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# Inertia Identification and Auto-Tuning of Induction Motor Using MRAS

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**Abstract**-During the operation of an induction motor (IM) drive system, parameters' vibration and external load disturbance will influence the effectiveness of control. A method using Model Reference Adaptive System (MRAS) algorithm is presented in this paper for an online identification of load inertia. Further, by using the result of identification, automatic adjustments to the speed controller can be made. A scheme for tuning the speed controller automatically is introduced. Simulation results in both time domain and frequency domain show the usefulness of the proposed method.

**Key words:** induction motor, inertia identification, MRAS, auto-tuning, frequency analyze

## 1. Introduction

The system performance of the IM is influenced by uncertainties such as unpredictable plant parameter variations, external load disturbances and unmodeled nonlinear dynamics of the plant. In order to get high performance, accurate values of the machine's parameters are necessary[1]. Drive moment of inertia mainly affects the mechanical response of the system. When the moment of inertia increases, the system response will be slower. Therefore, it is necessary to identify the moment of inertia of the drive.

Once the drive inertia is identified accurately, the parameters of the controller can be adjusted to compensate the inertia variation. System performance

can be improved in this way. In this paper, a method for identification of the drive inertia and auto-tuning the speed controller is presented. Analysis based on the simulation in both the time and frequency domain shows that the proposed scheme has good performance even with the dramatic variation of load inertia.

## 2. Control Strategy

The control method in this paper relies on the Field Orientated Control (F. O. C.).

In the synchronous frame, the rotor flux can be described as:

$$\varphi_{dr} = L_r i_{dr} + L_m i_{ds} \quad (1)$$

$$\varphi_{qr} = L_r i_{qr} + L_m i_{qs} \quad (2)$$

Rotor voltage as:

$$0 = pL_m i_{ds} + \omega_r L_m i_{qs} + (pL_r + R_r) i_{dr} + \omega_r L_r i_{qr} \quad (3)$$

$$0 = -\omega_r L_m i_{ds} + pL_m i_{qs} - \omega_r L_r i_{dr} + (pL_r + R_r) i_{qr} \quad (4)$$

Stator voltage as:

$$u_{ds} = (pL_s + R_s) i_{ds} + pL_m i_{dr} \quad (5)$$

$$u_{qs} = (pL_s + R_s) i_{qs} + pL_m i_{qr} \quad (6)$$

Where

$$p = d/dt$$

$i_{ds}$ ,  $i_{qs}$  are the d-axis and q-axis stator current

$i_{dr}$ ,  $i_{qr}$  are the d-axis and q-axis rotor current

$R_r$ ,  $L_r$  are the resistance and inductance of rotor

$L_m$  is the magnetizing inductance

$\omega_r$  is the rotor angular speed

$\varphi_{dr}$ ,  $\varphi_{qr}$  is the d-axis and q-axis rotor flux

$u_{ds}$ ,  $u_{qs}$  are the d-axis and q-axis stator voltage

$R_s, L_s$  are the resistance and inductance of stator

In the F.O.C. algorithm,  $\varphi_{dr}=\text{constant}$ ,  $\varphi_{qr}=0$ . This algorithm maintains efficiency in a wide range of speeds and takes into consideration torque changes with transient phases by controlling the flux directly from the rotor coordinates.

Fig. 1 shows the system configuration of a field oriented controlled IM drive system. This basic control structure includes an inner flux control loop and a speed control loop.

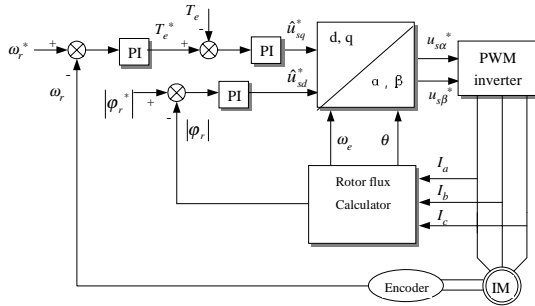


Fig.1 Block diagram of the control system

### 3. Parameter Identification Using MRAS

The discrete form of mechanical movement equation in the IM drive system can be described as:

$$\omega_r[i] = \omega_r[i-1] + \frac{T}{J}(T_e[i-1] - T_r[i-1]) \quad (7)$$

The load torque in (7) is assumed to change slowly. In a servo system that requires a fast response, sampling period tends to be shorter. Therefore, this assumption is reasonable. This leads to the following equation:

$$\omega_r[i] = 2\omega_r[i-1] - \omega_r[i-2] + \frac{T}{J}(T_e[i-1] - T_r[i-2]) \quad (8)$$

Where

$J$  is the inertia to be identified,

$T$  is the sampling period,

$T_e$  is the electromagnetic torque.

Equation (8) can be regarded as the difference equation of the reference mode. Using Landau's discrete time-recursive parameter identification [2], the MRAS algorithm is designed as:

$$\hat{y}[i] = 2y[i-1] - y[i-2] + \hat{b}[i-1]U[i-1] \quad (9)$$

$$\varepsilon[i] = y[i] - \hat{y}[i] \quad (10)$$

$$\hat{b}[i] = \hat{b}[i-1] + \beta \frac{U[i-1]}{1 + \beta U[i-1]^2} \varepsilon[i] \quad (11)$$

$$U[i-1] = T_e[i-1] - T_r[i-2] \quad (12)$$

Where

$b$  is the variable of  $T/J$  to be identified,

$y$  is the measured speed signal,

$\beta$  is the adaptive gain,

$\hat{b}$  is the identified variable,

$\hat{y}$  is the estimated speed.

Equation (9) is regarded as the difference equation for an adjustable model, and (11) as the adaptation mechanism.

### 4. Auto-Tuning of Speed Controller

Proportional integral (PI) controller is one of the most widely used controllers in industry. In this case, it is used both as flux and speed controllers.

By choosing the appropriate PI controller parameters, the close-loop time constant of flux loop is in the region of 10 times smaller than that of the speed control loop. Therefore, the flux loop in Fig. 1 can be designed before the speed loop.

The simplified system is a typical 3-order one, and the open-loop transfer function of speed control loop is:

$$G_{op}(s) = \frac{K_{op}(\tau s + 1)}{s^2(Ts + 1)} \quad (13)$$

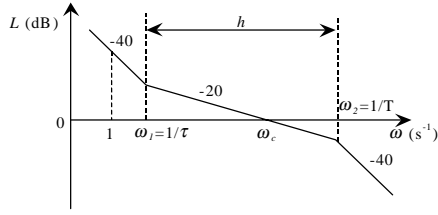
There are several methods to design such a system [3]. Some of them consider the time domain characteristics such as rising time and overshoot, while others consider frequency domain characteristics such as cutting frequency and phase margin, etc.

In the simulation of this paper, a method using "Least  $M_r$  criteria" is applied. Here,  $M_r$  is the peak value of the close-loop amplitude frequency response characteristics.

The Bode diagram of open-loop transfer function (13) is shown in Fig. 2.

In the figure below,  $\omega_1$  and  $\omega_2$  are turn frequencies,  $\omega_c$  is the cutting frequency.  $h = \tau/T = \omega_2/\omega_1$  is called Mid-frequency Width. The middle frequency  $(\omega_2 - \omega_1)$  is a

critical factor in determining the control system's dynamic performance, so  $h$  is a key parameter.



**Fig. 2** Open-loop amplitude frequency response characteristics of the typical 3-order system

It is proved that in order to obtain the least close-loop amplitude frequency response characteristics peak value  $M_{rMin}$ , the following relations should be satisfied:

$$\frac{\omega_2}{\omega_c} = \frac{2h}{h+1} \quad (14)$$

And

$$\frac{\omega_c}{\omega_1} = \frac{h+1}{2} \quad (15)$$

With the corresponding least peak value  $M_r$ :

$$M_{rMin} = \frac{h+1}{h-1} \quad (16)$$

Once  $h$  is given, the open-loop transfer function is obtained, and the parameter of speed controller can be calculated.

## 5. Simulation Results

### 5.1 Parameter Identification:

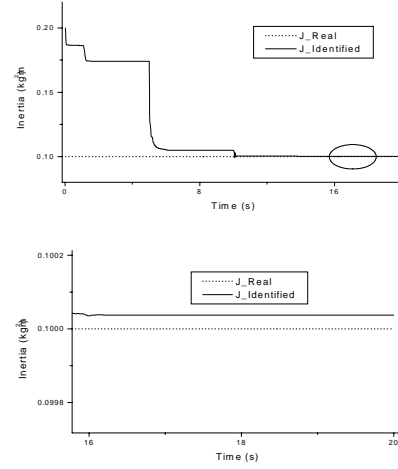
The parameters of the squirrel cage motor used in simulation and experiment are the same. They are depicted in Table 1.

**Table 1**

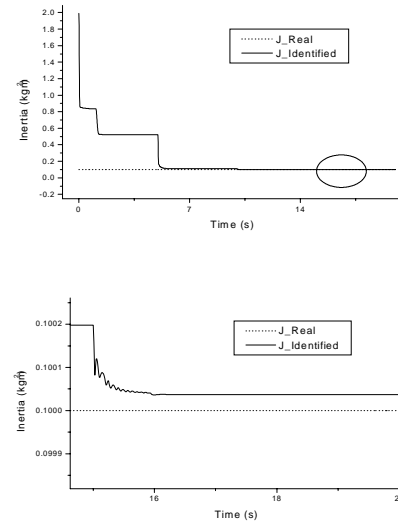
| Parameters of the Induction Motor |                   |            |                                 |
|-----------------------------------|-------------------|------------|---------------------------------|
| Pole                              | 4                 | $J$        | $0.1 \text{ kg}\cdot\text{m}^2$ |
| $R_s$                             | $2.92 \Omega$     | $R_r$      | $1.92 \Omega$                   |
| $L_s$                             | $0.371 \text{ H}$ | $L_r$      | $0.371 \text{ H}$               |
| $L_m$                             | $0.358 \text{ H}$ | $\omega_0$ | $1430 \text{ r/min}$            |

The simulation results of identification with different initial values are shown in Fig. 3 and Fig. 4 respectively. The nether figure is the detail of the circled area in the

upper figure.



**Fig. 3** Simulation of inertia identification result with initial value of  $2J_n$  and  $\beta=5$



**Fig. 4** Simulation of inertia identification result with initial value of  $20J_n$  and  $\beta=5$

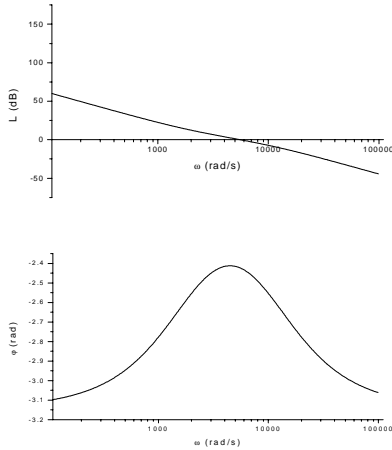
The figures above demonstrate that all the identification results converge to the real value quickly with the appropriately chosen simulation parameters.

### 5.2 Speed Controller Auto-tuning:

A PI controller is used to adjust the speed control loop so that it has suitable poles and zeros. In the frequency domain, it means appropriate cutting frequency and margin. Both the time domain response and frequency response of the system is simulated.

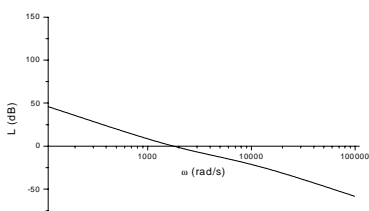
The Bode diagram of the desired system is shown in

Fig. 5. The upper one is a plot of amplitude frequency response characteristics and the nether one is phase frequency response characteristics. This desired system is a typical 3-order one, with mid-frequency width  $h=5$ .

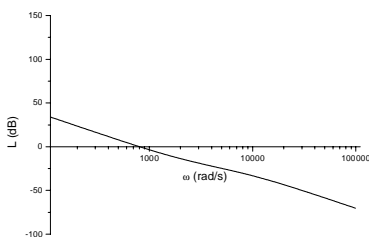


**Fig. 5** Bode diagram of desired 3-order system

Bode diagrams with  $J=5J_n$  and  $J=20J_n$  before tuning the speed controller are shown below. Because the phase frequency response characteristics remains unchanged, it is not shown here. The figures demonstrate that the mid-frequency width remains the same, while the cutting frequency becomes smaller with the increasing of  $J$ . This result implies the deterioration of the system's high frequency performance, thus making the system response slower.



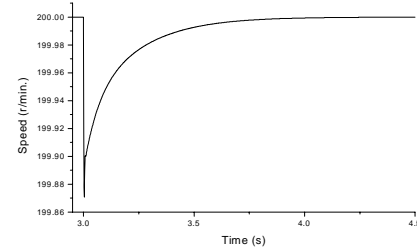
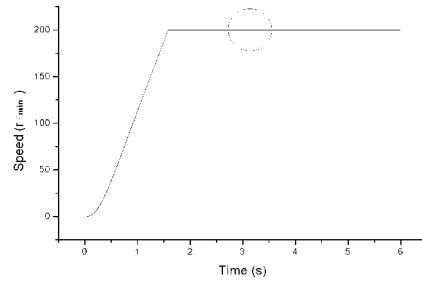
**Fig. 6** Bode diagram of system with  $J=5J_n$  before tuning the controller



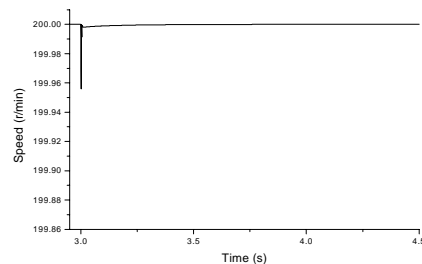
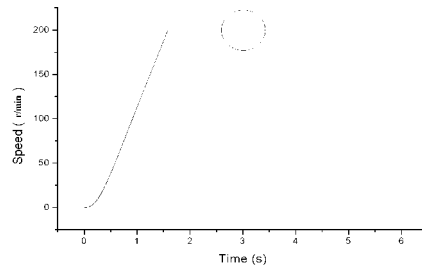
**Fig. 7** Bode diagram of system with  $J=20J_n$  before tuning the controller

In the simulation, the motor is speed up with no load, then at a certain time, a load torque is added to the motor shaft. In this case, the load torque added after the time of 3.0s is  $4.5N \cdot m$ , i.e.  $J=5J_n$ .

Fig.8 and Fig.9 show the simulation results of the load step responds without and with auto-tuning of the speed controller respectively. The nether figure is the detail of the circled area in the upper figure.



**Fig. 8** simulation result of load step response without auto-tuning



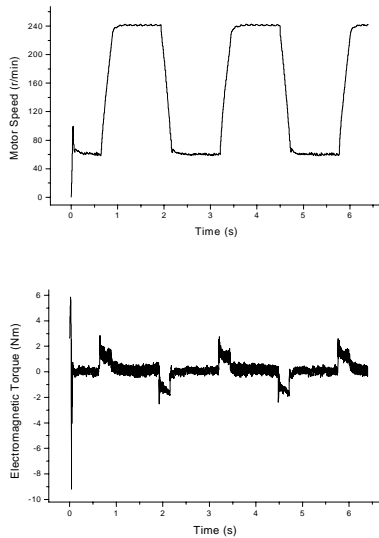
**Fig. 9** simulation result of load step response with auto-tuning

Obviously, in the results with auto-tuning, the overshoot and the recover time are both much less than those of the results without auto-tuning. The system performance is greatly improved.

## 6. Identification Experiment

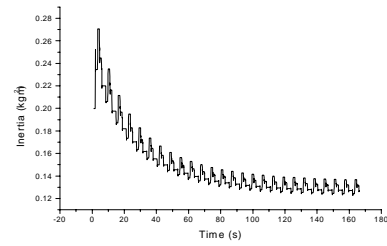
Identification experiments are implemented on a flexible control system of IM. The core of the dual-CPU control system is a personal computer and a TMS320C30 DSP (Digital Signal Processor) developing board. User interface, PWM pulse generation, A/D data acquisition and DSP control are all done by the host personal computer. DSP deals with control algorithm, parameter estimation and state observation.

The waveforms of real speed and electromagnetic torque in the experiment system of identification are demonstrated in Fig. 10. The speed waveform is satisfactory.

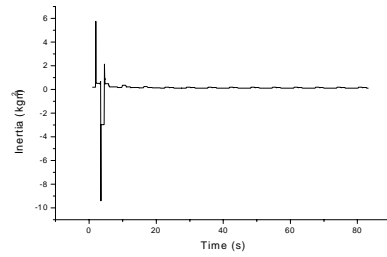


**Fig. 10** Speed and torque waveforms of the IM control system

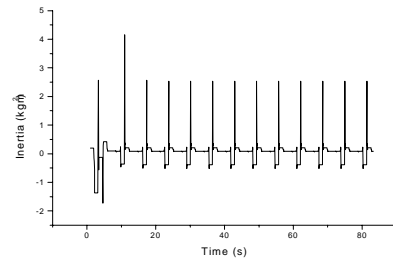
Experiment results of inertia identification with different adaptive gains on an IM are demonstrated in Fig. 11 — Fig.13. All the experiments are under the condition of no load. The initial value of inertia is set as  $0.2\text{kg}\cdot\text{m}^2$ .



**Fig. 11** Inertia identification result with  $\beta=1.0$



**Fig. 12** Inertia identification result with  $\beta=5.0$



**Fig. 13** Inertia identification result with  $\beta=20.0$

From the figure above, it can be seen that all the identification results converge to the same value, approximately  $0.13\text{kg}\cdot\text{m}^2$ , and with the increase of  $\beta$ , the process of identification becomes faster, but there may be some vibrations.

## 7. Conclusion

A method using MRAS to improve the performance of the IM drive system under parameter variation is presented. There are two steps: first to identify the load inertia, and then tune the speed controller automatically. The design method is based on frequency analysis. The system's speed step and load step responses are used to examine the validity of the proposed auto-tuning

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algorithm.

The frequency response analysis shows that the identification result is very effective for the design of speed controller, the system has good performance after auto-tuning. It is proved by the simulation results.

The proposed identification method is confirmed by the experiment on an IM system.

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