Signal Processing 74 (1999) 317–322

Locally optimum detector for correlated random signals in a weakly dependent noise model

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Received 24 April 1998; received in revised form 18 August 1998

Abstract

In this paper, we consider a discrete-time random signal detection problem under the presence of additive noise exhibiting weak dependence. We derive the test statistic of the locally optimum detector under a weakly dependent noise model. The performance characteristic of the locally optimum detector is analyzed and compared with that of the square-law detector in term of asymptotic relative efficiency. © 1999 Elsevier Science B.V. All rights reserved.

Zusmmenfassung

In dieser Arbeit wird ein Problem der Detektion eines zeitdiskreten Zufallssignals in additivem Rauschen mit schwacher Abhängigkeit betrachtet. Die Teststatistik des lokal optimalen Detektors wird für ein Rauschmodell mit schwacher Abhängigkeit abgeleitet. Die Leistungsfähigkeit des lokal optimalen Detektors wird analysiert und mit jener des quadratischen Detektors in Hinblick auf asymptotische relative Effizienz verglichen. © 1999 Elsevier Science B.V. All rights reserved.

Résumé

Dans cet article, nous considérons un problème de détection de signaux aléatoires en temps discret, en présence de bruit additif présentant une dépendance faible. Nous dérivons une statistique de test du détecteur localement optimal sous un modèle de bruit faiblement dépendant. La performance du détecteur localement optimum est analysée et comparée avec celle du détecteur à loi quadratique, en termes d'efficacité relative asymptotique. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Locally optimum detector; Weakly dependent noise; Random signals

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PII: S0165-1684(98)00219-9

1. Introduction

The signal detection problem in noisy observations has been considered in many previous studies. Among the various signal detection problems, weak signal detection has been of much interest in detection theory and applications. Among the typical investigations on locally optimum (LO) detectors are those considered in [1,4,5,11].

It has been commonly assumed that the additive noise samples are statistically independent. In practice, however, this assumption is often violated, and the optimum detectors designed under this assumption are no longer optimum in practice. Such a situation becomes more realistic as the sampling rate gets higher. Thus, investigations on signal detections in dependent noise should be considered. Among the investigations on the general optimum detection problem under various dependent noise models, including the ϕ-mixing noise model, m-dependent noise model, and transformation noise model, are those in [2,3,7,9]. When the dependence of noise is weak, we can use a simpler model. In [6,8], the first order moving average (MA) of an i.i.d. random process is considered as a weakly dependent noise model. In these studies, however, detection schemes only for known signals were considered.

In this paper, we will investigate the LO detection for *random signals* under a weakly-dependent noise model. The weakly dependent noise will be modeled as the first order MA of an i.i.d. random process.

2. Observation model

In this paper, we will consider the detection of discrete-time random signals in weakly dependent noise environment. Let H_0 be the null hypothesis and H_1 be the alternative hypothesis. Then, the observation model can be written as

$$\begin{aligned} &H_0\colon X_i = W_i, \quad i = 1,2,\dots,n, \\ &H_1\colon X_i = \theta s_i + W_i, \quad i = 1,2,\dots,n, \end{aligned} \tag{1}$$

where $\{X_i\}$ are the observations, $\{W_i\}$ are the weakly-dependent noise components, θ is the signal strength parameter, $\{s_i\}$ are the random signal components with mean zero and variance $\{\sigma_i^2\}$. Then, the detection problem becomes a problem of the hypothesis decision based on the n observations, $\{X_i\}$.

Weakly dependent noise can generally be modeled by the Volterra expansion with Volterra kernels and independent random processes [10]. This model, however, is almost intractable to handle because of the infinitely many terms of the expansion. In [6,8], simple first-order bilateral and unilateral MAs of an i.i.d. random process are used to model the weakly dependent noise, respectively. These two MA models are simple and good approximations to a weakly dependent noise. In addition, it was shown that the LO detector designed under one MA noise model can be applied with slight changes in the other one with almost the same performance [6]. In this paper, we will assume that the weakly dependent noise W_i, $i = 1,2,\dots,n$, are the unilateral MA of i.i.d. random variables:

$$W_i = e_i + \rho e_{i-1} u_{i-2}, \tag{2}$$

where e_i, $i = 1,2,\dots,n$, are the i.i.d. random variables with common p.d.f. f_e. The p.d.f. f_e is even symmetric with bounded continuous derivatives and satisfies the regularity condition [4]. This model is not only an analytically tractable model but also a good representation of practical dependent noise when the dependence is weak. Here, ρ is called the dependence parameter determining the correlation coefficient of W_i, and u_i is the unit step sequence, i.e., $u_i = 0$ when $i < 0$ and $u_i = 1$ when $i \geqslant 0$.

Let $\boldsymbol{X}$, $\boldsymbol{w}$, $\boldsymbol{e}$ and $\boldsymbol{s}$ be the n-tuple vectors representing $(x_1,x_2,\dots,x_n)$, $(W_1,W_2,\dots,W_n)$, $(e_1,e_2,\dots,e_n)$ and $(s_1,s_2,\dots,s_n)$, respectively, and $f_w(\boldsymbol{w})$, $f_e(\boldsymbol{e}) = \prod_{i=1}^n f_e(e_i)$ and $f_s(\boldsymbol{s})$ be the p.d.f.s of $\boldsymbol{w}$, $\boldsymbol{e}$ and $\boldsymbol{s}$, respectively. Then, under H_1 we have

$$\begin{aligned} f_w(\boldsymbol{w}) = {} & f_e(X_1 - \theta s_1) f_e(X_2 - \theta s_2 - \rho(X_1 - \theta s_1)) \\ & \cdots f_e(X_n - \rho X_{n-1} + \cdots + (-\rho)^{n-1} X_1 \\ & - \theta(s_n - \rho s_{n-1} + \cdots + (-\rho)^{n-1} s_1) \end{aligned}$$

$$= \prod_{i=1}^{n} f_e(Y_i - \theta c_i)$$

$$= f_e(\boldsymbol{Y} - \theta \boldsymbol{c}), \tag{3}$$

where $\boldsymbol{Y} = (Y_1, Y_2, \dots, Y_n)$, $Y_i = \sum_{k=0}^{i-1}(-\rho)^k X_{i-k}$, $\boldsymbol{c} = (c_1, c_2, \dots, c_n)$ and $c_i = \sum_{k=0}^{i-1}(-\rho)^k s_{i-k}$.

3. The locally optimum detector

Let us define

$$\phi(\boldsymbol{X}|\theta) = \int_{\mathbb{R}^n} f_w(\boldsymbol{X} - \theta \boldsymbol{s}) f_s(\boldsymbol{s})\,\mathrm{d}\boldsymbol{s}, \tag{4}$$

where $\mathbb{R}^n$ is the set of all n-tuples of real numbers. Then, the LO test statistic can be calculated by [4]

$$T_{\mathrm{LO}}(\boldsymbol{X}) = \frac{\mathrm{d}^v \phi(\boldsymbol{X}|\theta)/\mathrm{d}\theta^v|_{\theta=0}}{\phi(\boldsymbol{X}|0)}, \tag{5}$$

where v is the order of the first nonzero derivative of $\phi(\boldsymbol{X}|\theta)$ at $\theta = 0$. From Eqs. (3) and (4), it is easily seen that

$$\left.\frac{\mathrm{d}^2\phi(\boldsymbol{X}|\theta)}{\mathrm{d}\theta^2}\right|_{\theta=0} = \int_{\mathbb{R}^n} \left.\frac{\mathrm{d}^2 f_e(\boldsymbol{Y} - \theta\boldsymbol{c})}{\mathrm{d}\theta^2}\right|_{\theta=0} f_s(\boldsymbol{s})\,\mathrm{d}\boldsymbol{s}$$

$$= \int_{\mathbb{R}^n} f_e(\boldsymbol{Y}) f_s(\boldsymbol{s}) \left[\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} c_i c_j g_{\mathrm{LO}}(Y_i) \times g_{\mathrm{LO}}(Y_j) + \sum_{i=1}^{n} c_i^2 h_{\mathrm{LO}}(Y_i)\right] \mathrm{d}\boldsymbol{s} \tag{6}$$

and

$$\phi(\boldsymbol{X}|0) = \int_{\mathbb{R}^n} f_e(\boldsymbol{Y}) f_s(\boldsymbol{s})\,\mathrm{d}\boldsymbol{s} = f_e(\boldsymbol{Y}), \tag{7}$$

where $g_{\mathrm{LO}}(x) = -f_e'(x)/f_e(x)$ and $h_{\mathrm{LO}}(x) = f_e''(x)/f_e(x)$. Then, the LO test statistic can be obtained as

$$T_{\mathrm{LO}}(\boldsymbol{Y}) = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} E_s\{c_i c_j\} g_{\mathrm{LO}}(Y_i) g_{\mathrm{LO}}(Y_j) + \sum_{i=1}^{n} E_s\{c_i^2\} h_{\mathrm{LO}}(Y_i), \tag{8}$$

where $E_s\{\cdot\}$ is the expectation over $\boldsymbol{s}$.

It is easily seen that the test statistic (8) is the same as that obtained for independent noise except that we use weighted averages Y_i of the output samples and correlation coefficients of the weighted averages c_i of the signal components. It is also seen that $n-1$ memories are required to implement the LO detector. Thus, it is clear that the implementation of the LO detector becomes more inefficient as the sample size gets larger. We can, however, obtain finite memory approximations, which are easy to implement and have less memory requirement, to the exact LO detector from (8) by ignoring higher-order terms of ρ. The performance of the exact LO detector is then the upper bound of that of those finite memory detectors. Due to the fact that $|\rho|$ is small, the performance of the finite memory detectors is expected to be acceptable, which was shown in [6] for the known signal case.

4. Performance analysis

In this section, we will analyze the performance characteristics of the LO detector under the weakly dependent noise model. The performance of the LO detector for known signals in weakly dependent noise was studied and shown to be better than those of the linear correlator and the sign correlator in [6]. In this paper, the performance of the LO detector will be compared with that of the square-law (SQ) detector whose test statistic is

$$T_{\mathrm{SQ}} = \sum_{i=1}^{n} X_i^2. \tag{9}$$

In comparing the asymptotic performance of two detectors, the asymptotic relative efficiency (ARE) is generally employed. Under some regularity conditions [4] the $\mathrm{ARE}_{1,2}$ of detector D_1 with respect to detector D_2 can be expressed as $\mathrm{ARE}_{1,2} = \xi_1/\xi_2$, where ξ_i is the efficacy of D_i calculated as

$$\xi_i = \lim_{n\to\infty} \frac{[\mathrm{d}^v E\{T_i|\mathrm{H}_1\}/\mathrm{d}\theta^v|_{\theta=0}]^2}{nV\{T_i|\mathrm{H}_0\}}, \quad i = 1,2. \tag{10}$$

In Eq. (10), T_i denotes the test statistic of the detector D_i, $E\{T_i|\mathrm{H}_1\}$ denotes the expected value of T_i under the alternative hypothesis, and $V\{T_i|\mathrm{H}_0\}$ denotes the variance of T_i under the null hypothesis.

Theorem 1. *The efficacy of the LO detector is*

$$\xi_{\mathrm{LO}} = 2I_1^2(f_{\mathrm{e}})[\langle E_s^2(\boldsymbol{c},\boldsymbol{c})\rangle - \langle E_s^2(\boldsymbol{c}^2)\rangle] + I_2(f_{\mathrm{e}})\langle E_s^2(\boldsymbol{c}^2)\rangle, \tag{11}$$

where

$$\langle E_s^2(\boldsymbol{c},\boldsymbol{c})\rangle = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} E_s^2\{c_i c_j\} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} Q_{2,i,k}(-\rho) \times Q_{2,j,l}(-\rho) r_s(i,j) r_s(k,l), \tag{12}$$

$$\langle E_s^2(\boldsymbol{c}^2)\rangle = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} E_s^2\{c_i^2\} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} Q_{4,i,j,k,l}(-\rho) \times r_s(i,j) r_s(k,l), \tag{13}$$

$$I_1(f) = \int\left(\frac{f'(y)}{f(y)}\right)^2 f(y)\,\mathrm{d}y, \tag{14}$$

$$I_2(f) = \int\left(\frac{f''(y)}{f(y)}\right)^2 f(y)\,\mathrm{d}y, \tag{15}$$

$$Q_{2,i,j}(x) = \frac{x^{2\max(i,j)-i-j} - x^{2n-i-j+2}}{1-x^2}, \tag{16}$$

$$Q_{4,i,j,k,l}(x) = \frac{x^{4\max(i,j,k,l)-i-j-k-l} - x^{4n-i-j-k-l+4}}{1-x^4} \tag{17}$$

and

$$r_s(i,j) = E_s\{s_i s_j\}. \tag{18}$$

Theorem 2. *The efficacy of the SQ detector is*

$$\xi_{\mathrm{SQ}} = \frac{4((1-\rho^2)\langle E_s(\boldsymbol{c}^2)\rangle + 2\rho\langle E_s(\boldsymbol{s},\boldsymbol{c})\rangle)^2}{(1+\rho^2)^2 m_4 - (1-\rho^2)^2\sigma_{\mathrm{e}}^4}, \tag{19}$$

where

$$\langle E_s(\boldsymbol{c}^2)\rangle = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} E_s\{c_i^2\} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{2,i,j}(-\rho) r_s(i,j), \tag{20}$$

$$\langle E_s(\boldsymbol{s},\boldsymbol{c})\rangle = \lim_{n\to\infty}\frac{1}{n}\sum_{i=2}^{n} E_s\{s_i c_{i-1}\} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=2}^{n}\sum_{j=1}^{i-1}(-\rho)^{i-j-1} r_s(i,j), \tag{21}$$

$$m_4 = \int x^4 f_{\mathrm{e}}(x)\,\mathrm{d}x \tag{22}$$

and

$$\sigma_{\mathrm{e}}^4 = \left(\int x^2 f_{\mathrm{e}}(x)\,\mathrm{d}x\right)^2. \tag{23}$$

The proofs of Theorems 1 and 2 are shown in Appendix A.

Now, let us consider some examples to show the asymptotic performance of the LO detector more explicitly.

Example 1. Let $r_s(i,j) = r^{|i-j|}$, where $0 < |r| < 1$ and $f_{\mathrm{e}}(x) = (1/\sqrt{2\pi})\mathrm{e}^{-x^2/2}$. Then, we have $I_1(f_{\mathrm{e}}) = 1$, $I_2(f_{\mathrm{e}}) = 2$, $m_4 = 3$, $\sigma_{\mathrm{e}}^4 = 1$,

$$\langle E_s^2(\boldsymbol{c},\boldsymbol{c})\rangle = \frac{(1-\rho r)K(\rho,r)}{(1+\rho r)^3(1-\rho^2)^3(1-r^2)}, \tag{24}$$

$$\langle E_s^2(\boldsymbol{c}^2)\rangle = \frac{(1-\rho r)^2}{(1-\rho^2)^2(1+\rho r)^2}, \tag{25}$$

$$\langle E_s(\boldsymbol{c}^2)\rangle = \frac{1-\rho r}{(1-\rho^2)(1+\rho r)} \tag{26}$$

and

$$\langle E_s(\boldsymbol{s},\boldsymbol{c})\rangle = \frac{r}{1+\rho r}, \tag{27}$$

where

$$K(\rho,r) = (1+\rho^2 r^2)(1-r^2)(1-\rho^2) + 2(r-\rho)^2(1+\rho r)^2. \tag{28}$$

Then, from Theorems 1 and 2, the $\mathrm{ARE}_{\mathrm{LO,SQ}}$ is

$$\mathrm{ARE}_{\mathrm{LO,SQ}} = \frac{(1-\rho r)(1+4\rho^2+\rho^4)K(\rho,r)}{(1-\rho^2)^3(1+\rho r)^3(1-r^2)}. \tag{29}$$

Example 2. Let $r_s(i,j) = r^{|i-j|}$, where $0 < |r| < 1$ and $f_e(x) = e^{-x}/(1+e^{-x})^2$. Then, we have $I_1(f_e) = \frac{1}{3}$, $I_2(f_e) = \frac{1}{5}$, $m_4 = \frac{7}{15}\pi^4$ and $\sigma_e^4 = \frac{1}{9}\pi^4$. Then, from Theorems 1 and 2, the $\mathrm{ARE}_{\mathrm{LO,SQ}}$ is

$$\mathrm{ARE}_{\mathrm{LO,SQ}} = \frac{\pi^4(4+13\rho^2+4\rho^4)}{2025} \times \left(\frac{10(1-\rho r)K(\rho,r)}{(1+\rho r)^3(1-\rho^2)^3(1-r^2)} - \frac{(1-\rho r)^2}{(1-\rho^2)^2(1+\rho r)^2} \right). \quad (30)$$

In Figs. 1 and 2 , the $\mathrm{ARE}_{\mathrm{LO,SQ}}$ derived in the two examples are plotted for various values of ρ when the additive noise is the first-order MA of the i.i.d. Gaussian process and the i.i.d. symmetric logistic process, respectively.

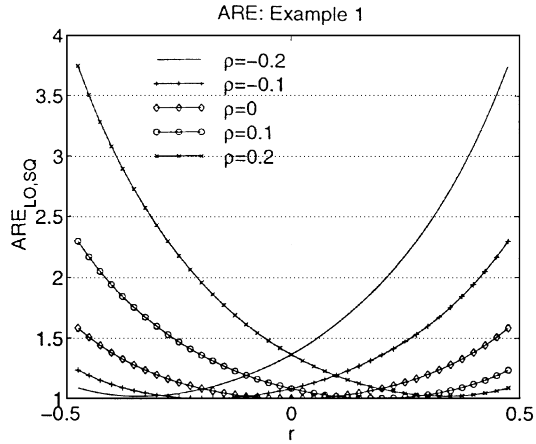

Fig. 1. $\mathrm{ARE}_{\mathrm{LO,SQ}}$ for various values of ρ when the noise is the first-order MA of the i.i.d. Gaussian process.

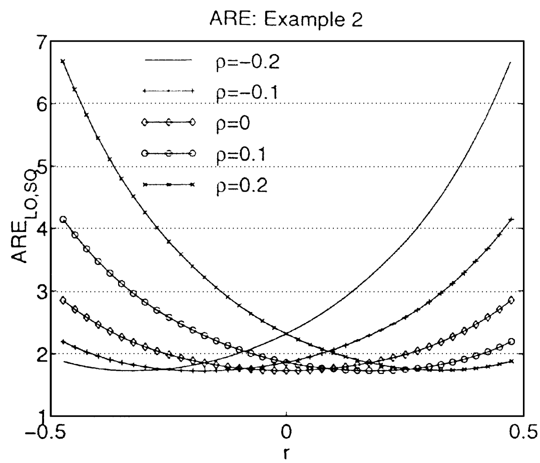

Fig. 2. $\mathrm{ARE}_{\mathrm{LO,SQ}}$ for various values of ρ when the noise is the first-order MA of the i.i.d. symmetric logistic process.

5. Concluding remark

In this paper, we considered the LO detection of random signals in additive weakly dependent noise. The test statistic of the LO detector for random signals in weakly dependent noise was derived and shown to have the same structure as that for independent noise with additional weighted averaging of output samples. The asymptotic performance of the LO detector was analyzed and compared to that of the SQ detector in terms of ARE. It was shown that the LO detector outperformed the SQ detector more as the correlation coefficient of the signal differed more from that of the noise.

Acknowledgements

This research was supported by a 1997 Grant from the Hallym Academy of Science, Hallym University, for which the authors would like to express their thanks.

Appendix A.

Proof of Theorem 1. Using $x_i = y_i + \rho y_{i-1}$, $y_0 = 0$, $E\{g_{\mathrm{LO}}(Y_i)|\mathrm{H}_0\} = 0$ and $E\{g_{\mathrm{LO}}(Y_i)|\mathrm{H}_0\} = 0$, it is easily seen that

$$\left.\frac{\mathrm{d}^2 E\{T_{\mathrm{LO}}(\boldsymbol{Y})|\mathrm{H}_1\}}{\mathrm{d}\theta^2}\right|_{\theta=0}$$

$$= V\{T^2_{\mathrm{LO}}(\boldsymbol{Y})|\mathrm{H}_0\}$$

$$= E\left\{ 2\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} E_s^2\{c_i c_j\} g^2_{\mathrm{LO}}(Y_i) g^2_{\mathrm{LO}}(Y_j) + \sum_{i=1}^{n} E_s^2\{c_i^2\} h^2_{\mathrm{LO}}(Y_i)|\mathrm{H}_0 \right\}$$

$$= 2I_1^2(f_e)\left[\sum_{i=1}^{n}\sum_{j=1}^{n} E_s^2\{c_i c_j\} - \sum_{i=1}^{n} E_s^2\{c_i^2\}\right] + I_2(f_e)\sum_{i=1}^{n} E_s^2\{c_i^2\}. \quad (31)$$

Thus, the ξ_{LO} is

$$\xi_{LO} = \lim_{n\to\infty} \frac{V\{T_{LO}(\boldsymbol{Y})\}}{n} = 2I_1^2(f_e)[\langle E_s^2(\boldsymbol{c},\boldsymbol{c})\rangle - \langle E_s^2(\boldsymbol{c}^2)\rangle] + I_2(f_e)\langle E_s^2(\boldsymbol{c}^2)\rangle. \quad \square \quad (32)$$

Proof of Theorem 2. Using $c_i = s_i - \rho c_{i-1}$, $c_0 = 0$, $E\{y_i^2 h_{LO}(y_i)\} = 2$ and $E\{y_i g_{LO}(y_i)\} = 1$, it is easily seen that

$$\left.\frac{d^2E\{T_{SQ}|H_1\}}{d\theta^2}\right|_{\theta=0} = E\{T_{SQ}(\boldsymbol{X})T_{LO}(\boldsymbol{Y})|H_0\}$$

$$= E\left\{\sum_{i=1}^{n} (y_i + \rho y_{i-1})^2 T_{LO}(\boldsymbol{Y})|H_0\right\}$$

$$= E\left\{\sum_{i=1}^{n} y_i^2 E_s\{c_i^2\} + 4\rho\sum_{i=1}^{n} y_i^2 E_s\{c_i c_{i-1}\} + \rho^2\sum_{i=1}^{n} y_{i-1}^2 E_s\{c_{i-1}^2\}|H_0\right\}$$

$$= 2\sum_{i=1}^{n} E_s^2\{c_i^2\} + 4\rho E_s\{c_i c_{i-1}\} + 2\rho^2\sum_{i=1}^{n} E_s^2\{c_{i-1}^2\}$$

$$= 2\sum_{i=1}^{n} E_s^2\{c_i^2\} + 4\rho E_s\{s_i c_{i-1}\} - 2\rho^2\sum_{i=1}^{n} E_s^2\{c_{i-1}^2\} \quad (33)$$

and

$$V\{T_{SQ}|H_0\} = E\{T_{SQ}^2|H_0\} - E^2\{T_{SQ}|H_0\} = n[(1+\rho^2)^2 m_4 - (1-\rho^2)^2\sigma_e^4] - \rho^2[(2+\rho^2)m_4 - (2-\rho^2)\sigma_e^4]. \quad (34)$$

Thus, ξ_{SQ} is

$$\lim_{n\to\infty} \frac{[d^2E\{T_{SQ}|H_1\}/d\theta^2|_{\theta=0}]^2}{nV\{T_{SQ}\}} = \frac{4((1-\rho^2)\langle E_s(\boldsymbol{c}^2)\rangle + 2\rho\langle E_s(\boldsymbol{s},\boldsymbol{c})\rangle)^2}{(1+\rho^2)^2 m_4 - (1-\rho^2)^2\sigma_e^4}. \quad \square \quad (35)$$

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