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"DELTA-STAR" TRANSFORMATION FOR CALCULATING THE RELIABILITY OF COMPLEX SYSTEMS*

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Abstract — The paper describes precise formulae for transformation from a star connection to a delta connection of equivalent reliability. An example of calculation is given. © 1997 Elsevier Science Ltd. All rights reserved.

Key words: Probability of no-failure operation, star connection, delta connection, transformation, calculation.

WE SHALL consider as an unrenewable system one which cannot be restored for some reason in the time period considered [1]. It is necessary to provide an estimate of ultimate reliability and specified performance when designing such systems.

We shall assume that all elements contained in a system may fail independently of one another, elements of the system may only be in two states: serviceable or unserviceable; failure flows and restoration flows of the elements are the simplest event flows; the capacity of the elements is unlimited.

The probabilities of no-failure operation R_n and R_m for n series-connected and m parallel-connected elements, respectively, are determined as follows [2]:

$$R_n = \prod_{i=1}^{n} p_i = 1 - \prod_{i=1}^{n} (1 - q_i); \qquad (1)$$

$$R_m = 1 - \prod_{j=1}^{m} q_j = 1 - \prod_{j=1}^{m} (1 - p_j), \qquad (2)$$

where p_i and p_j are the probabilities of no-failure operation of the ith and jth elements, and q_i and q_j, are the probabilities of failures of the ith and jth elements.

The circuit connections of a system do not always comprise series, parallel or mixed connections of the elements. More complex circuit connections exist. The elements in these circuit connections are connected so that direct determination of the equivalent probabilities of no-failure operation using only formulae (1) and (2) is impossible.

We shall consider as a complex circuit connection a connection of elements which contains only one group of elements with a bridge structure [3]. The method of "delta-star" conversion is used for transformation of such circuits (Fig. la, *b).* When transition from a delta connection to a star connection of equivalent reliability is carried out, the following formulae are used [4]:

$$p_i = \sqrt{\frac{p_{AC}p_{BC}}{p_{AB}}}; \quad p_j = \sqrt{\frac{p_{BC}p_{AB}}{p_{AC}}}; \quad p_k = \sqrt{\frac{p_{AC}p_{BC}}{p_{AB}}}, \quad (3)$$

где $p_{AB} = 1-(1-p_2)(1-p_1p_3)$; $p_{AC} = 1-(1-p_1)(1-p_2p_3)$; $p_{BC} = 1-(1-p_3)(1-p_1p_2)$.

Formulae (3) are correct when the following conditions are satisfied:

$$p_{AB}p_{AC} < p_{BC};\quad p_{BC}p_{AB} < p_{AC};\quad p_{AC}p_{BC} < p_{AB}.$$

It is necessary to know the exact formulae for transformation from a star connection to a delta connection of equivalent rehability when reliability analysis of complex circuits is carried out. For this it is necessary to solve the following system of nonlinear algebraic equations:

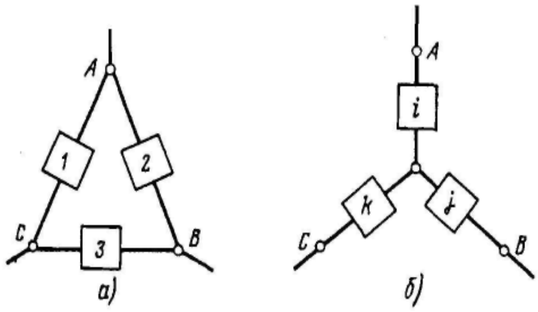

FIG. 1. Circuit connections of the elements:
(a) delta connection, *(б)* star connection

$$\left.\begin{aligned} a &= q_1(q_2 + q_3 - q_2 q_3); \\ b &= q_2(q_1 + q_3 - q_1 q_3); \\ c &= q_3(q_1 + q_2 - q_1 q_2), \end{aligned}\right\} \qquad (4)$$

где $a=q_i+q_k-q_i q_k;\quad b=q_i+q_j-q_i q_j;\quad c=q_j+q_k-q_j q_k.$

Let us find the failure probability of the delta elements q_1, q_2 and q_3 if the failure probabilities of the star elements q_i , q_j and $q_к$. are known.

We shall divide the right-hand and left-hand sides of equation system (4) by the product $q_1 q_2 q_3$ and introduce new variables:

$$\gamma = 1/q_1;\quad r = 1/q_2;\quad m = 1/q_3;\quad \gamma rm = t.$$

System of equations (4) will take the form

$$\left.\begin{aligned} at &= m + r - 1; \\ bt &= m + \gamma - 1; \\ ct &= - r + \gamma - 1; \\ \gamma rm &= t. \end{aligned}\right\} \qquad (5)$$

After summation of the right-hand sides and left-hand sides of the first three equations of system (5) and division of both sides of the equation derived by 2, we obtain the following equation:

$$m + r + \gamma - 1{,}5 = 0{,}5\,(a + b + c)\,t. \qquad (6)$$

Subtracting progressively the first three equations of system (5) from equation (6), we derive

$$\left.\begin{aligned} \gamma - 0{,}5 &= 0{,}5\,(b + c - a)\,t\,; \\ r - 0{,}5 &= 0{,}5\,(a + c - b)\,t\,; \\ m - 0{,}5 &= 0{,}5\,(a + b - c)\,t\,; \\ \gamma r m &= t\,. \end{aligned}\right\} \qquad (7)$$

From system of equations (7) we find

$$\gamma=\frac{(b+c-a)\,t+1}{2};\quad r=\frac{(a+c-b)\,t+1}{2};\quad m=\frac{(a+b-c)\,t+1}{2}. \quad (8)$$

Then

$$q_1=\frac{2}{(b+c-a)t+1};\ q_2=\frac{2}{(a+c-b)t+1};\ q_3=\frac{2}{(a+b-c)t+1}. \quad (9)$$

Substituting the values of γ, r and m from equation (8) in the fourth equation of system (7) and carrying out corresponding transformations, we obtain a cubic equation of the form

$$t^3 + \alpha_1 t^2 + \alpha_2 t + \alpha_3 = 0\,, \qquad (10)$$

где

$$\alpha_1 = \frac{1}{a+b-c} + \frac{1}{a+c-b} + \frac{1}{b+c-a};$$

$$\alpha_2 = \frac{a+b+c-8}{(a+b-c)\,(a+c-b)\,(b+c-a)};$$

$$\alpha_3 = \frac{1}{(a+b-c)\,(a+c-b)\,(b+c-a)}.$$

Substitution of the expression

$$t = y - \alpha_1/3$$

in equation (10) converts it into an incomplete form:

$$y^3 + py + q = 0\,. \qquad (11)$$

The roots of equation (11) are found by known techniques [5].

After determination of the values of t, let us substitute them in formula (9). In this case we shall obtain three values of the quantities q_v q_2 and q_v From each group of values we select only those which satisfy the condition

$$0 < q_i < 1\,,\quad \text{где}\quad i = \overline{1\,,3}\,.$$

We find the probability of no-failure operation of the elements of a delta connection of equivalent reliability:

$$p_1 = 1 - q_1; \quad p_2 = 1 - q_2; \quad p_3 = 1 - q_3. \quad (12)$$

In [6] approximate formulae are given for direct and reverse transformation of "delta-star" connections (Fig. 1*a, b)*:

$$p_i \approx 1 - q_1 q_2; \quad p_j \approx 1 - q_2 q_3; \quad p_k \approx 1 - q_1 q_3. \quad (13)$$

The formulae for transition from a star connection to a delta connection of equivalent reliability have the following form:

$$p_1 \approx 1 - \sqrt{\frac{q_i q_k}{q_j}}; \quad p_2 \approx 1 - \sqrt{\frac{q_i q_j}{q_k}}; \quad p_3 \approx 1 - \sqrt{\frac{q_j q_k}{q_i}}. \quad (14)$$

Formulae (14) are correct when

$$q_i q_k < q_j; \quad q_i q_j < q_k; \quad q_j q_k < q_i. \qquad (15)$$

When relationships (15) are not satisfied, for transition from a star connection to a delta connection of equivalent reliability, equation (11) must be solved.

Example. Let us find the probability of no-failure operation for the circuit connection in Fig. 2, using exact and approximate formulae for transition from a star connection to a delta connection of equivalent reliability (Fig. la, *b)*.

The failure rates of the elements have the following values:

$$\lambda_1 = 2{,}45 \cdot 10^{-4}\ 1/ч; \quad \lambda_2 = 1{,}9 \cdot 10^{-4}\ 1/ч;$$

$$\lambda_3 = 2{,}01 \cdot 10^{-4}\ 1/ч; \quad \lambda_4 = 2{,}29 \cdot 10^{-4}\ 1/ч;$$

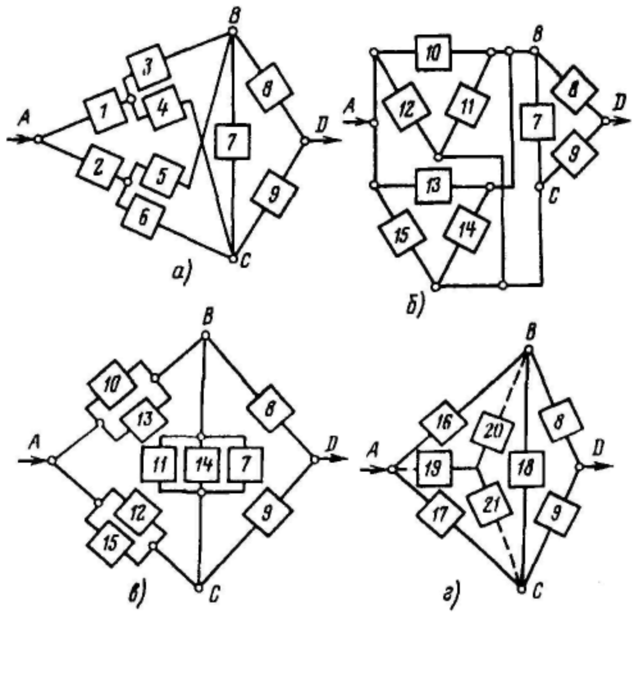

FIG. 2. Method of reduction of a complex connection comprising a mixed connection of elements

$$\lambda_5 = 2{,}5 \cdot 10^{-4}\ 1/ч; \quad \lambda_6 = 2{,}32 \cdot 10^{-4}\ 1/ч;$$

$$\lambda_7 = 2 \cdot 10^{-4}\ 1/ч; \quad \lambda_8 = 1{,}8 \cdot 10^{-4}\ 1/ч;$$

$$\lambda_9 = 2{,}09 \cdot 10^{-4}\ 1/ч; \quad p_i(t) = e^{-\lambda_i t}; \quad i=\overline{1,9}.$$

Let us apply the "delta-star" transformation (Fig. 2,*a*) twice to the circuit connection shown in Fig. *2,b,* using formulae (14). From the equivalent circuit given in Fig. *2b* we go to the equivalent circuit shown in Fig. 2c, and using formula (2) we derive the equivalent probabilities of no-failure operation of the elements contained in the arms of the delta connection *ABC.* By transforming the obtained delta connection *ABC* (Fig. *2d)* to the star connection of equivalent reliability by means of formulae (13), we obtain a mixed connection of elements. Then, by means of formulae (1) and (2) we obtain one equivalent element, the probability of no-failure operation of which corresponds to the reliability of the initial circuit connection (Fig. *2d).* A diagram of change in the probability of no-failure operation of the connection as a function of time is shown in Fig. 3 (curve 2). In a similar manner the reliability of the circuit connection is determined when an exact formula is used for transition from a star connection to a delta connection of equivalent reliability (by solving equation (11)).

The function of the probability of no-failure operation that was obtained by means of exact transformation from a star to a delta connection is shown in Fig. 3 (curve *1*).

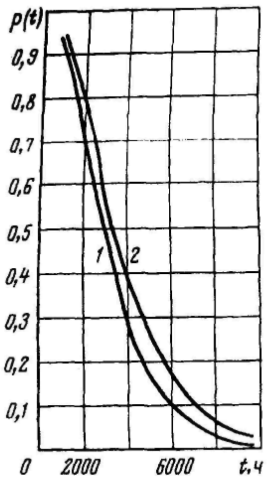

FIG. 3. Diagrams of the probabilities of no-failure operation of an initial circuit, obtained by various methods: (7) using direct and reverse "star-delta" transformations; (2) using approximate "star-delta" transformations

It can be seen from the diagram in Fig. 3 (curves *1* and 2) that, when the circuit analysed operates for less than 1000 h, the analytical method considered gives practically equal estimates of the probability of no-failure system operation. As the time of system operation increases, a considerable discrepancy in the calculated results is observed in the range 2000 h $< t <$ 5000 h. When the time of operation increases to more than 10,000 h, the calculation results practically coincide.

The accuracy of the problem considered has been checked by means of a logic-stochastic method by the referee of this paper, I. A. Ryabinin. It was found that, with t = 2000 h R(2000) = 0.707655. Similar results may be obtained using the diagram in Fig. 3; for t = 2000 h R(2000) = 0.71. In accordance with the calculations carried out by means of the method proposed R(2000) = 0.709974. In such a way, the accuracy of the calculation method using formulae (l)-(3) and (9) practically coincides

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