Skew Effect Parameters of AC Machines with Skewed Slots

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Abstract—The Skewing of rotor slots to eliminate the ill effects of slot harmonics causes a non-uniform distribution of flux density along the stack. It reduces the coupling between the stator and rotor mmfs introducing additional skew leakage. Three different equivalent circuits are derived from first principles to consider the skew leakage reactance as a parameter. The reduction in coupling between the harmonic mmfs increases harmonic leakage. A valid modification to the existing method is suggested to estimate the leakage reactance under saturated conditions for a skewed rotor induction motor.

Index Terms— Equivalent circuits of skewed machines, saturation effects, Skewing, skew leakage reactance.

I. INTRODUCTION

Small and medium size squirrel cage induction motors are normally constructed with rotors having skewed slots mainly with a view to minimizing the crawling and cogging torques as a result of slot harmonics, which cannot be eliminated by distribution and short chortling of the windings. Skewing of me slots in turn reduces the magnetic coupling between the stator and rotor mmfs resulting in extra leakage in the motor. The skew also causes a non-uniform distribution of air gap flux along the stuck. This distribution is modified by the saturation of the magnetic circuit as well as the axial fluxes that exist at the periphery of the air gap surface along the stack [1], [2]. A knowledge of the non-uniform distribution would be essential for a complete understanding of all flux dependent phenomena occurring in the machine.

Extra leakage flux and associated leakage reactance should be a accounted for in the equivalent circuit as a parameter in a systematic manner so that the performance of the motor having skewed slots can be predicted accurately at the design stage itself.

On the basis of the non-uniform mmf distribution along the stack several methods are available for estimating the skew leakage reactance taking in to account the leakage flux saturation in the motor [3], [4]. Angst [3] extended the method of Agarwal and Alger [5] to skewed rotor induction motors. The method presented considers all the components of leakage flux that constitute to the saturation of leakage flux. However the effect of saturation of main flux along the stack is neglected in his analysis. Moreover arbitrariness in fixing the saturated flux density for the idealized magnetization curve further adds to the underestimation.

The literature provides several alternate equivalent circuits to account for the skew leakage reactance. Butler and Birch [6] based on coupled circuit theory, discussed in detail the validity and practical utility of the alternate methods available for taking in to account the effects of skew in the variation of the parameters of the equivalent circuit. The effect of the slot skew on the parameters of the equivalent circuit can be deduced from basic principles by defining different stator to rotor turns ratios for the purpose of referring rotor current to the stator circuit [4j.

The aims of the present paper are three fold:

1. To determine and study the non-uniform distribution of the flux density distribution along the stack.

2. In the light of the above to arrive at suitable representation of the skew leakage reactance in the equivalent circuit.

3. To consider the saturation effects on the leakage reactance including skew leakage reactance and suggest a favorable method to determine the saturated leakage reactance of the motor as a function of voltage at all loads.

II. VARIATION OF AIR GAP FLUX DENSITY ALONG THE STACK

In a skewed rotor induction motor resultant air gap mmf is a function of axial length owing to varying phase difference between the stator and rotor mmfs (Fig. I) along the stack. The performance of the magnetic circuit changes along the stack length, as there is variation in the relative positions of stator and rotor teeth. Both mmf and the permeances variations cause axial variations in the air gap flux density distribution. The flux density distribution is affected to a large extent by saturation of the magnetic circuit. Also the axial flux introduced by the peripheral component of the bar currents the distribution. However it was established that the permeance variations along the axis and the axial flux referred to above have very little effect on the flux distribution along the stack. The air gap mmf is the resultant of stator and rotor mmfs. In a skewed rotor induction motor the air gap mmf at an axial location x from the centre of the stack can be written as

$$A_r(x) = A_{1r} + A_{2t(r)}[e^{j\mu\beta x/L}] \quad 0 \le x \le L/2 \qquad (1)$$

The stalor mmf

$$A_{1r} = \sqrt{2}I_1 \frac{m_1}{\pi}\frac{1}{r}\frac{T_{ph}}{p}k_{dpr} \qquad (2)$$

The rotor mmf of the t^{th} harmonic is induced in the rotor by the r^{th} harmonic of the stator

$$t = \frac{N_2}{P}K_2 + r \quad K_2 = \pm 1, \pm 2, \ldots. \qquad (3)$$

$$A_{2t(r)} = \frac{N_2}{P\pi}\frac{1}{t}\frac{\sqrt{2}I_1}{2}$$

and the corresponding flux density in the air gap is

$$B_{r(x)} = \frac{\mu_0}{g}A_{r(x)} \qquad (4)$$

and flux per pole then becomes

$$\phi_{r(x)} = \int_{-L/2}^{+L/2}\int_{y=0}^{\pi} B_{r(x)} \sin\frac{r\pi}{P}y\,dy\,dx \qquad (5)$$

substituting for $B_{r(x)}$ and integrating we get

$$\phi_{r(x)} = \frac{2\sqrt{2}\mu_0}{\pi}\frac{L}{rg}\frac{m_1}{r}\frac{T_{ph}}{rP}k_{dpr}\left[I_1 + \frac{N_2 K_{dtr}}{2m_1 T_{ph} k_{dpr}} I_{2r}\right] \qquad (6)$$

The flux along the stack can be estimated along the stack using the above equation.

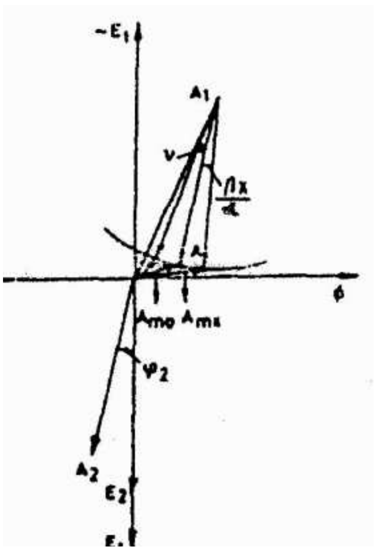

Fig. 1 Phasor diagram showing various mmfs at an axial location x from the center of the slack.

Further it is also possible to obtain the resultant mmf distribution along the stack using the phasor diagrams of the stator and the rotor mmfs at various axial locations. Fig.2 depicts such a phasor diagram at an axial location x from the center of the stack. The phase difference between the stator and the rotor mmfs at a location x from the center of the stack is

$$\varphi_{12x} = \gamma + \beta x/L \tag{7}$$

where y is the angle between the mmfs at the center of the slack and b is the angle of the skew in electrical degrees. For the rotation in the same direction of the skew the value of x from the observer is assumed to be positive (Fig.l). The effective air gap mmf

$$A_{mx} = \left[A_1^2 + A_2^2 - 2A_1A_2\cos\phi_{12x} \right]^{1/2} \tag{8}$$

The values of the primary mmf A_1 , secondary mmf A_1 and y are determined from the conventional equivalent circuit of the induction motor assuming that it represents the characteristics of the machine at the center of the stack. From the magnetization curve of the machine, the corresponding flux density distribution is obtained for any operating condition. The variation of the flux density and the air gap mmf along the core length for the blocked rotor an the full load conditions of the motor are depicted in Fig.3

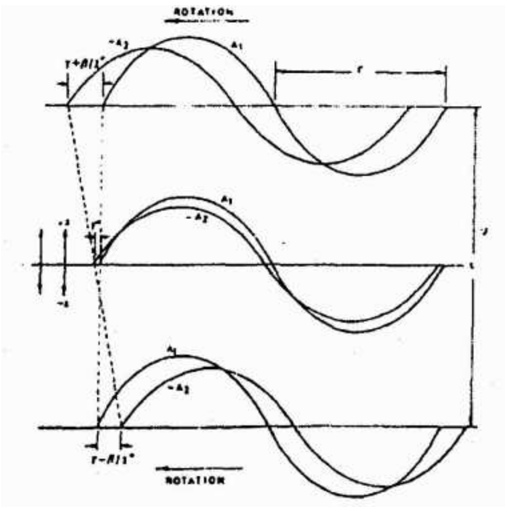

Fig.2 Convention used to determine ehe sign of X measured from the center

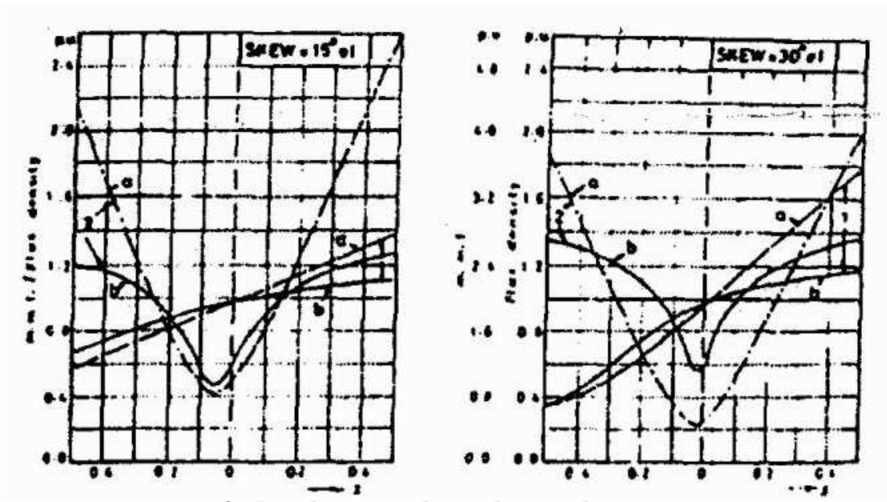

Fig 3 Variation of Flux density along the stack.

III. EQUIVALENT CIRCUIT OF A SKEWED ROTOR INDUCTION MOTOR

Voltage and current relationships of the stator circuit using (6), derived for the flux along the stack of the machine the induced emf in the stator due to the r^{th} harmonic component *to* is obtained from

$$E_{1r} = -jX_{mr}\left[I_1 + \frac{N_2 K_{skr} I_{2r}}{2m_1 T_{ph} k_{dpr}} \right] \tag{9}$$

Where X_w can be recognized as the main reactance of the stator winding giving by,

$$X_{mr} = \frac{4\mu_0}{\pi} m_1 f \frac{T_{ph}^2}{p} \frac{\tau L}{g} \left(\frac{k_{dpr}}{r} \right)^2 \qquad (10)$$

The stator voltage equation can now be written as

$$V_1 = \left(R_1 + jX_1\right) I_1 - \sum_{r=1}^{\infty} E_{1r} \qquad (11)$$

Voltage current relationships of the rotor circuit referring to Fig. 4, which represents the developed view of the rotor the V-I relationships of the ohmic resistance and leakage reactance of the rotor mesh formed by the two consecutive bars and the corresponding end ring segments are given respectively by

$$V_{h1} = R_{2r} I_{2r} \left(2\sin\left(rp\alpha_s/2\right)\right)$$

$$V_{h2} = -jX_{2r} I_{2r} \left(2\sin\left(rp\alpha_s/2\right)\right) s_r \text{ where, } \alpha_s = \frac{2\pi}{N_2}$$

$$R_{2r} = R_{2br} + 2R_{2re}\left(1/\left(4\sin^2\left(\alpha_s pr/2\right)\right)\right)$$

$$X_{2r} = X_{2br} + 2X_{2re}\left(1/\left(4\sin^2\left(\alpha_s pr/2\right)\right)\right)$$

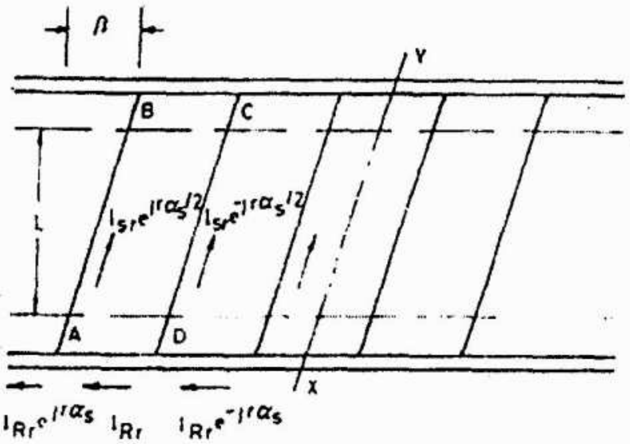

Fig.4 Developed view of the rotor.

Derivation of the equivalent circuits

In the conventional equivalent circuit, the extra skew leakage reactance due to skew is represented both in the stator and rotor circuits using the current transformation ratio as

$$a_{sr} = \frac{I'_{2r}}{I_{2r}} = \frac{N_2}{2m_1} \frac{1}{T_{ph} k_{dpr}} \qquad (13)$$

The stator v-i relationship, under the assumption that R_2 and x_2 are negligibly small compared to x_{h2r} for higher order harmonic frequeneies are obtained as

$$V_1 = \left(R_1 + jX_1\right) I_1 + jI_1 \sum X_{mr}\left(1 - k_{skr}\right) + jI_1 \sum^{\infty} \left(X_{mr}\left(1 - k_{skr}\eta_r^2\right)\right) k_{skr} + jX_m k_{skr}\left(I_1 + I'_2\right) \qquad (14)$$

Similarly rotor equation can be written as

$$0 = I'_2\left(\frac{R'_{2r}}{s_r} + jX'_{s2r}\right) + jX_{mr}k_{skr}\left(I_1 + I'_2\right) + jX_{mr}I'_2\left(\frac{1}{\eta^2} - k_{skr}^2\right) \qquad (15)$$

The equivalent circuit as shown in Fig. 5

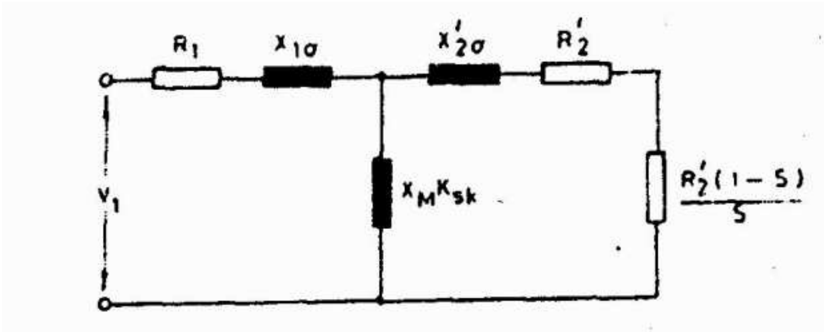

Fig 5 Conventional equivalent circuit of a skewed rotor induction motor.

The simplified v-i relationships are

$$V_1 = I_1\left(R_1 + jX_{s1}\right) - jI_2'X_{sr} + j\left(I_1 + I_2'\right)X_{sd}$$
$$+ jX_m\left(I_1 + I_2'\right) + \Delta V_1 + \Delta V_2$$
$$\text{where, } \Delta V_1 = -jI_2'\sum_{r=1}^{\infty} X_{mr}\left(1 - \eta_r^2 X_r^2\right)$$
$$\Delta V_2 = j\left(I_1 + I_2'\right)\sum_{r\neq 1}^{\infty} X_{mr}\left(1 - \eta_r^2 X_r^2\right)$$
$$V_2 = 0 = I_2'\left(\frac{R_2'}{s} + jX_{s2}'\right) + jX_m\left(I_1 + I_2'\right) \qquad (16)$$

The equivalent circuit is depicted in Fig.6

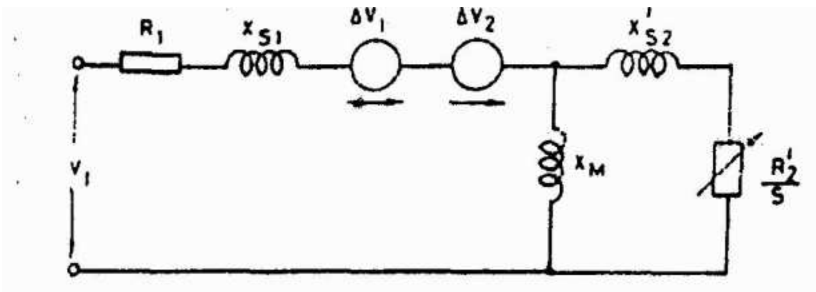

Fig. 6 Equivalent circuit of a skewed rotor induction motor considering the effect of skew in the slator circuit only.

The effect of skew is introduced in terms of skew factor

$$k_{skr} = \frac{\sin\left(r\beta/2\right)}{r\beta/2} \qquad (17)$$

From the development of the equivalent circuits it is clear that the skew has an effect similar to that of a distributed winding because of the similarity of the expressions for skew factor and distribution factor. The equations also show that the skew besides introducing skew leakage, increases the harmonic leakage of the motor. While performing similar to distributed winding a difference is obvious that the distribution of the winding reduces the effect of individual phase belt harmonics. The skew on the other hand increases the harmonic leakage. This can be attributed to the basic nature of the skew that it reduces coupling between the stator and rotor mmfs. The interaction between the stator and rotor mmfs becomes reduced and as a consequence the harmonic leakage increases. This effect is more on the rotor side than on the stator side. There can be an increase of 10% to 15% in harmonic leakage as a whole in the machine besides an extra component called skew leakage reactance.

IV. SATURATION EFFECTS IN A SKEWED ROTOR INDUCTION MOTOR

Recognizing the saturation as a total flux phenomenon the saturation factors arc evolved taking into consideration the total leakage flux in the saturated path. The total flux entering the area A_{S1} (Fig. 8b) is

$$\phi_{s1} = \phi_{rs} + \phi_{z} + \phi_{sks(x)} \tag{18}$$

Using the well adopted expressions for ϕ_{n}, the slot leakage, $\Phi.$, zigzag leakage and $\phi_{M<)}$ the skew leakage at a location x from the centre of stack along the axis. Thus we have

$$\phi_{x1} = \frac{\mu_0 6\sqrt{2}k_{w1} I_1 T_{ph}}{N_1}\frac{g}{3}\left\{\begin{aligned} &\frac{d_{10}}{w_{10}} + \frac{2d_{11}}{w_{10}+w_{11}} + \frac{t'_{10}a_2 b_1}{8g_e} \\ &+ \frac{K_{R2}K_d p_1 t'_{20} a_2 b_1}{K_{R1}K_{\phi 2} 8g_e} \\ &+ \frac{K_{\phi 1} a_2 t'_{10} k}{2.1 g_e k_{R1}}\frac{2}{L}\sigma_{s(x)} \end{aligned}\right\} \tag{19}$$

In this expression skew leakage flux is expressed as

$$\phi_{sk(x)} = \nabla AT_m \mu_0 \left(\frac{t_{10}}{8g_e}\sigma_{s(x)}\right) \tag{20}$$

where, $\sigma_{x(x)}$ is the saturation factor of the useful in the air gap. The expression for ϕ_{x1} (Eq. 19) can be abbreviated as

$$\frac{\phi_{x1}}{k_1} = I_1\left[h_1 + h_3 + h_2 \cdot \frac{2x}{L} \cdot \sigma_{s(x)}\right] \tag{21}$$

can be recognized from Eq. 19. The current at which the saturation starts is denoted by /,' and it is given as

$$I'_1 = \frac{B_s A_{s1}}{K_1\left(h_1 + h_3 + h_2\sigma_{s(x)}\right)} \tag{22}$$

Knowing the value of I,' the value of X_S in the flux diagram can be determined from

$$x_s = \frac{L}{2}\left[\frac{I'\left(h_1 + h_3 + h_2\sigma_{s(x)} - I_1\left(h_1 + h_3\right)\right)}{I_1 h_2 \sigma_s (L/2)}\right] \tag{23}$$

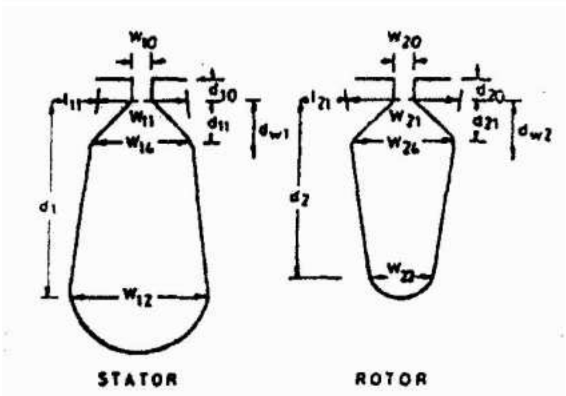

Fig. 7(a) Slot dimensions

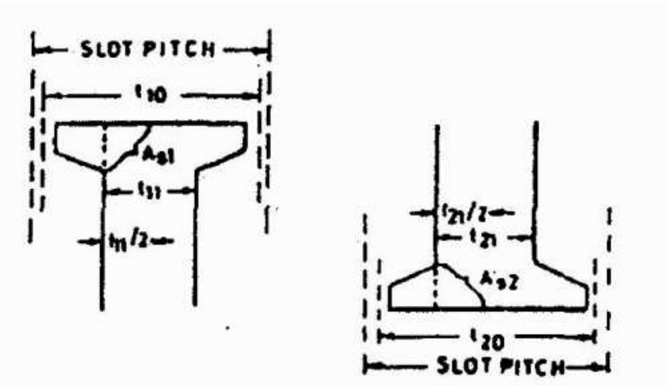

Fig. 7 b Saturating areas of stator and rotor teeth.

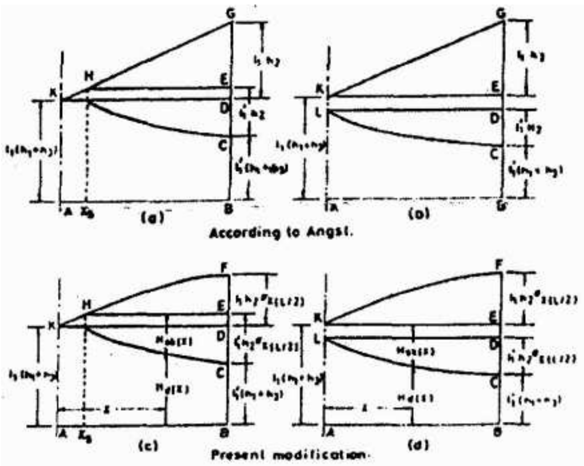

Fig.8 Leakage flux factors

The saturation factors for slot and zigzag leakage fluxes

SF, = area ABCLK/area ABDK in Fig.8c

= area ABCL / area ABEK in Fig.8d

The saturation factors for skew leakage reactance

SF_A, = area KLCDEK / area KDFHK in Fig.8c

= area LCD / area KEFK. in Fig.8d

The areas are calculated using trapezoidal rule and the saturation factors are determined and saturated leakage reactance is calculated. Similar procedure is used for rotor parameters. Fig. 7a gives the dimensions of the slots and slot shapes.

V. RESULTS

A 15KW, 3-phase, delta connected, 400V induction motor is used as a test machine. Two rotors are used in the investigations - one rotor having a slot skew of one slot pitch (15° electrical) and the other having a skew of two slot pitches (30° electrical). The dimensions required for the predetermination are made available. The blocked rotor current was measured at different voltages and also were predetermined using the method of Angst [3] and the present modification. The results are shown in figures 9, 10, 11,12. The figures confirm that the method due Angst overestimates the blocked rotor current, the error being more at larger skew angles on the other hand the present method predetermines the values of the blocked rotor current which are in good agreement with the experimental results (Fig. 9). Various saturation factors and saturated leakage reactance are depicted as functions of line current as predetermined by the present method (Figs. 10, 11, 12).

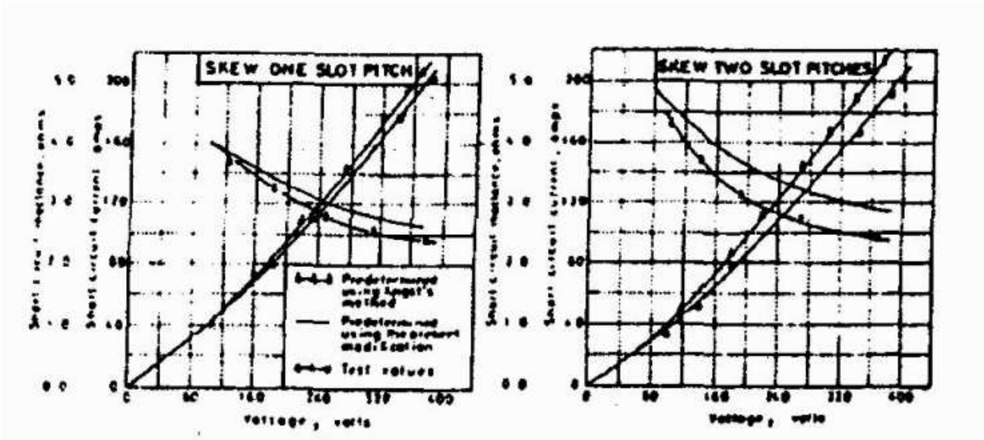

Figure 9. Voltage vs Short circuit current and reactance

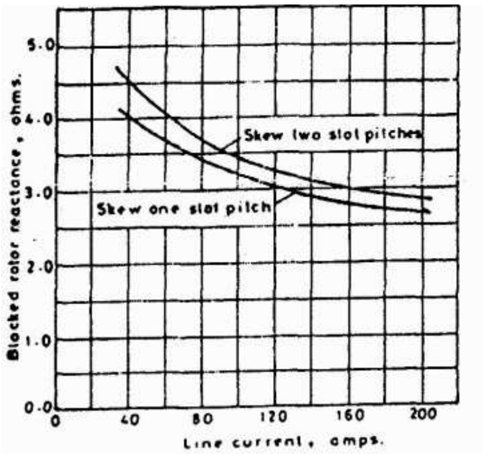

Figure 10. Saturated blocked rotor reactance Vs current

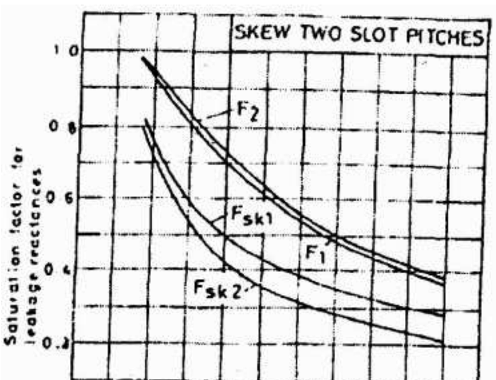

Figure 11. Variation of saturation factors as function of current

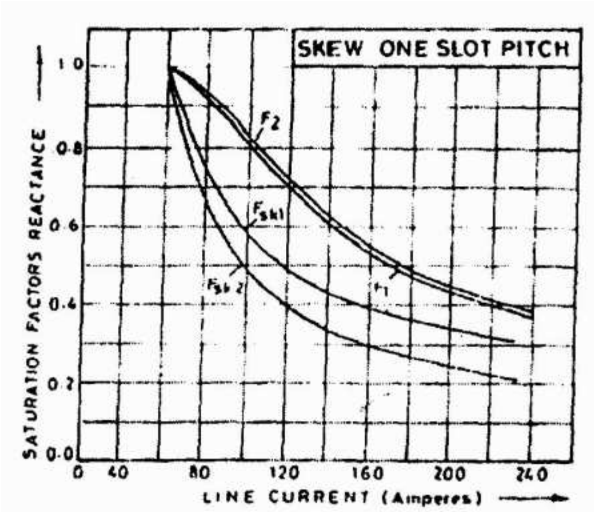

Figure 12. Variation of saturation factors for leakage reactance with current.

VI. CONCLUSIONS

Skew introduces flux variation along the stack of the motor. Flux gets saturated as the ends of the stack are reached. This variation can be predicted using phasor diagram. Skew effectively reduces the magnetic coupling between the stator and rotor mmfs. Consequently, it introduces additional skew leakage. It increases the harmonic leakage. Three kinds of equivalent circuits arc possible to take the skew leakage reactance into consideration simply by changing the transformation ratio of the motor. Saturation factors are determined considering saturation as total flux phenomenon. The main flux saturation along the stack affects the saturation factor of skew leakage. The method is general can be suitably implemented in the design analysts program.