Planning UMTS Base Station Location: Optimization Models With Power Control and Algorithms

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***Abstract*—Classical coverage models, adopted for second-generation cellular systems, are not suited for planning universal mobile telecommunication system (UMTS) base station (BS) location because they are only based on signal predictions and do not consider the traffic distribution, the signal quality requirements, and the power control (PC) mechanism. In this paper, we propose discrete optimization models and algorithms aimed at supporting the decisions in the process of planning where to locate new BSs. These models consider the signal-to-interference ratio as quality measure and capture at different levels of detail the signal quality requirements and the specific PC mechanism of the wideband CDMA air interface. Given that these UMTS BS location models are nonpolynomial (NP)-hard, we propose two randomized greedy procedures and a tabu search algorithm for the uplink (mobile to BS) direction which is the most stringent one from the traffic point of view in the presence of balanced connections such as voice calls. The different models, which take into account installation costs, signal quality and traffic coverage, and the corresponding algorithms, are compared on families of small to large-size instances generated by using classical propagation models.**

***Index Terms*—Code-division multiple access (CDMA), optimization algorithms, optimization models, planning, power control (PC), universal mobile telecommunication system (UMTS).**

I. INTRODUCTION

WITH THE extraordinary success of mobile communication services, service providers have been affording huge investments for network infrastructures. Due to the high costs and the scarcity of radio resources, an accurate and efficient mobile network planning appears of outmost importance. With the rapid growth of network size and number of users, efficient quantitative methods to support decisions for base station (BS) location have become essential. This need is now even more acute with the advent of third-generation systems, such as universal mobile telecommunication system (UMTS), due to the increased complexity of the system and the number of parameters that must be considered [1]–[3].

The problem of planning second-generation cellular systems adopting a time-division multiple access (TDMA)-based access scheme has usually been simplified by subdividing it into a coverage planning problem and a frequency planning problem which are driven by a coverage and, respectively, a capacity criterion [1], [3], [4]. In the coverage planning phase, BSs are placed so that the signal strength is high enough in the area to be served [5]–[9]. This step only makes use of propagation models, such as for instance Hata's model, to predict the signal levels (see, e.g., [10]). In the frequency planning phase, a set of channels has to be assigned to each BS [11], [12], taking into account the traffic requirements and the service quality measured as the signal-to-interference ratio (SIR).

With the wideband code-division multiple access (W-CDMA) air interface of UMTS, this two-phase approach is not appropriate mainly because the bandwidth is shared by all active connections and no actual frequency assignment is strictly required. The access scheme allows for a more flexible use of radio resources and the capacity of each cell (e.g., the number of connections) is not limited *a priori* by a fixed channel assignment as in TDMA systems, but it depends on the actual interference levels which determine the achievable SIR values. As these values depend on both traffic distribution and BS positions, BS location in UMTS networks cannot only be based on coverage but it must also be capacity driven [3]. Indeed, interference levels are functions of the emitted powers which, due to a *power control* (PC) mechanism, depend on the mobile station positions. Since the power available for transmission is limited, mobile stations that are far away from the BS may not reach the minimum SIR when the interference level is too high. Therefore, the area actually covered by each BS is heavily affected by the traffic distribution and its size can vary when the interference level changes (this is the so-called *cell breathing* effect). It is worth emphasizing that, since interference levels depend both on the connections within a given cell and on those in neighboring cells, the SIR values and the capacity are highly affected by the traffic distribution in the whole area.

The planning phase of cellular networks usually takes as input the following kind of information related to the service area: 1) a set of candidate sites where BSs can be installed; 2) the traffic distribution estimated by using empirical prediction models; and 3) the propagation description based on approximate radio channel models or ray tracing techniques. The main purpose of planning is then to select the sites where to install the BSs taking into account different aspects such as costs, signal quality, and service coverage.

As we shall see in Section II, there has been so far little work on optimizing BS locations for UMTS networks and, to the best of our knowledge, none of the proposed models considers the impact of traffic distribution, signal quality requirements, and PC mechanism at an adequate level of detail. In this paper, we

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propose and investigate discrete optimization models aimed at supporting the decisions in the process of planning where to locate new BSs. Our models differ in how closely they capture the peculiarities of the signal quality constraints and the PC mechanism of the UMTS W-CDMA air interface. The focus here is on the uplink direction (mobile to BS), which turns out to be the most stringent one from the system capacity point of view in the presence of full-duplex balanced connections such as voice calls (see, e.g., [13] and [14]). Models for the downlink direction are presented in [15] and [39].

In Section II, we discuss the main radio network planning issues pertaining to UMTS BS location and comment on previous work. In Section III, we propose and analyze mathematical programming formulations of this general problem for the W-CDMA setting considering the two most common ways to model the PC mechanism. In Section IV, we describe three heuristics: randomized greedy and reverse greedy procedures as well as a tabu search (TS) algorithm. Computational results obtained with realistic instances are reported and discussed in Section V. Finally, Section VI contains some concluding remarks. Preliminary versions of part of this work were presented in [16] and [17].

II. Radio Planning Issues for UMTS

UMTS [18] is the third-generation mobile communication system standardized by ETSI, the European Telecommunications Standard Institute, and is also considered by ITU (International Telecommunication Union) among the standards for the International Mobile Telephone standard 2000 (IMT-2000) family.

One of the two access schemes to be used in the assigned spectrum is based on W-CDMA and frequency-division duplexing. The main characteristic of CDMA is its flexibility in the use of radio resources. In particular, there is no *a priori* limit on the number of simultaneous connections per cell (hard capacity) as with TDMA systems, and resources are dynamically assigned according to interference levels and traffic distribution (soft capacity) [13]. However, this clearly implies an increased complexity in the network planning process and more involved access control procedures. Ad hoc planning and optimization strategies for the CDMA technology are, thus, needed to actually exploit this additional flexibility [3].

Spreading codes used for signals transmitted in downlink by the same BS are mutually orthogonal, while codes used for signals emitted by different stations (base or mobile) can be considered as pseudorandom due to the scrambling sequence [19]. In an ideal environment, the despreading process performed at the receiving end can completely avoid interference of orthogonal signals and reduce that of nonorthogonal ones by the *spreading factor* (SF), which is the ratio between the spread signal rate and the user rate. In wireless environments, due to multipath propagation, the interference of orthogonal signals cannot be completely avoided and the SIR is given by

$$\mathrm{SIR} = \mathrm{SF}\frac{P_{\mathrm{received}}}{\alpha I_{\mathrm{in}} + I_{\mathrm{out}} + \eta} \tag{1}$$

where P_{received} is the received power of the signal, I_{in} is the total interference due to the signals transmitted by the same BS (intracell interference), I_{out} that due to the signals emitted by the other BSs (intercell interference), α is the orthogonality loss factor $(0 \leq \alpha \leq 1)$, and η the thermal noise power. In the uplink case, no orthogonality must be accounted for and $\alpha = 1$.

Since the quality of the received signal, usually expressed in terms of *bit error rate*, mainly depends on the SIR, it is common to consider quality constraints requiring that the SIR exceeds a minimum value τ which may vary according to the communication service considered (voice, video, packet data, etc.). For the sake of simplicity, in the sequel, we refer to the minimum SIR before despreading as $\mathrm{SIR}_{\min} = \tau/\mathrm{SF}$.

A simplified and commonly adopted model [20] assumes that the interference due to the neighboring cells (I_{out}) can be expressed as a fraction f of the interference due to the other transmissions in the same cell, so that the SIR can be expressed as

$$\mathrm{SIR} = \mathrm{SF}\frac{P_{\mathrm{received}}}{I_{\mathrm{in}}(1+f)} \tag{2}$$

where the thermal noise is omitted since it is assumed to be much smaller than the interference. This simplified model is accurate when the traffic distribution among cells is homogeneous, while it is inappropriate in all the other cases where the contribution to intercell interference is different for each cell. Values of f in the 0.3–0.5 range are usually considered.

A. PC and Capacity Constraints

As mentioned above, the SIR depends on the received powers of the considered signal and of the interfering ones. These in turn depend on the transmitted powers and the attenuation of the radio link between sources and receivers. According to propagation conditions, the transmitted power can be adjusted by the PC mechanism so as to minimize interference and guarantee quality. Two PC mechanisms are commonly considered: one based on the received power and the other one on the estimated SIR. In the first one, the transmitted power is adjusted so that the power received on each channel is equal to a given target value P_{target}. Similarly, in the second one, the transmitted power is set so that the SIR is equal to a target value $\mathrm{SIR}_{\mathrm{target}}$. The latter mechanism, adopted for UMTS dedicated channels [18], is more complex since the power emitted by each station depends on that emitted by all the others, but more efficient since it allows for the use of lower powers [21]. Therefore, from a planning prospective, assuming a power-based PC mechanism instead of an SIR-based one leads to a conservative dimensioning which may allocate more radio resources than necessary.

Both in the case of power-based or SIR-based PC mechanisms, the transmitted powers are adjusted considering some power limits. In particular, a limit on the maximum power used for each radio channel must be considered both for uplink and downlink. Moreover, for the downlink case only, a constraint on the total power emitted by the BS must be added. Therefore, the actual power emitted on a channel is the minimum between that provided by the PC mechanism and the maximum value.

From a planning point of view, the effect of the power bounds and SIR constraints is to limit the capacity of the system. In the presence of power-based PC, as new users are added to the system, the SIR values of all the other users decrease until one falls below the lowest acceptable quality level $\mathrm{SIR}_{\min}$. No additional user can then be served. In the presence of SIR-based PC, signal quality is guaranteed by keeping the SIRs at a constant target value $\mathrm{SIR}_{\mathrm{target}} \geq \mathrm{SIR}_{\min}$. When new users are added, the emission powers required to keep the SIRs equal to $\mathrm{SIR}_{\mathrm{target}}$ increase until the power limit is exceeded and, hence, the SIR falls below the target value. In both power-based and SIR-based cases, the capacity is affected by user positions and propagation conditions. Indeed, the capacity depends on the interference generated by user-BS transmissions which, in turn, depends on the emitted powers, which are strongly related to the relative positions and the radio link propagation factors. Since each user-BS transmission generates interference not only in its own cell but also toward all the other cells, it is as if each user involved in a connection "absorbs" a fraction of the capacity from all BSs.

B. Related Work

Classical coverage optimization models do not consider SIR constraints but only constraints on the received power levels in the service area. In [22], the traffic distribution is described by means of *demand nodes* which represent the center of an area characterized by a given traffic demand (usually expressed in Erlang). Using the demand node characterization, the coverage problem is then defined by considering the signal level in each node from all BSs and requiring that at least one level is above a fixed threshold. A common objective of the optimization process is that of finding the smallest set of BSs covering all demand nodes (see, e.g., [6] and [7]). In [23] and [24], traffic capacity constraints are also added for each BS. A different coverage model is adopted in [5], where the position of transmitters is selected from continuous three-dimensional space so as to minimize the sum of path losses of the links to all receivers.

A few recent works address network planning problems for CDMA systems and, in particular, for UMTS. However, some of them still rely on a classical coverage approach: In [25], a simple model based on the minimum dominating set problem is considered while in [26], the traffic capacity is also taken into account and the resulting classical capacitated facility location problem is tackled with a TS algorithm. In [27], a different approach is adopted: A maximum independent set of vertices is searched for in a graph in which vertices represent candidate sites and edges correspond to pairs of sites whose BSs would have coverage areas with too much overlap.

In [28], a simplified model for locating BSs in CDMA-based UMTS networks, which partially takes into account interference, is proposed and a polynomial time approximation scheme is presented. However, only intracell interference is considered while the crucial aspect of the interference among BSs (intercell one) is neglected. As we shall see in Section III-A, even if the intercell interference is assumed to be a nonzero fraction of the intracell interference, in the uplink case, each interference constraint amounts to impose a simple upper bound on the maximum number of active connections with the corresponding BS.

III. BS Location Models

Since, to the best of our knowledge, some crucial issues of the planning problem for CDMA-based UMTS networks have not yet been captured, we propose and investigate different mathematical programming models for the UMTS BS location problem that account for intercell interference in the SIR constraints and for the PC mechanism. As previously mentioned, the focus here is on the uplink direction with power-based as well as SIR-based PC mechanism.

In this work, we make two simplifying assumptions. First, we assume that each connection is assigned to a single BS. Therefore, we do not explicitly account for soft-handover which allows a mobile terminal to be simultaneously connected with a set of BSs. It is worth noting, however, that our assumption is on the conservative side from a planning point of view since soft-handover tends to increase the SIR values. A simple way to account for this feature is that of including an additional margin on the SIR constraints (i.e., of selecting a lower $\mathrm{SIR}_{\min}$). Second, we assume that the number of available spreading codes is higher that the number of connections assigned to any BS. This assumption is clearly satisfied in the uplink direction since there is a very large number of nonorthogonal codes.[1]

A. Basic Model

Consider a territory to be covered by a UMTS service. Assume that a set of *candidate sites* $S = \{1, \ldots, m\}$ where a BS can be installed, is given and that an installation cost c_j is associated with each candidate site $j, j \in S$. A set of *test points* (TPs) $I = \{1, \ldots, n\}$ is also given. Each TP $i \in I$ can be considered as a centroid where a given amount of traffic d_i (in Erlang) is requested and where a certain level of service (measured in terms of SIR) must be guaranteed [6]. The required number of simultaneously active connections for TP i, denoted by u_i, turns out to be a function of the traffic demand, i.e., $u_i = \phi(d_i)$. The actual definition of the function ϕ is a degree of freedom of the planning process. It can simply correspond to the average number of active connections or to the number of simultaneous connections not exceeded with a given probability p. The connection activity factor can be considered as well.

The propagation information is also supposed to be known. In particular, let $g_{ij}, 0 < g_{ij} \leq 1$ be the propagation factor of the radio link between TP $i, 1 \leq i \leq n$ and a candidate site $j, 1 \leq j \leq m$. The propagation gain matrix $G = [g_{ij}]_{1 \leq i \leq n, 1 \leq j \leq m}$ is estimated according to approximate propagation models such as those proposed by Hata or to more precise but computationally intensive ray tracing techniques (see, e.g., [10]).

In the W-CDMA UMTS BS location problem, one wishes to select a subset of candidate sites within the set S where to install BSs, and to assign the TPs to the available BSs taking into account the traffic demand, the signal quality requirements in terms of SIR and the installation costs.

Let us define the two following classes of decision variables:

$$y_j = \begin{cases} 1, & \text{if a BS is installed in } j \\ 0, & \text{otherwise} \end{cases}$$

[1] In the downlink direction, where at most SF orthogonal codes are used, standard cardinality contraints can be easily added to the model.

for $j \in S$ and

$$x_{ij} = \begin{cases} 1, & \text{test point } i \text{ is assigned to BS } j \\ 0, & \text{otherwise} \end{cases}$$

for $i \in I$ and $j \in S$. The core of the basic integer programming model that we propose for the uplink case is the classical uncapacitated facility location model

$$\min \sum_{j=1}^{m} c_j y_j + \lambda \sum_{i=1}^{n} \sum_{j=1}^{m} u_i \frac{1}{g_{ij}} x_{ij} \tag{3}$$

subject to

$$\sum_{j=1}^{m} x_{ij} = 1, \quad i \in I \tag{4}$$

$$x_{ij} \le y_j, \quad i \in I, \ j \in S \tag{5}$$

$$x_{ij}, y_j \in \{0,1\}, \quad i \in I, \ j \in S. \tag{6}$$

The first term in the objective function corresponds to the total installation cost. Since $1/g_{ij}$ is proportional to the power emitted from TP i when assigned to BS j, the second term aims at favoring assignments which require a smaller total emission power. $\lambda \ge 0$ is a tradeoff parameter between these two objectives. Constraints (4) make sure that each TP i is assigned to a single BS. Constraints (5) impose that TPs are only assigned to sites where a BS is installed. Note that by restricting the assignment variables x_{ij} to take on binary values, it is required that in every feasible solution, all active connections must be assigned to a single BS.

To account for the power limit on the user terminals, we need to include, for each pair of TPs $i \in I$ and candidate site $j \in S$, the following constraint:

$$\frac{P_{\text{target}}}{g_{ij}} x_{ij} \le P_{\max} y_j \tag{7}$$

where $P_{\max}$ is the maximum emission power and P_{target}/g_{ij} corresponds to the emission power required by a mobile station in TP i to guarantee the target received power P_{target} at site j. Note that if $g_{ij}P_{\max}/P_{\text{target}} < 1$, the TP i cannot be assigned to candidate site j due to power limits, and therefore, the variable x_{ij} can be omitted from the model. Otherwise, constraint (7) is implied by the corresponding constraint (5).

The fundamental aspect to be taken into account is the quality of the signal received by each BS. As mentioned in Section II, the simplest way to express the quality constraints is either to neglect the intercell interference or to consider that it amounts to a given fraction of the intracell interference as given in formula (2) for nonzero values of the parameter f. For each connection, the quality constraint $\text{SIR} \ge \text{SIR}_{\min}$ can then be rewritten as

$$\text{SF} \frac{P_{\text{received}}}{I_{\text{in}}(1+f)} \ge \text{SIR}_{\min} \tag{8}$$

which is equivalent to

$$\frac{I_{\text{in}}}{P_{\text{received}}} \le \frac{\text{SF}}{\text{SIR}_{\min}(1+f)}. \tag{9}$$

Considering a power-based PC mechanism, the power received P_{received} at BS j from each mobile station in a TP assigned to it is equal to P_{target} and the quality constraint amounts to imposing an upper bound on the number $\sum_{i=1}^{n} u_i x_{ij}$ of connections that can be assigned to that BS [20]. Specifically, we have

$$\sum_{i=1}^{n} u_i x_{ij} \le \frac{\text{SF}}{\text{SIR}_{\min}(1+f)} + 1. \tag{10}$$

For the typical values $f = 0.4, \text{SF} = 128$ and $\text{SIR}_{\text{target}} = 6$ dB used in the literature, we obtain an upper bound of 23.97 on the maximum number of connections that can be served by any single BS. Thus, for each candidate site $j \in S$, the signal quality constraints can be rewritten as

$$\sum_{i=1}^{n} u_i x_{ij} \le 23 y_j. \tag{11}$$

The resulting *basic model*, which amounts to

$$\min \sum_{j=1}^{m} c_j y_j + \lambda \sum_{i=1}^{n} \sum_{j=1}^{m} u_i \frac{1}{g_{ij}} x_{ij} \tag{12}$$

subject to

$$\sum_{j=1}^{m} x_{ij} = 1, \quad i \in I \tag{13}$$

$$x_{ij} \le \min\left\{1, \frac{g_{ij}P_{\max}}{P_{\text{target}}}\right\} y_j, \quad i \in I, \ j \in S \tag{14}$$

$$\sum_{i=1}^{n} u_i x_{ij} \le 23 y_j, \quad j \in S \tag{15}$$

$$x_{ij}, y_j \in \{0,1\}, \quad i \in I, \ j \in S \tag{16}$$

falls within the class of standard capacitated facility location problems which have been extensively studied in the operations research literature (see [29]). Note that to obtain a formulation which does not involve the variables x_{ij} such that $g_{ij}P_{\max}/P_{\text{target}} < 1$, it suffices to proceed as follows. For each TP i, let $S_i \subseteq S$ denote the set of all candidate sites to which TP i can be assigned to while respecting the power limit $P_{\max}$. Symmetrically, for each candidate site j, let $I_j \subseteq I$ denote the set of all TPs i that can be assigned to j while respecting the power limit. Then replace the summation over all m candidate sites j in the second term of the objective function (12) and in constraints (13) by a summation over S_i, and the summation over all n TPs in constraints (15) by one over I_j. Finally, substitute all constraints (14) by $x_{ij} \le y_j$ for all $i \in I$ and $j \in S_i$.

Unfortunately even medium-size instances of these nonpolynomial (NP)-hard capacitated location problems turn out to be out of reach of state-of-the-art optimization algorithms. But, even more importantly, the capacity constraints (15) do not capture the distinctive features of the W-CDMA technology and, as we shall see in Section V-A, in most cases the above basic model provides meaningless solutions.

B. Enhanced Model With Power-Based PC

To make the model more realistic, intercell interference needs to be considered explicitly and independently from intracell interference. The use of pseudorandom spreading codes implies that, for a specific uplink connection between TP i and BS j, there is no significant difference in the two types of interference. In other words, $\alpha = 1$ in the SIR formula (1) and for each connection, the quality constraint amounts to $P_{\text{received}}/(I_{\text{in}} + I_{\text{out}} + \eta) \geq \text{SIR}_{\min}$, where $\text{SIR}_{\min} = \tau/\text{SF}$ is the minimum SIR before despreading.

In the presence of a power-based PC mechanism, the thermal noise η is omitted as in the other works focusing on this type of PC mechanism (see, e.g., [20]) and for each candidate site $j \in S$ the signal quality constraint can be expressed as follows:

$$\frac{P_{\text{target}}}{\sum_{h=1}^{n} u_h g_{hj} \sum_{t=1}^{m} \frac{P_{\text{target}}}{g_{ht}} x_{ht} - P_{\text{target}}} \geq \text{SIR}_{\min} y_j \tag{17}$$

where P_{target} is by definition the power received from each assigned TP. It is not difficult to verify that constraint (17) enforces that, if a BS is installed in site $j \in S$ (i.e., $y_j = 1$), the SIR value in j must exceed the given $\text{SIR}_{\min}$. For any single connection assigned to the BS located in site j, the numerator of the left-hand-side term is the power of the relevant signal received in j while the denominator amounts to the total interference due to all other connections. Indeed, the double summation term expresses the total power received at site j from all TPs $h \in I$, from which the received power P_{target} of the relevant signal is substracted. More specifically, for any TP h, the quantity P_{target}/g_{ht} amounts to the emission power required at TP h to guarantee a received power value of P_{target} at site t. Note that, since $\sum_{t=1}^{m} x_{ht} = 1$, the only term of the inner summation (over index t) that is nonzero corresponds to the site to which TP h is actually assigned. Thus, if this site is denoted by $t(h)$, the outer summation can be rewritten as $\sum_{h=1}^{n} u_h g_{hj} \ (P_{\text{target}}/g_{ht(h)})$ where $g_{hj} \ (P_{\text{target}}/g_{ht(h)})$ is the power received at site j from TP h and u_h is the number of connections required from TP h. Clearly, the contribution to the outer summation of any TP h assigned to site j amounts to $u_h P_{\text{target}}$ since $g_{hj} = g_{ht(h)}$.

Multiplying both sides of the inequality (17) by the denominator of its left-hand side and dividing the left and right sides by P_{target}, we obtain for each candidate site $j \in S$ the bilinear constraint

$$1 \geq \text{SIR}_{\min} y_j \left(\sum_{h=1}^{n} \sum_{t=1}^{m} u_h \frac{g_{hj}}{g_{ht}} x_{ht} - 1 \right). \tag{18}$$

Thus, the *enhanced model*, assuming a power-based PC mechanism, amounts to the following nonlinear mathematical program:

$$\min \sum_{j=1}^{m} c_j y_j + \lambda \sum_{i=1}^{n} \sum_{j=1}^{m} u_i \frac{1}{g_{ij}} x_{ij} \tag{19}$$

subject to (20)–(23), shown at the bottom of the page, where, for each pair of TP i in I and candidate site j in S, constraints (21) corresponds to the most stringent constraint among (5) and (7). As mentioned in Section III-A for the basic model, a formulation involving only the variables x_{ij} such that $g_{ij} P_{\max}/P_{\text{target}} \geq 1$ can be obtained by using appropriate summation sets S_i and I_j.

To tackle this problem with state-of-the-art mixed integer programming (MIP) solvers like CPLEX 7.0 [30], constraints (18) and, hence, (22) can be linearized as follows:

$$1 + M(1 - y_j) \geq \text{SIR}_{\min} \left(\sum_{h=1}^{n} \sum_{t=1}^{m} u_h \frac{g_{hj}}{g_{ht}} x_{ht} - 1 \right) \tag{24}$$

for a large enough value of M. Indeed, constraint (24) amounts to (18) when $y_j = 1$ and, due to the value of M, it is always satisfied when $y_j = 0$. In the linearized version of the enhanced model, the nonlinear constraints (22) are replaced by the corresponding inequalities (24).

It is worth emphasizing that constraints (22) and (24) are always satisfied when $y_j = 0$, regardless of the way the TPs are assigned to the BSs, and when $y_j = 1$, these constraints can be restated as

$$\sum_{h=1}^{n} \sum_{t=1}^{m} a_{ht}^{j} x_{ht} \leq \frac{1}{\text{SIR}_{\min}}. \tag{25}$$

Here, we define $a_{ht}^{j} = u_h - 1$ for one of the TP h assigned to BS j and $a_{ht}^{j} = u_h$ for all other TPs assigned to BS j. For all TPs h assigned to other BSs, we have $0 \leq a_{ht}^{j} \leq u_h$. This is clearly in contrast with standard capacity constraints arising in classical capacitated facility location problems that can be expressed, when $y_j = 1$, as

$$\sum_{h=1}^{n} d_h x_{hj} \leq s_j \quad j = 1, \ldots, m \tag{26}$$

where the "demand" d_h of "client" h does not depend on the "facility" to which "client" h is assigned (there is no summa-

$$\sum_{j=1}^{m} x_{ij} = 1, \quad i \in I \tag{20}$$

$$x_{ij} \leq \min \left\{ 1, \frac{g_{ij} P_{\max}}{P_{\text{target}}} \right\} y_j, \quad i \in I, \ j \in S \tag{21}$$

$$y_j \left(\sum_{h=1}^{n} \sum_{t=1}^{m} u_h \frac{g_{hj}}{g_{ht}} x_{ht} - 1 \right) \leq \frac{1}{\text{SIR}_{\min}}, \quad j \in S \tag{22}$$

$$x_{ij}, y_j \in \{0, 1\}, \quad i \in I, \ j \in S \tag{23}$$

tion on the second subscript of the x variables) and, for each "facility" j, the constraint only involves the "clients" assigned to it [29]. In the context of the UMTS BS location problem, it is as if each TP h "absorbs" part of the capacity of every BS and not only that of the BS it is assigned to. Moreover, the amount of capacity requested from BS j depends on the "distance" (gain) between TP h and BS j, as well as on the "distance" (gain) between TP h and the BS to which it is assigned.

By analyzing the quality (SIR) constraints (17), one can establish a simple but important property of the underlying assignment subproblem of the BS location problem [31].

Property: Given any set of active BSs $\bar{S} \subseteq S$ and set I of TPs, only assignments of TPs to "closest" active BSs need to be considered, where "closeness" is meant in terms of the required emission power.

This derives from the fact that if any one of the TPs is not assigned to one of its "closest" BSs, the SIR level at each active BS can only increase when that TP is reassigned to a closer BS.

Thus, the UMTS BS location problem under consideration turns out to be substantially different from standard capacitated facility location problems, where, for any given set of active BSs, the optimal assignment is not known *a priori* and needs to be determined.

C. Model With SIR-Based PC

Assuming a more sophisticated SIR-based PC mechanism yields a more involved mathematical programming model since the emission power p_i, required to connect each TP $i \in I$ to the candidate site it is assigned to, must be considered as an explicit variable. Indeed, if TP i is assigned to site j, p_i is not taken to be equal to P_{target}/g_{ij}, so as to guarantee a given prescribed received power P_{target} for every active connection. In the presence of an SIR-based PC mechanism, the emission power values p_i can be freely selected provided they do not exceed the maximum emission power $P_{\max}$ and that the SIR level of each active connection is not lower than a prescribed $\text{SIR}_{\text{target}}$. Besides the n new power variables p_i, the core model is then as in (3)–(6) except that in the second term of the objective function $1/g_{ij}$ is replaced by the actual emission power p_i. To account for the power limit on the user terminals, constraints (7) are replaced by $0 \le p_i \le P_{\max}$. Moreover, for each pair of TP $i \in I$ and candidate site $j \in S$, the signal quality constraint is now

$$\frac{p_i g_{ij}}{\sum_{h=1}^{n} u_h g_{hj} \sum_{t=1}^{m} p_h x_{ht} - p_i g_{ij} + \eta} \ge \text{SIR}_{\text{target}} x_{ij} \tag{27}$$

where $\text{SIR}_{\text{target}} \ge \text{SIR}_{\min}$. Note that $p_i g_{ij}$ is the power of the signal received at BS j from TP i. If TP i is assigned to site j (i.e., $x_{ij} = 1$), this third-order nonlinear constraint makes sure that the SIR level of the corresponding connection is high enough. Unlike in the power-based PC case, in this context, the thermal noise η is usually included to guarantee convergence of the closed-loop PC mechanism used in real systems [21].

Thus, the *SIR-based PC model* amounts to the following mixed mathematical program:

$$\min \sum_{j=1}^{m} c_j y_j + \lambda \sum_{i=1}^{n} \sum_{j=1}^{m} u_i p_i x_{ij} \tag{28}$$

subject to (29)–(33), shown at the bottom of the page, with binary variables x_{ij} and y_j as well as real variables p_i. Note that, unlike in the power-based PC case, there is here a signal quality constraint (31) for each pair of TP i in I and candidate site j in S, and obviously only those with $x_{ij} = 1$ are relevant.

D. Different Planning Objectives

Although the generic objective function proposed in (3) (and the corresponding (12), (19), and (28) in the three models) takes into account the installation costs and the total emission power; the appropriate choice depends on the specific planning objectives. According to the traffic requirements and distribution, the number of candidate sites and their locations as well as the mobile station maximum power, the signal quality constraints (15), (22), and, respectively, (31) can be infeasible. In real-world instances where traffic patterns are based on short-to-mid term predictions, it is likely that the system will be required to serve all traffic. On the other hand, when traffic patterns are based on long-term predictions, a multiperiod network planning strategy can be adopted [32] and, in the first stages, one has to cope with solutions that do not cover all traffic. In this case, it is reasonable to aim also at maximizing the fraction of traffic that is actually served. This can be achieved by relaxing the assignment constraints (13), (20), and, respectively, (29)

$$\sum_{j=1}^{m} x_{ij} \le 1, \quad i \in I \tag{34}$$

so as to allow some TPs not to be assigned. The additional term $\sum_{i=1}^{n} u_i \sum_{j=1}^{m} x_{ij}$ can then be included in the objective function with an appropriate negative weight parameter.

$$\sum_{j=1}^{m} x_{ij} = 1, \quad i \in I \tag{29}$$

$$x_{ij} \le y_j, \quad i \in I,\ j \in S \tag{30}$$

$$x_{ij} \left(\sum_{h=1}^{n} u_h g_{hj} \sum_{t=1}^{m} p_h x_{ht} - p_i g_{ij} + \eta \right) \le \frac{p_i g_{ij}}{\text{SIR}_{\min}}, \quad i \in I,\ j \in S \tag{31}$$

$$x_{ij}, y_j \in \{0, 1\}, \quad i \in I,\ j \in S \tag{32}$$

$$0 \le p_i \le P_{\max}, \ i \in I \tag{33}$$

IV. Heuristic Algorithms

Since the UMTS BS location models proposed in the previous section contain the uncapacitated facility location problem as a special case, they turn out to be NP-hard. To obtain good approximate solutions within a reasonable amount of computing time, we have first devised greedy and reverse greedy randomized procedures (see [33]) that construct a solution (i.e., a subset of candidate sites where to activate BSs) by iteratively adding or, respectively, removing BSs from the current solution. TS algorithms have also been developed for the power-based as well as SIR-based PC models.

We first describe the algorithms for the power-based PC model. From Section III-B, we know that in this case, assigning each TP to a "closest" active BS yields the largest possible SIR value for each BS. Moreover, in this setting all connections to a given BS j have the same SIR level, denoted by SIR_j. Given any set of active BSs, if all traffic requests can not be covered, the central procedure of our algorithms aims at satisfying the largest fraction of demands [see the relaxed constraints (34)]. This subproblem amounts to a special case of the multidimensional knapsack problem which is difficult (NP-hard) to solve optimally [34]. Since in our context good solutions are needed in a short amount of time, we proceed as follows.

Given a set $\bar{S}$ of active BSs, each TP is first assigned to one of its "closest" BSs j in $\bar{S}$. Then, TPs assigned to active BSs with $\mathrm{SIR}_j < \mathrm{SIR}_{\min}$ are considered by nonincreasing value of emission power $u_i P_{\mathrm{target}}/g_{ij}$ and deleted one at a time until all active BSs have an SIR of at least $\mathrm{SIR}_{\min}$.

Thus, TPs which cause higher interference are first deleted while the number of BSs affected by the deletion is not considered.

A. Randomized Greedy Procedures

In the first randomized greedy algorithm (Add), BSs are added iteratively to $\bar{S}$ starting from $\bar{S} = \emptyset$. At each iteration there is a current set $\bar{S} \subset S$ of sites (possibly empty) in which BSs have already been installed. For each remaining candidate site $j \in S \setminus \bar{S}$, the above assignment procedure is then applied to $\bar{S} \cup \{j\}$ so as to obtain a corresponding vector $\mathbf{x}$. The characteristic vector $\mathbf{y}$ of subset $\bar{S} \cup j \subseteq S$ is simply defined as $y_t = 1$ for all $t \in \bar{S} \cup \{j\}$ and $y_t = 0$, otherwise. For each of these potential solutions, specified by the set of active sites $\bar{S} \cup \{j\}$ and a corresponding pair $(\mathbf{x}, \mathbf{y})$, the following utility function is evaluated:

$$U_a(\bar{S} \cup \{j\}) = \sum_{h=1}^{n} \sum_{t=1}^{m} u_h x_{ht} - \omega \sum_{t=1}^{m} c_t y_t \qquad (35)$$

where the first term amounts to the fraction of traffic that is currently covered and the second one expresses the total cost for installing the BSs in the sites selected so far. The tradeoff parameter $\omega, \omega > 0$ allows us to assign higher priority to maximize the first objective than to minimize the second one. At each iteration, one $j' \in S \setminus \bar{S}$ is randomly selected among the ρ_a fraction of those that yield the largest value of U_a, where ρ_a is a given parameter $0 < \rho_a < 1$. The procedure stops when the addition of a new BS worsens the current solution value according to utility function U_a.

In the randomized reverse greedy algorithm (Remove), BSs are removed iteratively starting from $\bar{S} = S$. Given the current $\bar{S} \subseteq S$, for each candidate site $j \in \bar{S}$, the above procedure is applied to $\bar{S} \setminus \{j\}$ so as to obtain corresponding $\mathbf{x}$ and $\mathbf{y}$ vectors. Then, the following utility function is evaluated:

$$U_r(\bar{S} \cup j) = \sum_{h=1}^{n} \sum_{t=1}^{m} u_h x_{ht} - \omega \sum_{t=1}^{m} c_t y_t + \nu F(\mathbf{x}) \quad (36)$$

where $F(\mathbf{x})$ is the sum, over all BSs j that have so far been installed, of the number of additional connections they could service, namely, of $\lfloor \Delta_j / P_{\mathrm{target}} \rfloor$, where Δ_j is defined as the difference between the current SIR and $\mathrm{SIR}_{\min}$. $\nu > 0$ is the corresponding weight. At each iteration one $j' \in \bar{S}$ is randomly selected among the ρ_r fraction of those that yield the largest value of U_r, where the parameter ρ_r is $0 < \rho_r < 1$. As for the Add procedure, the Remove procedure stops when the removal of another BS worsens the current solution value according to utility function U_r.

Given the randomized nature of the Add and Remove procedures and their relatively low computational requirements, a *multistart* strategy is adopted. Specifically, the greedy procedure is run a predefined number of times and the best solution found during all the runs is returned as output.

B. TS Algorithm

TS is a *metaheuristic* that guides a local search procedure to explore the solution space of optimization problems beyond local optima. The idea is to use the history of the search process through an appropriate memory scheme to prevent cycling (running into feasible solutions that have already been generated) and to explore regions of the solution space that are promising in terms of the objective function. The modern TS paradigm goes back to the seminal work by Glover [35], [36] and is extensively discussed in [37].

The basic ingredients of a general TS strategy can be described as follows. Starting from an initial feasible solution $\mathbf{s}^0$, a set of neighboring solutions $N(\mathbf{s}^0)$ are generated by applying a set of possible "moves" to $\mathbf{s}^0$. Then the best solution in the "neighborhood" $N(\mathbf{s}^0)$ is selected as the next iterate $\mathbf{s}^1$ even if it does not strictly improve the value of the objective function and the process is repeated to generate a sequence of solutions $\{\mathbf{s}^k\}$.

In order to prevent cycling and to try to escape from local optima a list of "tabu moves" is maintained. The purpose of this list is to forbid the opposite move that has been made at a given step for a certain number of iterations. A move that is added to the list remains tabu for a number of iterations that corresponds to the length L of the list. According to the "aspiration criteria," tabu moves can be clearly made if they lead to an improved solution. The best solution encountered is stored as the algorithm proceeds and it is returned after a maximum number of iterations $\max_{\mathrm{it}}$. $\max_{\mathrm{it}}$ and L are two parameters of the method. Not surprisingly, the efficiency and performance of a TS procedure strongly depend on the way the moves are defined and how well they exploit the actual structure of the problem at hand.

As an initial solution, we consider the set of active BSs $\bar{S} \subseteq S$ provided by the Add or the Remove procedures together with an assignment $\mathbf{x}$ of the TPs to their closest active BS, whenever they are served. Note that the current solution is in a sense fully characterized by the set $\bar{S} \subseteq S$ of active BSs. Besides the *add* and *remove* moves used in the randomized greedy procedures, we do also define *swap* moves that amount to installing a new BS in one of the empty sites while deleting one of the active BSs. Exploring all possible swap moves for any given current solution would, of course, be very time consuming even for small-size instances. But, due to the actual structure of the problem, it is reasonable to focus on swaps between candidate sites that are relatively "close" to each other. For each active BS j, we initially consider the set $C \subseteq S$ of $\max_{\text{swap}}$ available sites that have the largest propagation gains with respect to j and that are in a sense the best candidates for a swap. Let q be a parameter such that $0 < q < 1$. The swaps involving the $\max_{\text{swap}} \cdot q$ best sites in C are systematically evaluated, while those corresponding to the remaining available sites in C are considered with a given probability $q', 0 < q' < 1$.

The objective function used to guide our TS algorithm is the utility function U_r given in (36). As to the implementation of the tabu list, BSs that are installed (disactivated) cannot be disactivated (reinstalled) during L iterations. Although this imposes stronger restrictions on the search process than just avoiding considering solutions that have already been generated, we have observed that it provides better experimental results. Indeed, by reducing the size of the neighborhood examined at each iteration, one allows the algorithm to explore a larger region of the solution space. The ability of a local search procedure to explore solution-space areas far away from each other is usually referred to as diversification [37].

In the experiments described in Section V, TS is applied in two different settings. On the one hand, TS iterations are used in a multistart Add or Remove context as a local search procedure to improve the solutions obtained with each one of the ten runs of the greedy method. Specifically, 200 iterations of TS are applied after each one of the ten runs of Add or Remove and the best solution encountered is returned. On the other hand, the same total number of iterations of TS, namely 2000, are carried out starting from a single initial solution provided by Add or Remove. Note that the advantage of Remove to generate a starting solution is that it allows for automatically checking whether all traffic demands can be satisfied.

C. Extension to SIR-Based PC Model

Although in the SIR-based case assigning TPs to closest BSs is no longer guaranteed to yield the best possible SIR values, we make this simplifying assumption which reflects a conservative point of view in the sense that there might exist assignments of TPs to active BSs involving a smaller number of active BSs or a lower total installation cost. Note that, from a planning point of view, delving into such details does not seem appropriate since the actual assignments used during operation are dynamically determined by the access procedure.

Given a current set of active BSs $\bar{S} \subseteq S$ and an assignment $\mathbf{x}$, to compute the emitted powers p_i it would be necessary to solve the system composed of constraints (27) for each active connection between TP and BS together subject to the bounds on the maximum power. Since the resulting system contains n constraints (27) in the n variables p_i, a solution satisfies all of them with equality. Assuming that TPs are assigned to closest active BSs, the system can then be simplified by reducing the number of variables as well as the number of equations. Indeed, for each given active BS j, the equations corresponding to all TPs i assigned to BS j are equivalent when considered as functions of $p_i g_{ij}$. Denoting the power received from any such TP i by $p'_j = p_i g_{ij}$, the single equation associated to each j in $\bar{S}$ can be rewritten as

$$\frac{p'_j}{\sum_{h=1}^{n} u_h g_{hj} \sum_{t=1}^{m} \frac{p'_t}{g_{ht}} x_{ht} - p'_j + \eta} = \mathrm{SIR}_{\text{target}}. \qquad (37)$$

The size of the resulting system is, thus, equal to the number of active BSs which is at most m and usually much smaller than the number n of TPs. This size reduction plays an important role in making our algorithms applicable to the more accurate model. Given the assignment $\mathbf{x}$ and the solution $\mathbf{p}'$ of the the above system, the emission power at the TPs are derived by setting $p_i = p'_j / g_{ij}$ if $p_i \leq P_{\max}$ and, otherwise, $p_i = P_{\max}$.

V. Computational Results

To evaluate the performance of the proposed algorithms, we have considered synthetic but realistic uplink instances generated by using Hatas propagation model [38]. For each instance, we consider a rectangular service area with dimensions $L \times W$, a number m of candidate sites in which to locate omnidirectional antennas, and a number n of TPs. Using a pseudorandom number generator each candidate site j and each TP i is assigned a position with uniform distribution in the service area. The matrix $\mathbf{G}$ is obtained by using Hata's formulas which give the attenuation A (loss) in decibels due to signal propagation. In particular, the attenuation for urban areas is given by

$$\begin{aligned} A_u = {} & 69.55 + 26.16 \log(F) - 13.82 \log(H_b) \\ & - [(1.1 \log(F) - 0.7) H_m - (1.56 \log(F) - 0.8)] \\ & + [44.9 - 6.55 \log(H_b)] \log d \end{aligned} \qquad (38)$$

where F is the signal frequency in megahertz, H_b and H_m are the heights of the base and the mobile station in meters, and d is the distance in kilometers, while the formula for rural areas is

$$A_r = A_u - 4.78[\log(F)]^2 + 18.33 \log(F) - 35.94. \qquad (39)$$

Clearly $g_{ij} = 1/10^{A(d_{ij})/10}$, where d_{ij} denotes the distance between TP i and candidate site j.

We considered three families of uplink instances with a service area of 400×400 m, 1×1 km, and 1.5×1.5 km, respectively. Different instances of these families are obtained using the pseudorandom generator. Two gain matrices $\mathbf{G}$ are considered for each configuration using the urban and the rural Hata's formulas with

$$H_b = 10\,m, \quad H_m = 1\,m, \quad F = 2000 \text{ MHz}. \qquad (40)$$

Small-size instances are characterized by a service area of 400×400 m, $m = 22$ candidate sites, $n = 95$ TPs, and $u_i = 1$ for all TPs $i \in I$. Medium-size instances are characterized by a service area of 1×1 km, $m = 120, n = 400$, and u_i uniformly distributed in $\{1, 2, 3\}$ for all $i \in I$. Large-size urban (LU) instances are characterized by a service area of 1.5×1.5 km,

TABLE I
RESULTS OBTAINED WITH ADD, REMOVE ($\rho_a = \rho_r = 0.3$), MULTISTART TS AND SINGLE-RUN TS FOR SMALL-SU AND RURAL (SR) INSTANCES WITH $m = 22, n = 95, u_i = 1$. FOR EACH ALGORITHM, FROM LEFT TO RIGHT: MINIMUM NUMBER OF BSs INSTALLED, THE AVERAGE NUMBER AND THE STANDARD DEVIATION OVER 50 RUNS

	Add			Remove			multistart TS (A)			multistart TS (R)			TS
SU-1	**5**	5.80	0.85	**4**	5.22	0.46	**4**	4.62	0.49	**4**	4.42	0.49	**4**
SU-2	**5**	5.22	0.41	**4**	4.74	0.44	**4**	4	0	**4**	4	0	**4**
SU-3	**4**	5.32	0.58	**4**	5.08	0.74	**4**	4.02	0.14	**4**	4	0	**4**
SU-4	**5**	6.72	0.96	**5**	5.58	0.64	**5**	5	0	**5**	5	0	**5**
SU-5	**4**	5.06	0.54	**4**	4.90	0.30	**4**	4	0	**4**	4	0	**4**
SR-1	**5**	5.74	0.69	**4**	5.34	0.65	**4**	4.50	0.50	**4**	4.44	0.49	**4**
SR-2	**4**	5.16	0.50	**4**	4.72	0.45	**4**	4	0	**4**	4	0	**4**
SR-3	**4**	5.18	0.51	**4**	4.72	0.49	**4**	4.02	0.14	**4**	4	0	**4**
SR-4	**5**	5.60	0.90	**5**	5.34	0.62	**5**	5	0	**5**	5	0	**5**
SR-5	**4**	5.04	0.45	**4**	5.02	0.37	**4**	4	0	**4**	4	0	**4**

$m = 200, n = 750$, and u_i uniformly distributed in $\{1, 2\}$ for all $i \in I$. Finally, all instances have uniform installation costs, namely $c_j = c$ for all $j \in S$.

The remaining parameters have been selected as follows: SF $= 128$ and $\tau = 4$ (so that $\mathrm{SIR}_{\min} = 0.03125$), $\mathrm{SIR}_{\mathrm{target}} = 6$ dB, $P_{\max} = 30$ dBm, $P_{\mathrm{target}} = -100$ dBm. In all the experiments reported here, $\omega = 1/(c\,m)$ and $\nu = 1/m^2$, which imply that in the utility functions (35) and (36), the first term has higher priority than the second term and that in (36) the second term has higher priority than the third one.

A. Shortcomings of the Model With Simplified SIR Formula

The first computational effort aimed at comparing the simplified power-based PC model (i.e., the basic model (12)–(16) with only the relevant variables x_{ij}) with the enhanced power-based PC model (19)–(23) in which intercell interference is explicitly considered. Setting $f = 0.4$ and the other parameters as mentioned above, in the simplified model the maximum number of users per BS, is bounded above by 23.97 and, hence, it is at most 23. The resulting models for the small instances have been solved to optimality by using CPLEX 7.0 MIP solver [30]. In all cases, the simplified model activates 5 BSs (which corresponds to the lower bound, since $95/23$ is about 4.1) while the optimal solution of the enhanced model always activates just four BSs.

To evaluate the quality of the solutions yielded by the simplified model, we consider the active BSs and the assignments of TPs specified by the optimal solution provided by CPLEX, and we compute the SIR value of each BS using a power-based PC mechanism as well as the power received by each BS using an SIR-based PC mechanism. Typical results obtained for one of the small instances of Table I [instance small-size urban (SU-3)] are reported in Fig. 1. SIR values and received powers are given for each of the five active BSs. Note that in the power-based PC setting, the power received is constant, while in the SIR-based setting the SIR is constant. In the power-based setting, SIR values often exceed the minimum level required to guarantee signal quality, while in the SIR-based setting, the SIR values are set to $\mathrm{SIR}_{\mathrm{target}} = 6$ and the actual received powers associated to a number of connections can be lower than P_{target}.

For a fair comparison between the solutions provided by the simplified and enhanced models, we changed the value of pa-

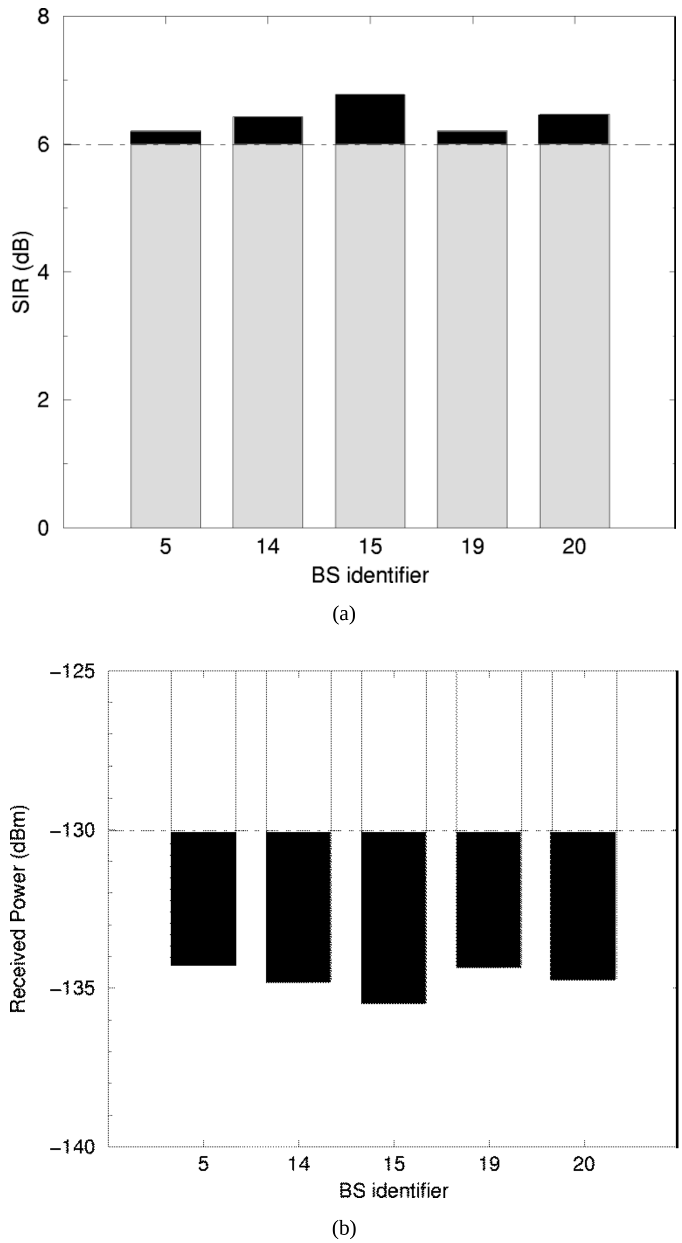

Fig. 1. Quality of the optimal solution (including five BSs) of the simplified model with $f = 0.4$ for instance SU-3 of Table I. (a) SIR values obtained with the power-based PC setting. (b) Received powers obtained with the SIR-based PC setting.

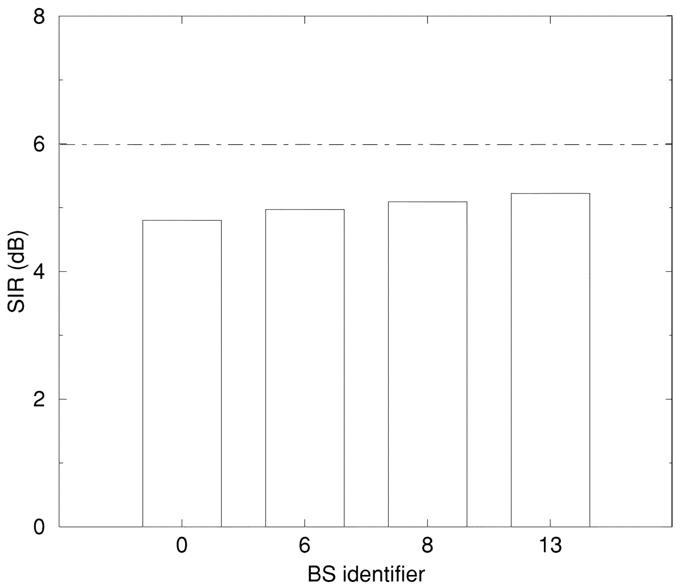

Fig. 2. Quality of the optimal solution (including four BSs) of the simplified model with $f = 0.35$ for instance SU-3: SIR values obtained with the power-based PC setting.

rameter f (setting it to 0.35) so that also the simplified model returns solutions with only four active BSs. The corresponding results obtained for the same instance of Fig. 1 are reported in Fig. 2. Notice that in the power-based PC setting, the SIR values drop far below any acceptable value, while in the SIR-based PC setting, it has not even been possible to find a feasible value for the received powers, meaning that the PC mechanism does not converge to a stable solution.

Interesting insight on the inadequacy of the location model with the simplified SIR constraints is provided by the distribution of the number of connections assigned to each active BS in the solutions of the other more sophisticated models. A typical example for a medium-size instance (instance MU-3) is shown in Fig. 3. In the the case of SIR-based PC model, a substantial fraction of BSs (here more than 37%) is assigned a number of connections that is larger than 23, which is the strict upper bound that would be imposed by the simplified power-based PC model. Solutions of the enhanced power-based PC model do also exhibit similar distributions.

All our experiments show that the model with the simplified SIR formula is not suitable for locating BSs in UMTS networks since it fails to capture some fundamental characteristics of third-generation systems.

B. Comparison of Heuristic Algorithms for Power-Based PC Model

In this subsection, the results obtained with the three algorithms for the enhanced power-based PC model (19)–(23) with no thermal noise $(\eta = 0)$ are reported and discussed.

Results for the small-size instances obtained with the randomized Add and Remove procedures $(\rho_a = \rho_r = 0.3)$ over 50 runs are reported in Table I. The solutions found always satisfy all 95 requested connections. Out of the ten instances mentioned in Table I, five have been generated using the Hata formula for urban areas (SU) and five using that for rural areas (SR). For the same instances, the optimal solutions, involving four BSs in all cases, have been obtained by using CPLEX 7.0. Although Add and Remove are heuristics with no guarantee to provide an optimal solution, they do so in half of these cases and the number of BSs installed never exceeds the minimum number by more than one. It is worth noting that the computation time required by Add or Remove to find a close-to-optimal solution for a single instance was less than 5 s on a Pentium III/700-MHz personal computer, while CPLEX 7.0 MIP solver required 5–20 min to find an optimal solution on an about twice as fast computer. As a matter of fact, solving even such small instances using a state-of-the-art MIP solver turns out to be a challenging task due to the very delicate choice of the value M. If the parameter M is too small, then some SIR constraints that should be omitted will not be disactivated and, therefore, the resulting solutions contain a larger number of BSs than optimal solutions of the actual problem. On the other hand, for too large values of M, one runs into numerical problems related to machine precision which tend to override some SIR constraints and, hence, to yield solutions with a too small number of BSs.

In Table I, multistart TS consists of 50 runs of Add (A) or Remove (R) followed by 200 iterations of TS while single-run TS consists of Remove followed by 2000 iterations of TS. For these small instances, a tabu list of length $L = 8$ is used, $\max_{\text{swap}} = 5, q = 1$ and $q' = 0$. As far as the best solutions are concerned, for these small instances the Remove procedure provides as good solutions as the TS variants. However, in multistart TS and single-run TS, the average quality of the solutions is better and the standard deviations are lower in all but one case. Due to this more robust behavior, a single run of multistart TS would suffice to yield the same best solutions for eight out of the ten instances.

Table II reports the results obtained with Add, Remove (both with 50 runs), multistart TS (Add and Remove), and single-run TS for ten medium-size instances. The number of TS iterations for multistart and single-run TS are as for small-size instances. For medium-size instances, a tabu list of length $L = 15$ is used, $\max_{\text{swap}} = 15$ and $q = q' = 0.3$. The solutions found provided in all but one cases full traffic coverage. The number of BSs installed by our Add and Remove randomized procedures differ at most by three. None of them consistently outperforms the other one. However, the standard deviation of the number of BSs in the solutions given by Remove (over the 50 runs) is usually higher than that of the solutions provided by Add. The solutions provided by the TS procedures require a few BSs less than Add and Remove. Although there is no significant difference among the three TS procedures, single-run TS provides in all but one cases the best solution and it always does so at lower computational cost.

The 50 runs of Add and Remove take in average 10 and, respectively, 30 s for the small-size instances, and 1:50 and, respectively, 1:20 h for the medium-size instances. For large-size instances more than 13 CPU hours are needed for Add and approximately 7 h for Remove. A single-run TS lasts in average 1:20 h for medium-size instances and 6 h for large-size ones. For solution quality comparison purposes, multistart TS was run with the same total number of TS iterations as single-run TS and

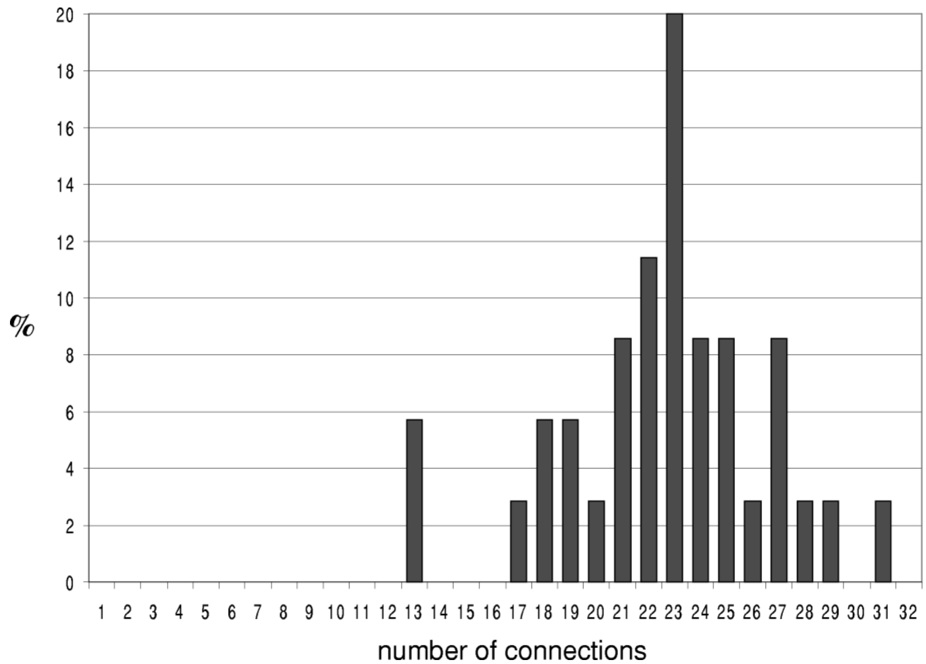

Fig. 3. Distribution of the number of TPs assigned to the various BSs for the medium-size instance MU-3.

TABLE II
RESULTS OBTAINED WITH THE ADD, REMOVE ($\rho_a = \rho_r = 0.3$), MULTISTART TS AND SINGLE-RUN TS FOR MEDIUM-SIZE URBAN (MU) AND RURAL (MR) INSTANCES WITH $m = 120, n = 400$, AND u_i UNIFORMLY DISTRIBUTED IN $\{1, 2, 3\}$. FOR EACH ALGORITHM, FROM LEFT TO RIGHT: MINIMUM NUMBER OF BSs INSTALLED, THE AVERAGE NUMBER, AND THE STANDARD DEVIATION OVER 50 RUNS.* NOT ALL THE TRAFFIC IS COVERED

	Add			Remove			multi TS (A)			multi TS (R)			TS
MU-1	**47***	50.30	1.53	**50**	56.16	7.36	**46**	46.90	1.04	**48**	48.50	0.50	**47**
MU-2	**46**	48.26	1.51	**46**	49.06	6.65	**43**	43.40	0.66	**43**	43.40	0.49	**43**
MU-3	**45**	46.40	1.18	**43**	48.98	4.06	**41**	41.00	0	**41**	41	0	**41**
MU-4	**45**	48.10	1.36	**44**	48.08	2.98	**42**	42.30	0.46	**42**	42.10	0.3	**42**
MU-5	**44**	47.54	1.37	**46**	49.86	3.26	**42**	42.30	0.46	**42**	42.10	0.3	**42**
MR-1	**44**	45.88	1.13	**42**	45.80	3.23	**40**	40.80	0.40	**41**	41	0	**40**
MR-2	**44**	47.72	1.48	**45**	48.60	5.72	**43**	43.10	0.30	**43**	43.30	0.46	**43**
MR-3	**43**	46.26	1.60	**44**	48.80	4.83	**41**	41.10	0.30	**41**	41.20	0.4	**41**
MR-4	**45**	48.34	1.57	**45**	49.08	4.5	**42**	42.10	0.30	**42**	42	0	**42**
MR-5	**44**	46.38	1.13	**46**	49.64	2.64	**42**	42.40	0.49	**42**	42.30	0.46	**42**

TABLE III
RESULTS OBTAINED WITH ADD AND REMOVE ($\rho_a = \rho_r = 0.3$) FOR LU INSTANCES WITH $m = 200, n = 750, u_i$ UNIFORMLY DISTRIBUTED IN $\{1, 2\}$. FROM LEFT TO RIGHT: FRACTION OF TRAFFIC COVERED, MINIMUM NUMBER OF BSs INSTALLED, AVERAGE NUMBER, AND STANDARD DEVIATION OVER TEN RUNS

	Add				Remove			
LU-1	1106/1107	**68**	70.80	1.89	1106/1107	**68**	100.34	17.41
LU-2	1133/1136	**72**	70.46	2.00	1136/1136	**73**	92.54	9.14
LU-3	1120/1124	**64**	67.46	2.15	1120/1124	**61**	73.14	13.10
LU-4	1130/1130	**63**	66.60	2.04	1130/1130	**69**	89.08	7.02
LU-5	1103/1103	**61**	64.66	2.02	1103/1103	**60**	64.40	2.68

this obviously led to higher (approximately twice as long) computing times.

As shown in Table II, multistart TS (ten runs) and single-run TS yield for all instances better solutions than the best ones obtained with Add and Remove. In many cases, these solutions are obtained within less than 500 local search iterations.

We have also applied Add and Remove (ten runs), multistart TS (ten runs with 200 iterations of local search each) as well as single-run TS (1000 iterations) to five LU instances. This choice of parameter values, with 1500 active connections in the average, is quite realistic for UMTS setting in medium-to-large cities. The results are reported in Tables III and IV. Unlike for the previous classes of instances, the traffic demand is not always satisfied. It is worth noting that Remove provides solutions with much higher average number of active BSs even though the best solution found is worse than the best one yield by Add only for two out of the five instances. In fact, the best solutions found with Remove turn out to be better for instances LU-3 as well as LU-5 and equivalent for LU-1. Moreover, for instance LU-2 the best solution provided by Remove contains an additional BS but it covers the whole traffic. The advantage of the Remove scheme to obtain the initial solutions is that it

TABLE IV

RESULTS FOR LU INSTANCES WITH $m = 200, n = 750, u_i$ UNIFORMLY DISTRIBUTED IN $\{1, 2\}$. FROM LEFT TO RIGHT: FRACTION OF TRAFFIC COVERED, MINIMUM NUMBER OF BSs INSTALLED, AVERAGE NUMBER, AND STANDARD DEVIATION OVER TEN RUNS

	multi TS (A)				multi TS (R)				TS
LU-1	1106/1107	**62**	63	0.89	1106/1107	**64**	66.9	1.64	**62**
LU-2	1136/1136	**63**	61.3	1.10	1136/1136	**63**	65.1	1.22	**63**
LU-3	1120/1124	**57**	57.5	0.67	1120/1124	**57**	58.1	1.14	**57**
LU-4	1130/1130	**57**	58	0.63	1130/1130	**58**	61.8	1.6	**58**
LU-5	1103/1103	**54**	54.7	0.78	1103/1103	**54**	54.7	1.01	**54**

naturally allows for testing whether all traffic demands can be satisfied.

According to Table IV, multistart TS and single-run TS always yield solutions of better quality than the best ones provided by Add and Remove. For these large-size instances, a tabu list of length $L = 15$ is used, $\max_{\text{swap}} = 15, q = 0.26$, and $q' = 0.3$. Note that for all five instances the improvement in terms of BSs installed is substantial.

C. Comparison Between Models With Power-Based and SIR-Based PC

We now discuss the results provided by the model with SIR-based PC (28)–(33) pointing out the differences with respect to those from the model with power-based PC (19)–(23). To obtain meaningful solutions with the SIR-based model, a nonzero thermal noise η must be considered. For comparison purposes, the same value of $\eta = -130$ dB is also used in the model with power-based PC.

Multistart TS is applied with ten runs and single-run TS with 2000 iterations. To maintain the computational times at a reasonable level, TPs have been assigned to closest BSs in terms of emitted powers, even though better performance (and possibly also lower costs) may be attained with different types of assignments. At each iteration, once the current solution is fully determined (i.e., a TP-BS assignment is derived for the current set of active BSs), the received powers are computed by solving the related linear system (37). This constitutes the heavy part of the computation. Tests have been conducted on the small-size instances and both algorithms provided solutions with four BSs, as in the enhanced power-based PC case. In order to emphasize the differences, experiments were also carried out with the medium-size instances. For each of the medium-size instances, multistart TS took about 15 h while single-run TS took about 9 h. These computing times have to be compared to the 2:30 and 1:30 h, respectively, for the same type of algorithms applied to the enhanced power-based PC. This clearly indicates the considerable additional computational load required to compute the power values at each iteration. Table V shows the numerical results for the ten medium-size instances. For multistart TS the minimum number of installed BSs is reported together with the average and the standard deviation (over ten runs), while for single-run TS the best solution is given.

Comparing the number of active BSs yielded by the enhanced power-based PC model with that obtained by the SIR-based PC model, we observe that the latter one allows for a saving of at least six BSs. This is due to the fact that, since the mobile stations can emit lower powers with respect to those in the power-based PC case, they generate lower interference toward

TABLE V

COMPARISON BETWEEN MODELS WITH SIR-BASED AND POWER-BASED PC: NUMBER OF INSTALLED BSs WITH TABU SEARCH ALGORITHMS

	SIR-based PC				**power-based PC**		
	multistart TS			TS	multistart TS		
Instance	best	average	st. dev.	best	best	average	st. dev.
MU-1	**39**	39	0	39	**52**	53.1	0.83
MU-2	**37**	37.1	0.3	36	**45**	45.5	0.5
MU-3	**35**	35.1	0.3	35	**43**	43.4	0.66
MU-4	**36**	36.7	0.46	36	**43**	44.1	0.94
MU-5	**36**	36.5	0.5	36	**43**	44.5	1.20
MR-1	**35**	35.9	0.3	36	**42**	42.5	0.5
MR-2	**37**	37	0	36	**45**	45.5	0.5
MR-3	**35**	35.1	0.3	35	**43**	43.3	0.46
MR-4	**36**	36.4	0.49	36	**43**	44.7	1.19
MR-5	**36**	36.6	0.49	36	**43**	43.9	0.83

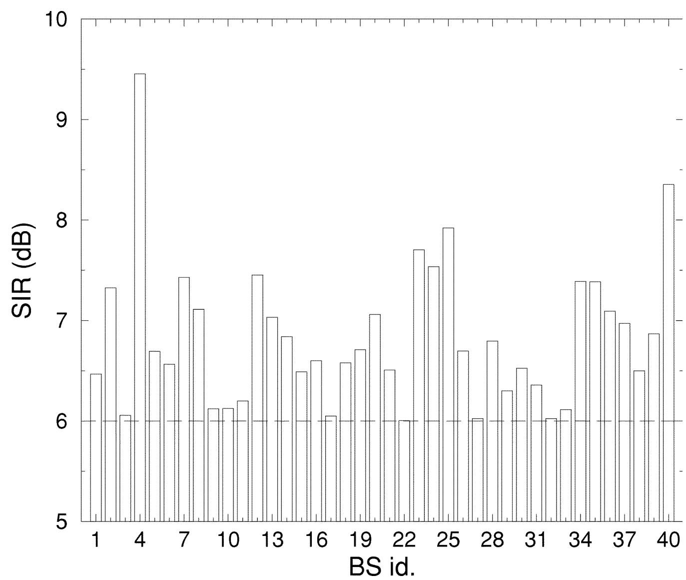

Fig. 4. Measured SIR values with the enhanced power-based PC model for instance MU-3.

all BSs they are not assigned to. Thus, the system can support more connections per BSs and, in practice, the capacity of the overall network is better exploited.

The above observations are also confirmed by comparing the SIR values at each BS and the received powers for each connection. Fig. 4 reports the SIR values measured in each active BS by using the enhanced power-based PC model. Notice that only a few BSs have an SIR equal to the required $\text{SIR}_{\text{target}}$ of 6 dB, and for many BSs the service quality is far above the required level. Thus, the emitted powers are higher than needed and as a result the interference levels are higher and the scarce radio resources are not used as efficiently as they could.

VI. Concluding Remarks

The main features characterizing the important problem of planning UMTS networks using the W-CDMA air interface have been discussed. Focusing on the uplink direction, we have investigated discrete optimization models for the UMTS BS location problem that capture at different levels of detail the peculiarities of the signal quality constraints and the PC mechanism for W-CDMA. Standard capacitated location models being inappropriate, we have proposed an enhanced power-based PC model as well as a more accurate SIR-based PC one.

Since solving even medium-size instances of these NP-hard problems is beyond the reach of state-of-the-art commercial optimization solvers, we have developed three heuristics for the power-based PC model. Our randomized greedy and reverse greedy procedures provide, in a reasonable amount of time, good approximate solutions for medium-to-large size realistic instances generated by using classical propagation models. A TS algorithm, which can be applied either in a multistart or single-run setting, allows us to further improve the approximate solutions obtained with these greedy procedures.

The three above algorithms have also been extended to the SIR-based model by assuming that TPs are assigned to a closest active BS. From the planning point of view, this is just a conservative assumption in the sense that there might exist solutions with less natural assignments but better objective function values.

Our experimental results show that the enhanced power-based PC model yields interesting solutions but those obtained with the SIR-based model use in a more efficient way the scarce radio resources and the computed SIR values are closer to the actual values in real systems.

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