Nonlinear PD regulation for ball and beam system

Wen *Yu*
Departamento de Control Automatico, CINVESTAV-IPN, Mexico D.F., Mexico
E-mail: yuw@ctrl.cinvestav.mx

Abstract The ball and beam system is one of the most popular laboratory experiments for control education. There are two problems for ball and beam control: (1) many laboratories use simple controllers such as PD control, but theory analysis is based on linear models, and (2) nonlinear controllers for the ball and beam system have good theory results, but they are seldom used in laboratories. Little effort has been made to analyse PD control with nonlinear models. In this paper we modify the normal PD control in two ways for the ball and beam system: parallel and serial PD regulations; then we analyse the stability of these types of PD regulations with the complete nonlinear model. Real experiments are applied to test our theory results. This paper gives a good example of how to apply nonlinear theory in the laboratory for control education.

Keywords ball and beam system; nonlinear compensation; PD control; stability

The ball and beam system is widely used because many important classical and modem design methods can be studied based on it. The system (shown in Fig. 1) is very simple – a steel ball rolling on the top of a long beam. One side of the beam is fixed, the other side is mounted on the output shaft of an electric motor and so the beam can be tilted by applying an electrical control signal to the motor amplifier. The position of the ball can be measured using a special sensor. It has a very important property – open loop unstable, because the system output (the ball position) increases without limit for a fixed input (beam angle). The control job is to automatically regulate the position of the ball by changing the position of the motor. This is a difficult control task because the ball does not stay in one place on the beam when $a \neq 0$, but moves with an acceleration that is proportional to the tilt of the beam.

This standard experiment can be approximated by a linear model, and many universities use it for education of classical control theory. Linear feedback control or PID control can be applied. The stability analysis is based on a linear state-space model or transfer function.' Resent results show that the stabilisation problem of the ball and beam can be solved by nonlinear controllers. Approximate input-output linearisation used state feedback to linearise the ball and beam system first, then a tacking controller based on the approximates system can stabilise the ball and beam system.[2] But this controller is very complex for real applications. In order to solve the transient performance problem, an energy shaping method uses a nonlinear static state feedback that is derived from the interconnection and damping assignment.[3] But it requires shaping of the kinetic and potential energies.[4] A sliding mode controller can overcome the problem associated with singular states.' But chattering in sliding mode is a big problem in application. Observer-based nonlinear control in Ref. 6 uses the same coordinate transformation as in Ref. 2 to design a nonlinear

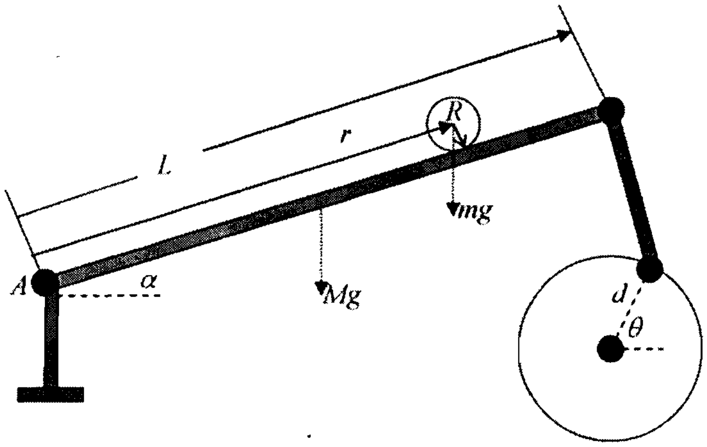


Fig. 1 *Ball and beam system.*

observer for the velocities of the ball and beam system. The controller is more complex than that in Ref. 2. Some intelligent controllers for ball and beam can also be found. such as fuzzy control,' sliding mode fuzzy control, neural control,' fuzzy neural control,' etc. These intelligent controllers are derived from some prior information or input-output data of the ball and beam system. There are two problems for ball and beam control: (1) many laboratories use simple controllers such as PD control, but theory analysis is based on linear models;' and (2) nonlinear controllers which are based on Lagrangian and kinetic energy–potential energy'" for ball and beam system have good theory results,'.' but they are complex and seldom used in real applications.

In this paper we modify the normal PD control in two ways for the ball and beam system: parallel and serial PD regulations. We analyse the stability of the PD control with the complete nonlinear model. To the best of our knowledge. stability analysis of PD control based on a nonlinear model of the ball and beam system has not yet been established in the literature. A real experiment is applied to test our theory results. We hope we can build a bridge between the nonlinear theory and control education in laboratories."

The ball and beam model and PD control

For the ball and beam system described schematically in Fig. 1, a ball is placed on a beam where it is allowed to roll with 1 degree of freedom along the length of the beam. A lever arm is attached to the beam at one end and a servo gear at the other. As the servo gear turns by an angle θ, the lever changes the angle of the beam by α. When the angle is changed from the horizontal position, gravity causes the ball to roll along the beam. The basic mathematical description of this system consists of d.c. servomotor dynamic and ball on the beam model.

Modelling the d.c. servomotor can be divided into electrical and mechanical subsystems. The electrical system is based on Kirchhoff's voltage law

$$U = L_m \dot{I}_m + R_m I_m + K_b \dot{\theta} \tag{1}$$

where U is input voltage, I_m is armature current, R_m and L_m are the resistance and inductance of the armature, K_b is back e.m.f. constant, and $\dot{\theta}$ is angular velocity. Compared to $R_m I_m$ and $K_b \dot{\theta}$, the term $L_m \dot{I}_m$ is very small. In order to simplify the modelling and as in most d.c. motor modelling methods, we neglected the term $L_m \dot{I}_m$.

The mechanical subsystem is

$$\frac{1}{K_g}\left(J_m \ddot{\theta} + B_m \dot{\theta}\right) = \tau_m \tag{2}$$

where K_g is gear ratio, J_m is the effective moment of inertia, B_m is viscous friction coefficient, and τ_m is the torque produced at the motor shaft. The electrical and mechanical subsystems are coupled to each other through an algebraic torque equation

$$\tau_m = K_m I_m$$

where K_m is the torque constant of the motor. Assuming that there is no backlash or electric deformation in the gears, the work done by the load shaft equals the work done by the motor shaft, $\tau = \frac{1}{K_g}\tau_m = \tau_m$, where τ is the torque on the frame of the ball and beam system. So the d.c. motor model is

$$\frac{R_m J_m}{K_m K_g}\ddot{\theta} + \left(K_b + \frac{R_m B_m}{K_m K_g}\right)\dot{\theta} = U \tag{3}$$

In the absence of friction or other disturbances, the dynamics of the ball and beam system can be obtained by Lagrangian method. Consider the ball and beam system with a coordinate reference frame about the A point (see Fig. 1). and a sphere with its centre aligned with the axis of the beam as in the figure below. The kinetic energy of the system is

$$T = T_1 + T_2$$

where T_1 and T_2 are kinctic energies of the beam and the ball; these kinetic energies include radial and circular motions. Since A is not moving from the coordinate kame, the rotational kinctic cncrgy of the beam is simply

$$T_1 = \frac{1}{2} J_1 \dot{\alpha}^2$$

where J_1 is the moment of inertia of the beam and $\dot{\alpha}$ is the angle velocity of the frame. The ball has kinetic energy

$$T_2 = \frac{1}{2}(mr^2)\dot{\alpha}^2 + \frac{1}{2}m\dot{r}^2 + \frac{1}{2}J_2\omega_2^2$$

where J_2 is the moment of inertia of the ball, $\dot{r}$ and ω_2 are radial and rotational velocities of the ball and m is its mass. Because $J_2 = \frac{2}{5}mR^2$, $\dot{r} = R\omega_2$, the rotational kinetic energy of the ball is $\frac{1}{2}\left(\frac{2}{5}m\dot{r}^2\right)$, so

$$T = \frac{1}{2}\left[(J_1 + mr^2)\dot{\alpha}^2 + \frac{7}{5}m\dot{r}^2\right]$$

The potential energy of the system is exhibited by the rolling ball alone

$$P = mgr\sin\alpha + Mg\frac{L}{2}\sin\alpha$$

where M is the mass of the frame, L is the longitude of the frame, and r is the position of the ball. The Lagrange equation is

$$\begin{aligned} L &= T - P \\ &= \frac{1}{2}\left[(J_1 + mr^2)\dot{\alpha}^2 + \frac{7}{5}m\dot{r}^2\right] - \left(mgr + \frac{L}{2}Mg\right)\sin\alpha \end{aligned}$$

Since there is no external force on the ball in the radial direction, Lagrange's equations of motion are formed as

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\alpha}}\right] - \frac{\partial L}{\partial \alpha} = \tau$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{r}}\right] - \frac{\partial L}{\partial r} = 0$$

Because $\frac{\partial L}{\partial \alpha} = -\left(mgr + \frac{L}{2}Mg\right)\cos\alpha$, $\frac{\partial L}{\partial \dot{\alpha}} = (J_1 + mr^2)\dot{\alpha}$, $\frac{\partial L}{\partial r} = mr\dot{\alpha}^2 - mg\sin\alpha$, $\frac{\partial L}{\partial \dot{r}} = \frac{7}{5}m\dot{r}$. So

$$(J_1 + mr^2)\ddot{\alpha} + 2mr\dot{r}\dot{\alpha} + \left(mgr + \frac{L}{2}Mg\right)\cos\alpha = \tau \tag{4a}$$

$$\frac{7}{5}\ddot{r} - r\dot{\alpha}^2 + g\sin\alpha = 0 \tag{4b}$$

Remark I The second equation of (4)can be derived directly from the force relation. In Fig. I

$$my = -mg + N\cos\alpha + F\sin\alpha$$
$$m\ddot{z} = -N\sin\alpha + F\cos a \tag{5}$$

where N is friction, F is rotational force, $FR = J\dot{\omega}$, $\omega = \frac{x}{R}$, $J = \frac{2}{5}mR^2$. *Multiplying with sin a and* cos *a; and summarising the two equations in (5) with F = 0, gives*

$$m\ddot{y}\sin\alpha + m\ddot{z}\cos\alpha = -mg\sin\alpha + \frac{2}{5}m\ddot{r}$$

Using the conditions $\ddot{y} = \frac{d}{dt}(\dot{r}\sin\alpha + r\dot{\alpha}\cos\alpha)$, $\ddot{z} = \frac{d}{dt}(\dot{r}\cos\alpha - r\dot{\alpha}\sin\alpha)$, $y = -r\sin a, z = -r\cos a$; *gives*

$$\ddot{y}\sin\alpha + \ddot{z}\cos\alpha = -\ddot{r} + r\dot{\alpha}^2$$

This expression is similar to the second equation of (4). When the system is near to a stable point, $\dot{\alpha} \approx 0$, *the acceleration of the ball is given by*

$$\ddot{r} = -\frac{5}{7}g\sin\alpha$$

Since a is a small angle, $\sin\alpha \approx \alpha$. *The approximation linear model for the ball and beam system becomes*

$$G(s) = \frac{b}{s^2} \tag{6}$$

In state space form, it is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ b\end{bmatrix}u$$
$$y = [1, 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

where $x_1 = r, x_1 = r$.

*Remark 2 The model **(4)** differs from the most commonly used ball and beam system as in **Ref.** 2, where the motor **is fixed** in the body centre of the beam. In our case **the fixed** point A is to one side of the beam. So the gravity of the beam cannot be neglected. Also, the beam angle a and motor position* θ *are not the same; we use Fig. 2 to calculate them. The arc distances in the two circles are equal, **i.e.**,*

$$\alpha L = \theta d \tag{7}$$

The control problem is to design a controller which computes the applied voltage U for the motor to move the ball in such a way that the actual position of the ball reaches the desired one. The controllers are constructed by introducing nonlinear compensation terms into the traditional PD controller. Two types of PD controller

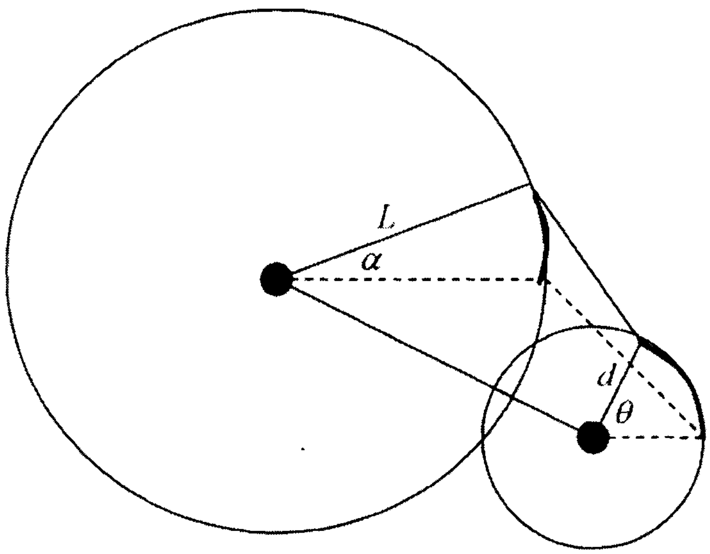


Fig. 2 *Relation between motor position and beam angle.*

will be designed for this system. The first one is serial PD control which is shown in Fig. 3 (a). The beam angle α (or motor position θ) can be controlled by PD controller $C1$. This constitutes the inner loop. The outer loop controls the ball position with PD controller C2. π is a compensator which can assure asymptotic stability.[10] The serial PD control has the following form

$$\begin{aligned} U &= k_{pm}(\alpha^* - \alpha) + k_{dm}(\dot{\alpha}^* - \dot{\alpha}) + \pi \\ \alpha^* &= k_{pb}(r^* - r) + k_{db}(\dot{r}^* - \dot{r}) \end{aligned} \tag{8}$$

where k_{pm} and k_{dm} are positive constants. which correspond to proportional and derivative coefficients for motor control; k_{pb} and k_{db} are proportional and derivative gains for the ball control.

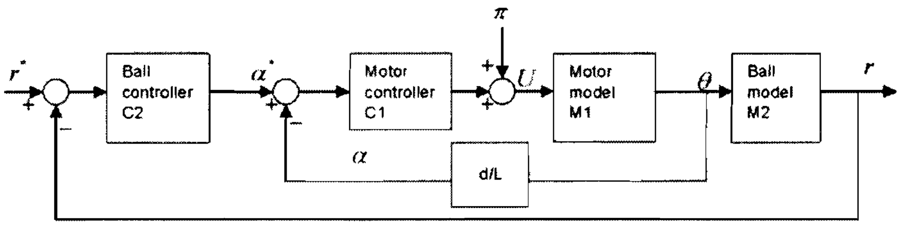
The second one is parallel PD control which is shown in Fig. **3** (b). Because the final position of the motor must be 0, such that the ball does not move, so $\alpha^* = 0$. The feedback control of motor position becomes -1. The parallel PD control has the following form

$$U = (-k_{pm}\alpha - k_{dm}\dot{\alpha}) + [k_{pb}(r^* - r) + k_{db}(\dot{r}^* - \dot{r})] + \pi \tag{9}$$

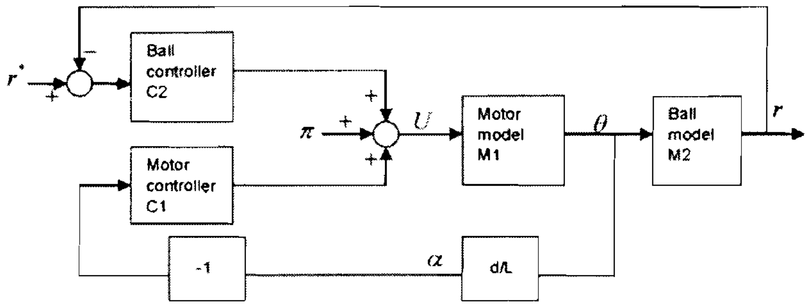
For regulation problems the control aim is to stabilise the ball in a desired position r^*, so $r^* = 0$. The two PD controllers (8) and (9) can be rewritten in a unique form

$$U = a_1\tilde{r} - a_2\dot{r} - a_3\ddot{r} - a_4\alpha - a_5\dot{\alpha} + \pi \tag{10}$$

where serial PD control $a_1 = k_{pm}k_{pb}$, $a_2 = (k_{pm} + k_{dm})k_{db}$, $a_3 = k_{dm}k_{db}$, $a_4 = k_{pm}$, $a_5 = k_{dm}$, and for parallel PD control $a_1 = k_{pb}$, $a_2 = k_{db}$, $a_3 = 0$, $a_4 = k_{pm}$, $a_5 = k_{dm}$, $a_i > 0$ ($i = 1 \ldots 5$).



(a) Serial control



(b) Parallel control

Fig. 3 *PD control for* **bull** *and beam system.*

Stability analysis of PD regulation

In this section, PD regulation for the ball and beam system is proposed. From cqn (2) we have

$$\frac{K_m}{R_m}\left(U - K_b\dot{\theta}\right) = \pi$$

The whole ball and beam system is given by (3), (4) and (7):

$$\begin{aligned} &(mr^2 + k_1)\ddot{\alpha} + 2mr\dot{r}\dot{\alpha} + \left(mgr + \frac{L}{2}Mg\right)\cos\alpha = k_2 U - k_3\dot{\alpha} \\ &k_4\ddot{r} - r\dot{\alpha}^2 + g\sin\alpha = 0 \end{aligned} \qquad (11)$$

where $k_1 = \frac{R_m J_m}{K_m K_e}\frac{L}{d} + J_1$, $k_2 = 1 + \frac{K_m}{R_m}$, $k_3 = \frac{L}{d}\left(\frac{K_m K_b}{R_m} + K_b + \frac{R_m B_m}{K_m K_g}\right)$, $k_4 = \frac{7}{5}$, $k_i > 0 (i = 1 \ldots 4)$. We define the system state as $x = [\alpha, r]^T$; the regulation error is

$$\tilde{x} = x^* - x$$

where x^* is the desired variable, $x^* = [\alpha^*, r^*]^T$. For the ball and beam system, in the balance position $\alpha^* = 0$, $\dot{x}^* = 0$. So $x^* = [0, r^*]^T$, r^* is the desired ball position.

It is difficult to apply the dynamic equation of the ball and beam system (11) and PD control (10) for the Lyapunov method directly. On the other hand, it is well known that we can prove the stability of robots with PD control by the Lyapunov method. In this paper we will transfer (11) and (10) into the form of the robot dynamics, then we will prove that the ball and beam system has similar properties as robots. The closed-loop system is obtained by substituting the control voltage U from the control law (10) into ball and beam system (11)

$$M(x)\ddot{x}+C(x,\dot{x})\dot{x}+G(x)=B\tilde{x}+D\pi$$

where $M(x)=\begin{bmatrix} k_1+mr^2 & k_2a_3 \\ 0 & k_4 \end{bmatrix}$, $C(x,\dot{x})=\begin{bmatrix} k_2a_5+k_3 & k_2a_2+2mr\dot{\alpha} \\ -r\dot{\alpha} & 0 \end{bmatrix}$,

$B=\begin{bmatrix} k_2a_4 & k_2a_1 \\ 0 & 0 \end{bmatrix}$, $D=\begin{bmatrix} k_2 \\ 0 \end{bmatrix}$, $G(x)=\begin{bmatrix} \left(mgr+\frac{L}{2}Mg\right)\cos a \\ g\sin\alpha \end{bmatrix}$. Before presenting the stability analysis, we give the following lemma.

Lemma 1 The following equations

$$\left(mgr+\frac{L}{2}Mg\right)\cos\alpha=k_2U$$
$$k_4\ddot{r}+g\sin\alpha=0 \tag{12}$$
$$U=a_1\tilde{r}-a_3\ddot{r}-a_4\alpha+\frac{1}{k_2}\left(mgr+\frac{L}{2}Mg\right)\cos\alpha$$

have an isolated solution $[\alpha, r]=[\alpha^*, r^*]$.

Proof: Substituting U into the first equation of (12), we have

$$k_2a_1\tilde{r}-k_2a_3\ddot{r}-k_2a_4\alpha=0. \tag{13}$$

From the second equation of (12), we can conclude $\ddot{r}=-\frac{1}{k_4}g\sin\alpha$. So (13) becomes

$$k_2a_1\tilde{r}+\frac{k_2a_3}{k_4}g\sin\alpha-k_2a_4\alpha=0.$$

It can be rewritten as

$$\sin\alpha=\frac{k_4a_4}{a_3g}\alpha-\frac{k_4a_1}{a_3g}\tilde{r} \tag{14}$$

The only possible solution for α is $\alpha=0$, otherwise the ball has to move. For any $a\neq 0$, $\tilde{r}$ cannot be a constant, so (14) has no solution. When $a=0$, form (14) we know $\tilde{r}=0$. Because $\alpha^*=0$, this allows us to conclude $[\alpha, r]=[a^*, r^*]$ is the unique solution for (12).

The stability of the closed loop system is stated in the following theorem.

Theorem 1 *The serial or parallel PD control as in (10) with a compensator as*

$$\pi = \frac{1}{k_2}\left\{[k_2 a_2 + (2m-3) r\dot{\alpha}]\dot{r} + \left(mgr + \frac{L}{2} Mg\right)\cos\alpha\right\} \tag{15}$$

can guarantee asymptotic stability of the ball and beam system (11).

Proof: Because $M(x)$ and B are positive definite matrices, we choose the following positive definite quadratic form as Lyapunov function candidate:

$$V(x, \dot{x}) = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}\tilde{x}^T B\tilde{x} + \frac{k_4}{2}\dot{r}^2. \tag{16}$$

Differentiating it with respect to time, and recalling that x^* is constant, yields

$$\frac{dV}{dt} = \dot{x}^T M(x)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x} - \dot{x}^T B\tilde{x} + k_4\dot{r}\ddot{r}.$$

Since $M(x)\ddot{x} = B\tilde{x} + D\pi - C(x, \dot{x})\dot{x} - G(x)$,

$$\dot{V} = \frac{1}{2}\dot{x}^T[\dot{M}(x) - 2C(x, \dot{x})]\dot{x} + \dot{x}^T[B\tilde{x} - G(x) + D\pi - B\tilde{x}] + k_4\dot{r}\ddot{r}$$

because

$$\dot{M}(x) - 2C(x, \dot{x}) = \begin{bmatrix} 2mr\dot{r} & -4mr\dot{\alpha} \\ 2r\dot{\alpha} & 0 \end{bmatrix} - \begin{bmatrix} 2k_2a_5 + 2k_3 & 2k_2a_2 \\ 0 & 0 \end{bmatrix}.$$

So

$$\dot{V} = -\frac{1}{2}\dot{x}^T Q\dot{x} + 2r\dot{x}^T \begin{bmatrix} m\dot{r} & -2m\dot{\alpha} \\ \dot{\alpha} & 0 \end{bmatrix}\dot{x} - \dot{x}^T G(x) + \dot{x}^T D\pi + k_4\dot{r}\ddot{r},$$

where $Q = \begin{bmatrix} 2k_2a_5 + 2k_3 & 2k_2a_2 \\ 0 & 0 \end{bmatrix}$

$$2r\dot{x}^T \begin{bmatrix} m\dot{r} & -2m\dot{\alpha} \\ \dot{\alpha} & 0 \end{bmatrix}\dot{x} = -2r\dot{\alpha}^2(m-1)\dot{r}$$

$$-\dot{x}^T G(x) = -\left(mgr + \frac{L}{2}Mg\right)\cos\alpha\dot{\alpha} + g\sin\alpha\dot{r} \tag{17}$$

$$\dot{x}^T D\pi = \dot{\alpha}k_2\pi$$

$$-\frac{1}{2}\dot{x}^T Q\dot{x} = -(k_2a_5 + k_3)\dot{\alpha}^2 - k_2a_2\dot{\alpha}\dot{r}$$

Using $k_4\ddot{r} - r\dot{\alpha}^2 + g\sin a = 0$

$$\dot{V} = -(k_2 a_5 + k_3)\dot{\alpha}^2 + \dot{\alpha}\left[k_2\pi + (-2m+3)r\dot{r}\dot{\alpha} - \left(mgr + \frac{L}{2}Mg\right)\cos\alpha - k_2 a_2 \dot{r}\right]$$

If we choose the compensator as

$$\pi = \frac{1}{k_2}\left[(2m-3)r\dot{r}\dot{\alpha} + \left(mgr + \frac{L}{2}Mg\right)\cos\alpha + k_2 a_2 \dot{r}\right]$$

$$\dot{V} \leq -(k_3 a_5 + k_4)\dot{\alpha}^2 \quad (18)$$

since $(k_3a_5 + k_4) > 0$. V is a negative-semidefinite function. Therefore, by invoking the Lyapunov direct method, it can be concluded that $[\alpha, r] = [0, r^*]$ $(\alpha^* = 0)$ is a stable equilibrium.

In order to prove asymptotic stability, we use LaSalle's theorem. In the region

$$\Psi = \{[\alpha, r]: V = 0)$$

the invariant set is obtained from the closed-loop system (11) when $\dot{\alpha} = 0$, that is (12). Furthermore, according to Lemma 1, (12) is satisfied for $[\alpha, r] = [0, r^*]$ $(\alpha^* = 0)$. Therefore, invoking LaSalle's theorem, we can be assured that the equilibrium $[\alpha, r] = [0, r^*]$ is asymptotically stable.[12] This means that

$$\lim_{t\to\infty} \tilde{x} = 0.$$

Remark 3 Since the velocities $\dot{r}$ and $\dot{\alpha}$ in (15) are very small in the regulation case, the main compensation is the gravities of the brill and beam

$$\pi \approx \frac{1}{k_2}\left(mgr + \frac{L}{2}Mg\right)\cos\alpha. \quad (19)$$

The controllers (8) or (9) with (19) are very simple and easy to implement. The control parameters of PD control are independent of system parameters, the compensator uses two motor parameters and the masses of the ball and beam. Although the pure PD controller (with $\pi = 0$) can also stabilise the system as in many laborutories' experimental proofs, the control performance under the pure PD control is very unsatisfactory (especially for the confutation of our type), due to the gravities of bull and beam.

Remark 3 *To the best of our knowledge, the theoretical analysis of a PD controller for the ball and beam system based on a complete nonlinear model has not yet been established in the literature. Many stability analyses are based on complete nonlinear controllers,[2,4] and these controllers have to use the nonlinear model of the ball and beam system. On the other hand, many laboratories use model-free controllers'[5,8] (e.g. the PD controller); the theoretical analyses use the simplified linear model as in (6). Because die PD controller is also a linear system, traditional control theory can be appliedfor stability analysis.*

Simulation and experimental case study

First we give some simulation examples to compare our controller with the other existing methods. For the simulation we chose $\frac{R_m J}{K_m K_g} = 0.01176$, $\frac{R_m B}{K_m K_g} + K_m = 0.58823$.[1] If we do not consider the energy effect, the whole dynamic equation is

$$\frac{7}{5}\ddot{r} - r\dot{\alpha}^2 = -g\sin\alpha, \quad 0.01176\ddot{\theta} + 0.45823\dot{\theta} = U, \quad \alpha = \frac{1}{16}\theta \tag{20}$$

The regulation results of normal PD control are given by

$$U = (-5.8\alpha - 0.1\dot{\alpha}) + [2.2(r^* - r) + 0.8(\dot{r}^* - \dot{r})]. \tag{21}$$

When the kinetic energy of the system is considered. the first equation of (20) becomes (11). We use the parameters m = 0.06, g = 9.8, M = 0.12, L = 0.6. The modified PD control is (21) with compensation $\pi = 0.2r + 0.1$ cons a. The simulation results are shown in Fig. 1. The normal PD control is suitable for a simplified model, but it does not work for the complete nonlinear model. The modified PD control proposed in this paper can work. The response is similar to that in Ref. 2, but the transient performance is worse than in Ref. 4. We note that the nonlinear controllers of Refs 2 and 4 need the complete hall and beam system rnodel. The only give simulation results. Our modified PD control does not require the nonlinear ball and beam system model. Its application is easier.

The experiment is carried out on the Quanser ball and beam system' (see Fig. 4). The beam is 60 cm long. The ball is about 60 g. The input to the system is the motor control voltage U; outputs are the positions of motor (θ) and ball (r). The power module is also Quanser, PA-0103 with ±12 V and 3 A output. The A/D-D/A board is based on a Xilinx FPGA microprocessor, which is a multifunction analogue and digital timing I/O hoard dedicated to real-time data acquisition and control in the Windows XP environment. The hoard is mounted in a PC Pentium-III 500 MHz host computer. Because the Xilinx FPGA chip supports real-time operations without introducing latencies caused by the Windows default timing system, the control programme is operated in Windows XP with Matlab 6.5/Simulink. The sampling time is about 10 ms.

The motor and ball controllers are both of the PD type and require direct velocity measurements, but they are unavailable. We use the derivative block of Simulink to calculate them. This requires that the position signals are smooth enough; so first-order low-pass filters are applied. For motor position we use the following first-order filter: $G_1(s) = \frac{10}{s+10}$. For ball position wc use the following first-order filter: $G_2(s) = \frac{17}{s+17}$. For the serial PD control (8) we use k_{pm} = 2, k_{dm} = 0.1, k_{pb} = 0.5, k_{db} = 0.1. For the parallel PD control (9) wc use k_{pm} = 2, k_{dm} = 0.5, k_{pb} = 0.4,

Fig. 4 ***Ball and beam control system.***

k_{db} = 0.1. The parameters for this experiment are $\frac{L}{d} = 16$, m = 0.06, g = 9.8, $\frac{R_m}{R_m + K_m} = 0.3$, M = 0.12, L = 0.6. We only use gravity compensation[19]. The compensator is

$$\pi = \frac{R_m}{R_m + K_m}\left\{\left[\frac{R_m}{R_m + K_m}(k_{pm} + k_{dm})k_{db} + (2m-3)r\dot{\alpha}\right]\dot{r} + \left(mgr + \frac{L}{2}Mg\right)\cos a\right\}$$

It can be approximated as $\pi \approx 0.3(0.588r + 0.353)\cos\alpha$ (see (19)). The response of the parallel PD control for the ball and beam system is shown in Fig. 5. The serial PD control has the same compensator as the parallel one. At time t = 200 ms, we move the ball 1 cm, to mimic an external disturbance. The response is shown in Fig. 6. When we use pure PD control, the response of the serial PD control without compensator is shown in Fig. 7. We can see that PD control with exact compensation is effective for the ball and beam system. The closed-loop system appears (from the step input) to exhibit second-order behaviour with a natural frequency around 1 rad/s. Faster filters are used (a rule of thumb would suggest at least 5 to 10 times faster than the fastest closed-loop modes). So the filter dynamics will not have a significant impact on the control.

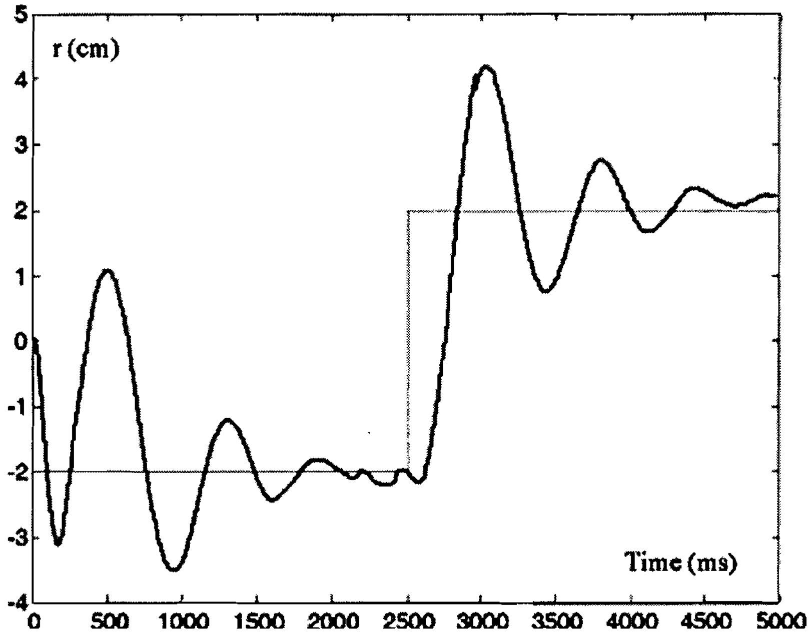


Fig. 5 *Parallel P D control with gravity compensation.*

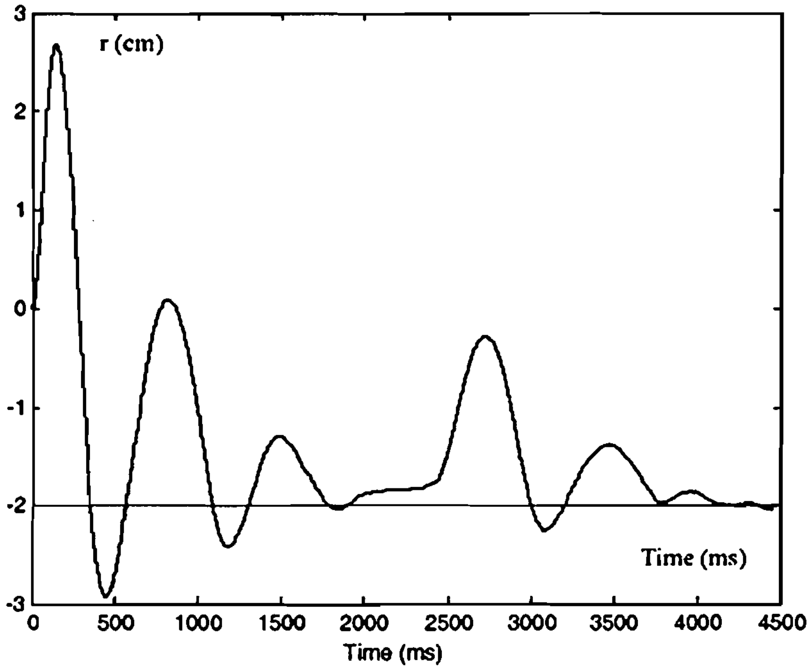


Fig. 6 *Serial P D control with nonlinear compensation.*

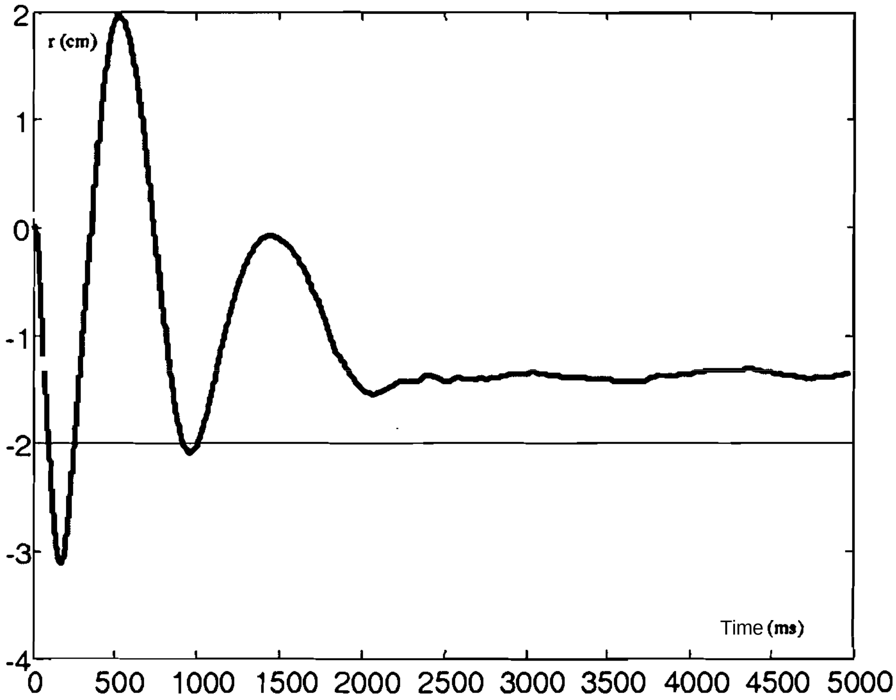


Fig. 7 *Serial PD control without nonlinear compensator.*

Conclusion

The main contributions of the paper are: (1) Two types of PD controllers with nonlinear compensation have been presented for regulation of the ball and beam system. (2) By using Lyapunov's direct method, we have shown that for a well-defined set of initial conditions, the ball remains on any point of the bar. (3) Experimental results are presented to illustrate the control system's stability and performance.

The results of this paper can be easily extended to the other mechanical plants. A great benefit to engineering educators is that this paper provides an approach to transferring complex theory problems found in textbooks into prototypes in the laboratory.

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