.
${ }_{n} \mathrm{~d}_{\mathrm{x}}$ : the number of deaths between the age x and $\mathrm{x}+\mathrm{n}$, out of the number of persons alive at the beginning of that age interval.
${ }_{n} L_{x}$ : the number of person-years that would be lived within the age interval ( x to $\mathrm{x}+\mathrm{n}$ ) by an assumed cohort of 100,00 births.
$T_{x}$ : the total of person-years that would be lived after the beginning of the indicated age interval (x) by the cohort of 100,000 births.
$\mathrm{e}_{\mathrm{x}}$ : the expectation of remaining life time (in years) that a person who survives to the beginning of the indicated age interval.

The life table functions are demonstrated in the accompanying Female Life Table for Arkansas.

TABLE 1
Female Life Table
Arkansas 2000

| Age | $\mathrm{n}_{\mathrm{x}}$ | ${ }_{\mathrm{n}} \mathrm{q}_{\mathrm{x}}$ | $\mathrm{l}_{\mathrm{x}}$ | $\mathrm{n}_{\mathrm{n}} \mathrm{d}_{\mathrm{x}}$ | ${ }_{\mathrm{n}} \mathrm{L}_{\mathrm{x}}$ | $\mathrm{T}_{\mathrm{x}}$ | $\mathrm{e}_{\mathrm{x}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $<1$ | 0.0090 | 0.0086 | 100000 | 862 | 99244 | 7771725 | 77.7 |
| 1 to 4 | 0.0005 | 0.0020 | 99138 | 194 | 396186 | 7672481 | 77.4 |
| 5 to 9 | 0.0002 | 0.0012 | 98943 | 122 | 494434 | 7276295 | 73.5 |
| 10 to 14 | 0.0003 | 0.0014 | 98821 | 137 | 493790 | 6781861 | 68.6 |
| 15 to 19 | 0.0007 | 0.0036 | 98684 | 360 | 492593 | 6288071 | 63.7 |
| 20 to 24 | 0.0008 | 0.0041 | 98325 | 404 | 490693 | 5795477 | 58.9 |
| 25 to 29 | 0.0010 | 0.0049 | 97921 | 481 | 488495 | 5304785 | 54.2 |
| 30 to 34 | 0.0013 | 0.0064 | 97440 | 621 | 485767 | 4816290 | 49.4 |
| 35 to39 | 0.0015 | 0.0077 | 96819 | 746 | 482373 | 4330523 | 44.7 |
| 40 to 44 | 0.0021 | 0.0104 | 96073 | 1002 | 478051 | 3848150 | 40.1 |
| 45 to 49 | 0.0030 | 0.0150 | 95070 | 1424 | 472065 | 3370098 | 35.4 |
| 50 to 54 | 0.0042 | 0.0209 | 93646 | 1956 | 463710 | 2898034 | 30.9 |
| 55 to 59 | 0.0071 | 0.0348 | 91690 | 3192 | 451054 | 2434323 | 26.5 |
| 60 to 64 | 0.0119 | 0.0581 | 88498 | 5143 | 430523 | 1983269 | 22.4 |
| 65 to 69 | 0.0193 | 0.0924 | 83355 | 7700 | 398741 | 1552746 | 18.6 |
| 70 to 74 | 0.0285 | 0.1335 | 75655 | 10100 | 354426 | 1154005 | 15.3 |
| 75 to 79 | 0.0424 | 0.1924 | 65556 | 12614 | 297626 | 799579 | 12.2 |
| 80 to 84 | 0.0687 | 0.2941 | 52942 | 15571 | 226635 | 501953 | 9.5 |
| $85+$ | 0.1357 | 1.0000 | 37371 | 37371 | 275317 | 275317 | 7.4 |

Table 1 is an abridged current life table. As such, it gives a cross-sectional view of mortality of the population based on the age-specific death rates. The table serves as an example of the life table functions.

Column 1: Age groups and intervals of life between two exact ages. The first age interval $(<1)$ represents the age-cohort of new born who have not yet reached their first birthday. The second age-cohort ( 1 to 4 ) is the period of life between the first birthday and the fifth birthday. In general, an age interval is the exact age interval between $x$ and $x+n$, where $n$ equals five years for the age intervals 5-84. The final age-interval ( $85+$ ) is the 85 and over age-cohort.

Column 2: ${ }_{\mathrm{n}} \mathrm{m}_{\mathrm{x}}$ is the age-specific death rate. These rates are calculated by dividing the number of deaths in an age-cohort ( x to $\mathrm{x}+\mathrm{n}$ ) by the midyear population of that age-cohort. For example in the $85+$ cohort, a death rate of .1357 means that out of every 10,000 persons in this age-cohort 1,357 die during a given year.

Column 3: ${ }_{\mathrm{n}} \mathrm{q}_{\mathrm{x}}$ is the proportion of persons in an age-cohort alive at the beginning of the age interval ( x ) who will die before reaching the end of the age-cohort. $(x+n)$. This is an estimate of the probability of an individual of the exact age $x$ will die during the next $\mathrm{x}+\mathrm{n}$ years. Thus, a value of .0064 in the 30-34
age-cohort means that for every 10,000 persons alive at the beginning of this age interval 64 will die before reaching their $35^{\text {th }}$ birthday.

Column 4: $1_{x}$ is a survivor rate or the number of persons alive at the beginning of an age interval out of the initial cohort of 100,000 newborn babies. The value of 97,440 in the 30 to 34 age-cohort means that out of the 100,000 new births, 97,440 persons will survive to reach an age of at least 30 .

Column 5: ${ }_{\mathrm{n}} \mathrm{d}_{\mathrm{x}}$ is the number of death in an age interval out of the cohort of 100,000 newborn babies. For example in the 30 to 34 age interval, 621 persons will die out of the 97,440 persons in this age-cohort before reaching the age of 35 .

Column 6: ${ }_{n} L_{x}$ is the number of person-years lived in the age interval $x$ to $x+n$ by the survivors of the assumed 100,000 births. In the 30 to 34 age interval, 96,819 persons would survive to their $35^{\text {th }}$ birthday and contribute 482,373 $(=96819 * 5)$ person-years. Of the 97,440 person reaching their $30^{\text {th }}$ birthday, 621 would die before reaching their $35^{\text {th }}$ birthday, but they would live varying periods of less than 5 years. In this case, this group contributed 1,672 person-years.

Column 7: $\mathrm{T}_{\mathrm{x}}$ is the total number of person-years lived beyond the age of x . Thus, for the 30 to 34 female age-cohort, females would live 4,816,290 person-years beyond their $30^{\text {th }}$ birthday.

Column 8: $\mathrm{e}_{\mathrm{x}}$ is the expected number of years on average that a person will live after reaching their x birthday. Thus, a female reaching their $30^{\text {th }}$ birthday can expect to live another 49.4 years.

## Overview: Life Tables for Arkansas: 2000

The life tables produced by the Demographic Research Group at the Institute for Economic Advancement at University of Arkansas, Little Rock are abridged current life tables. The tables are based on data from the Census 2000 enumeration, deaths in Arkansas from 1990 to 2000, and births over the 1997-2000 period. The website provides two ways to retrieve life table functions. If one wants a complete life table for a county or state, then the appropriate section of the website is the state and county life tables. In this section, life tables by gender and race for a county or at the state level can be extracted. For comparison of a life table function across counties, the life table functions section provides a means to extract county and state specific life table functions.

## Construction of Abridged Life Tables for Arkansas 2000

In order to construct the current county life tables for Arkansas by gender and race, many assumptions were made about the survival and mortality of the populations. This section of the document discusses these underlying assumptions; the computation formulas used to construct the life tables, and primary data sources. For ease of presentation, this discussion does not distinguish between race, gender, or county as was done in the actual construction of the life tables.

Age-specific death rate: Age-specific death rates are a ratio of the number of deaths in an age-cohort to the population of the age-cohort. The average annual number of deaths over the 1990-2000 was used to derive an estimate of deaths by age-cohort enumeration (Arkansas Department of Health). The age-specific population data were taken from Census 2000 enumeration (U.S. Bureau of the Census). In the computation of the agespecific death rates, the following formula was used

$$
{ }_{\mathrm{n}} \mathrm{~m}_{\mathrm{x}}=\left(\sum_{\mathrm{t}=1}^{\mathrm{T}}{ }_{\mathrm{n}} \mathrm{D}_{\mathrm{xt}} / \mathrm{T}\right) /{ }_{\mathrm{n}} \mathrm{POP}_{\mathrm{x}}
$$

where ${ }_{n} \mathrm{D}_{\mathrm{xt}}$ is the number of death in the age-cohort between the ages of x to $\mathrm{x}+\mathrm{n}$ in year $t$, T is the number of years (1990 to 2000), and ${ }_{\mathrm{n}} \mathrm{POP}_{\mathrm{x}}$ is the number of persons in the age-cohort.

Probability of dying: A method suggested by T.N.E. Greville (was used to compute this mortality rate for all age-cohorts except for the under one age-cohort (Shryock and Siegel, 1980). In this latter age-cohort, a birth-cohort method as outlined by Elizabeth Arias (2002) was utilized. This technique requires infant births, deaths, and a separation factor. A separation factor is defined as the proportion of infant deaths in a given year occurring to infants born in the previous year. Infant births for 1999 and 2000 were taken from the Arkansas Vital Statistics, and death rates were averaged over the 1990 to 2000 period (Arkansas Department of Health). For the separation factor by race and gender, the United States rates are applied to the Arkansas birth and death rates (Arias Elizabeth, 2002). To compute the probability of dying for age-cohorts $1-85+$ and under the assumption that the death rates reflect midyear death rates, the following formula was used

$$
{ }_{\mathrm{n}} \mathrm{q}_{\mathrm{x}}=\left({ }_{\mathrm{n}} \mathrm{~m}_{\mathrm{x}}\right) /\left(1 / \mathrm{n}+{ }_{\mathrm{n}} \mathrm{~m}_{\mathrm{x}}\left(.5+\mathrm{n} / 12\left({ }_{\mathrm{n}} \mathrm{~m}_{\mathrm{x}}-.095\right)\right) .\right.
$$

For the birth cohort method, the probability of the under one age-cohort was calculated as

$$
\mathrm{q}_{\mathrm{o}}=\left(\sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{D}_{\mathrm{ot}} / \mathrm{T}\right)\left((1-\mathrm{f}) / \mathrm{B}_{\mathrm{t}}+\mathrm{f} / \mathrm{B}_{\mathrm{t}-1}\right)
$$

where f is the separation factor, $\mathrm{B}_{\mathrm{t}}$ is the number of births at time t , and $\mathrm{D}_{\mathrm{ot}}$ is the number of infant death at time $t$, and $T$ stands for years (1990 to 2000).

Survivor Rate: to compute survivor rates by age-cohort, the traditional practice is to set the number of survivors in the first age-cohort at $100,000\left(=1_{0}\right)$, and then compute the number of remaining survivors at the beginning of age interval x with the formula

$$
1_{x}=l_{x-n}\left(1-n q_{x-n}\right) .
$$

Number of deaths: the expected number of deaths occurring between x to $\mathrm{x}+\mathrm{n}$ was computed from the survivor rate using the following formula

$$
{ }_{\mathrm{n}} \mathrm{~d}_{\mathrm{x}}=1_{\mathrm{x}-\mathrm{n}} \mathrm{q}_{\mathrm{x}} .
$$

Person-years lived: Under the assumption that the life table death rates reflect the population death rate, the number of person-years lived by age-cohort was calculated by one of three formulas:

Under one age-cohort

$$
\mathrm{L}_{\mathrm{o}}=\mathrm{fl}_{\mathrm{o}}+(1-\mathrm{f})\left(\mathrm{l}_{\mathrm{o}}\left(1-\mathrm{q}_{\mathrm{o}}\right)\right),
$$

Age-cohorts 5-85

$$
{ }_{\mathrm{n}} \mathrm{~L}_{\mathrm{x}}={ }_{\mathrm{n}} \mathrm{~d}_{\mathrm{x}} / \mathrm{n}_{\mathrm{n}} \mathrm{~m}_{\mathrm{x}}
$$

85+ age-cohort

$$
\mathrm{L}_{85+}=\mathrm{l}_{85+} / \mathrm{m}_{85+} .
$$

Total number of person-years beyond the age of $x$ : The value was determined by summing the ${ }_{n} L_{x}$ for age-cohorts above x . So, for the $\mathrm{i}^{\text {th }}$ age-cohort, the number of person- years lived beyond the age of $\mathrm{i}^{\text {th }}$ age-cohort.

$$
\mathrm{T}_{\mathrm{i}}=\sum_{\mathrm{i}=\mathrm{x}}^{\mathrm{W}}{ }_{\mathrm{n}} \mathrm{~L}_{\mathrm{i}}
$$

where W is the last age-cohort or in this case the $85+$ age-cohort.
Expectation of remaining life: To compute the expected number of years remaining for a person reaching age x , the following calculation was made

$$
\mathrm{e}_{\mathrm{x}}=\mathrm{T}_{\mathrm{x}} / \mathrm{l}_{\mathrm{x}} .
$$

Special Assumptions for missing data: In several instances during the construction of databases by gender and race, there were no values in some of the age-cohorts. This happens frequently in counties with small overall populations or where there are few if any persons of a particular ethnic group. In most cases, there would be no deaths or no persons in an ethnic group. When this happen, an appropriate state value was substituted for the missing value. The substitution made possible the completion of the life table computations. This means that in the Life Tables for Arkansas: 2000, the value for many of a county's life function has been replaced with a corresponding state value.

NA (not applicable): For several ethnic groups, the county counts were too small to ensure meaningful calculations. As a rule-of-thumb, when a life expectancy deviates by more than $10 \%$ of the state's life expectancy, the value was assumed not to be meaningful. In that case, a value of NA is reported for life expectancy, total personyears, the number of person-years, and the number of deaths.

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