Chapter 20

TEXTURE ANALYSIS

Image Analysis Sourcebook
Professor John Keith Bedown PhD

The Gray Level Surface.

One method for texture analysis is to use the Gray Level Surface Representation of an actual surface (Fig. 117). Here the (x,y) coordinates locate the point in the plane and the gray level (G) indicates the height of that coordinate point in the surface. Alternatively, the gray level at the point (x,y) can be used to represent the gray level itself.

The gray level image can therefore represent a surface topology or a microstructure.

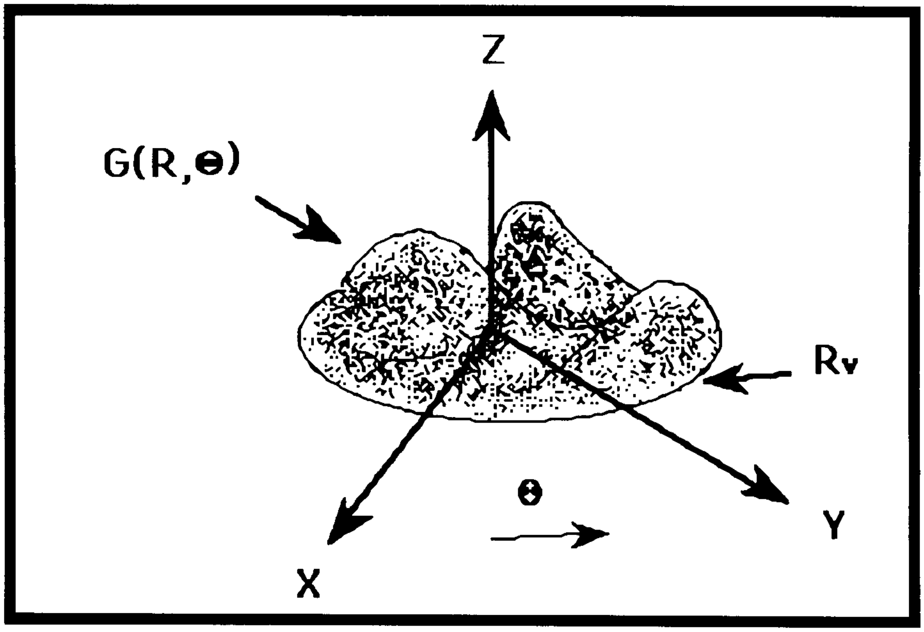

Fig. 117
The Gray Level Extended Surface. Cylindrical coordinates. G (R,θ) is the Gray Level Function, R is the radius of the fields of view, the angle is θ

Texture.

The definition of texture is:

> **Texture is defined as the pattern of the Gray Level Distribution (GLD) within the boundary of the field of view.**

Texture Definition

This operational definition is similar to the definitions for shape referred to in Chapter two in that it tells us what to do in order to analyze surface texture.

The first step is to note the gray levels at each coordinate (x,y) point of the image and determine the Gray Level Distribution (GLD). The next step is to elucidate the appropriate boundary function, in this case the Gray level boundary function. Following this we seek to extract invariant descriptors that represent significant image surface features.

The Gray Level Boundary Function.

Variational calculus has been used to determine the Gray Level Boundary function. This is the Bessel-Fourier boundary function. It has the form:

$$G(R,\theta) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(A_{m,n}Cosm\theta + B_{m,n}Sinm\theta)J_m(\Omega_{m,n}\frac{R}{R_v})$$

The Gray Level Boundary Functionin which

G(R,Q)	= the Gray Level Function, angle Q
Am,n,Bm,n	= the Bessel-Fourier coefficients
Jm	= the first kind mth order Bessel function
Ωm,n	= the zero root of the Bessel function
Rv	= the radius of the field of view

The important textural features include:

- Bessel-Fourier coefficients
- Moments of the Gray Level Distribution
- Partial rotational symmetry
- Partial translational symmetry
- Coarseness
- Contrast
- Roughness
- Regularity

Moments of the Gray Level Distribution.

Consider an image represented by a GLD as shown in Fig. 118.

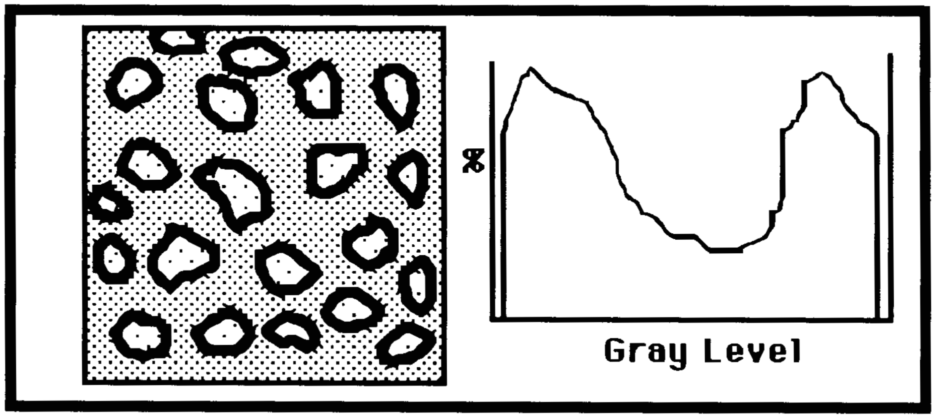

Fig. 118
An image with light objects is a darker background and a Gray Level Distribution (GLD)

If the total number of pixels N = n1 + n2 + n3 + ...+ nl .

In which the Gray levels are 1,2,3.....,l

Then the mean and standard deviation of the GLD are:

Some Statistical Properties of the Gray Level Distribution

$$\textit{The Mean, } \overline{G} = \sum_{j=1}^{I} i_j P_j$$

$$\textit{The Standard Deviation} = \sqrt{\sum_{j=1}^{I} (i_j - \overline{G})^2 P_j}$$

In which

$$P_j = n_i / N_i$$

$$n_i = \textit{number of pixels at gray level } i$$

and

$$\sum_{j=1}^{I} P_j = 1$$

Partial Rotational Symmetry.

In Chapter nine we saw how one can describe the partial rotational symmetry Cm of a closed boundary. For example, the three-fold partial rotational symmetry of a closed boundary is 0< Cm <1. For example, C3 of an equilateral triangle = 1 and the C3 of a square <1.

In a similar manner, one can describe the partial rotational symmetry of the image texture. For example, the images shown in Fig. 119 illustrate two-fold, three-fold and four-fold rotational symmetry of texture. This description is stated in terms of the Bessel-Fourier coefficients.

The texture is composed of discrete gray levels. An R-fold symmetry operation can be performed by comparing the gray level at (R,Q) with the gray level at (R,Q + ΔQ). The rotational angle can be chosen by changing the integer R (R = 1,2,3....m).

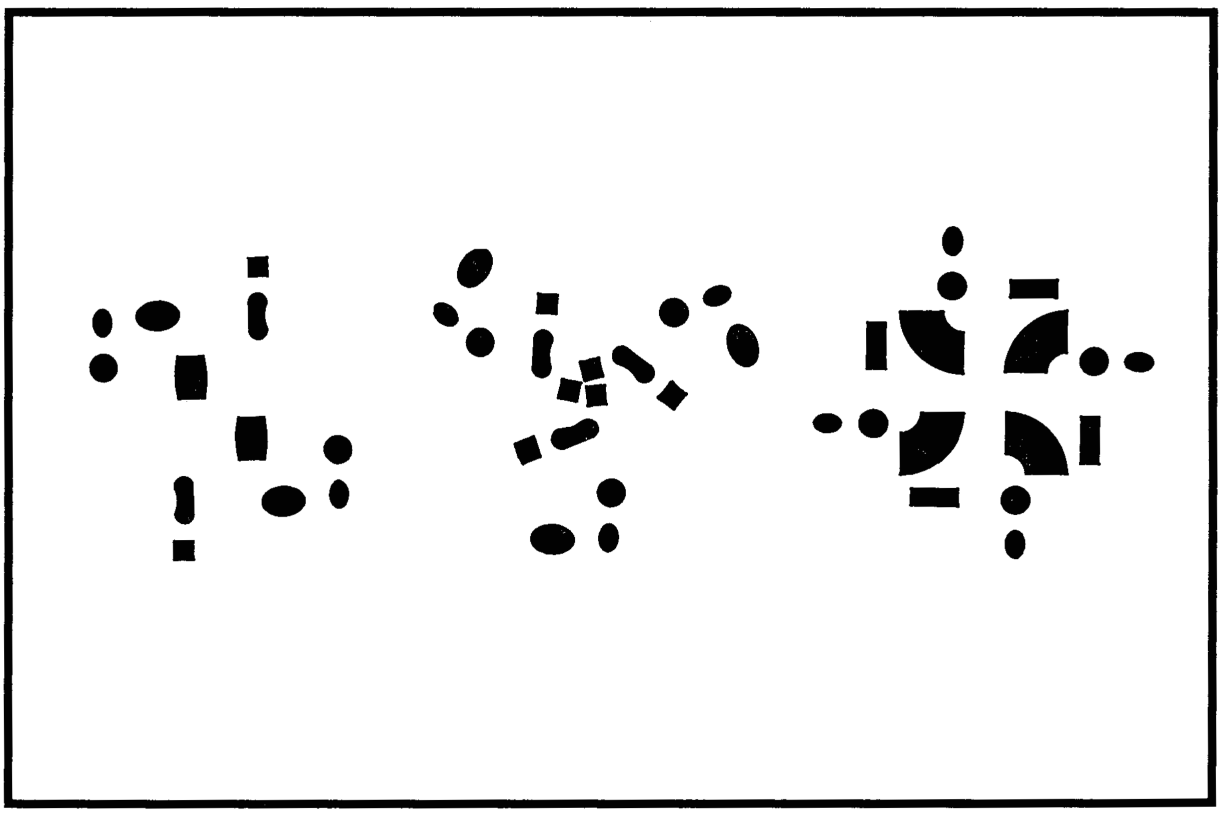

Fig. 119
Illustrating the concepts of ratational textural symmetry.
First: two-fold. Middle: three-fold. Last: four-fold.

This provides the index Crt, the Partial Rotational Symmetry Index of the texture as follows:

Crt the Partial Rotational Symmetry Index of the Texture

$$Crt = \frac{\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (H_{mn})R^2 \cos m(2\pi/R)\, J_m{}^2(\Omega_{m,n})}{\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (H_{mn})R^2 J_m{}^2(\Omega_{m,n})}$$

in which
Crt = the Partial Rotational Symmetry Index
R = 1,2...
(Hmn)R2 = (Am,n)R2 + (Bm,n)R2

Partial Translational Symmetry.

A Partial Translational Symmetry is recognized when gray levels are compared along the radius. So that $G(R,Q)$ is compared with $G(R + \Delta R,Q)$.

The Partial Translational Symmetry Index of the texture is $0< Ctt <1$ and is defined as follows:

Ctt The Partial Translational Symmetry Index of Texture

$$Ctt = \frac{\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \{H^2{}_{m,n} J^2{}_m(\Omega_{m,n}) - [A_{m,n}A_{m-1,n} + B_{m,n}B_{m-1,n}] J^2{}_m(\Omega_{m-1,n})\ \Delta r/2Rv}{2\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \{H^2{}_{m,n} J^2{}_m(\Omega_{m,n})}$$

In which Ctt is the Partial Translational Symmetry Index and the other symbols are as previously indicated.

The Coarseness.

The coarseness, Fcrs, is defined as the difference in gray level among four neighboring points around a specific (x,y) coordinate point.

Analysis has shown that the coarseness is related directly to the partial rotational and translational symmetries as follows:

The Coarsness of the Texture

$$F_{crs} = 4 - 2[C_{rt} + C_{tt}]$$

The Contrast.

In the case of a distribution in which the values of the variables are concentrated near the mean, the distribution is said to have a large kurtosis (Fig 120).

The contrast, Fcon, is defined as the kurtosis as follows:

The Contrast of the Texture

$$Fcon = \mu^4/\sigma^4$$

In which $\mu 4$ is the fourth moment of the GLD about the mean and s 2 is the variance.

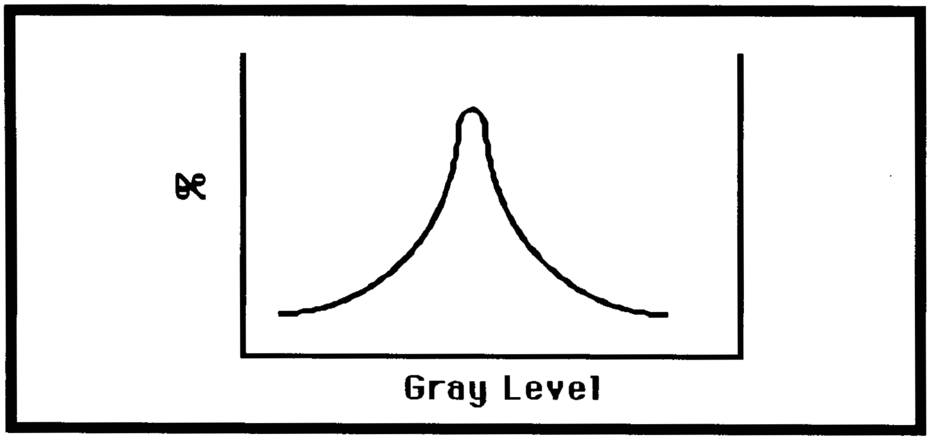

Fig. 120
Illustrating a GLD with a large kurtosis.

The Roughness.

The roughness, Frou, is related to the contrast and coarseness as follows:

The Roughness of the Texture

$$F_{rou} = F_{crs} + F_{con}$$

The Regularity.

The regularity, Freg, is a function of the variation of the textural elements over the image. It is defined as:

The Regularity of the Texture

$$F_{reg} = \sum_{t=1}^{n} C_{tt} + \sum_{r=1}^{m} C_{rt}$$

If there are repetitive patterns throughout the image, it has a high regularity. If the image varies widely with little or no repeating pattern, it has low regularity.

An image with high rotational and high translational symmetries will have high regularity.