DIGITAL I/Q IMBALANCE COMPENSATION IN A LOW-IF RECEIVER

Jack P.F. Glas
Bell Labs, Lucent Technologies
600 Mountain Avenue, Murray Hill, NJ 07974, USA
e-mail: JPFGlas@lucent.com

ABSTRACT

(Double) Low-IF receiver architectures avoid the need for expensive image-reject and channel selection filters and so reduce cost. This approach however introduces the need for an accurate in-phase and quadrature version of the first local oscillator.

In this document we present a way to digitally measure amplitude and phase imbalances and compensate for them. Simulations show image-reject values round 50 dB with amplitude imbalances of more than 10% and phase imbalances of more than 10 degrees. Although we focus on GSM-like applications, the algorithms are suitable for other Low-IF systems as well.

I INTRODUCTION

RF Front-end topologies using RF image-reject mixing will gain a lot of popularity in the future. They avoid the need for image-reject and SAW-filters and enable direct-conversion and (Double) Low-IF architectures [CS95b, BWS+97] that greatly reduce the bill of materials. Quadrature signal generation is however a critical part in these systems. I/Q imbalances cause interference that cannot be removed in later stages and so directly decrease the image-reject capabilities of the front-end.

A common way to acquire the quadrature signal is using an RC-CR circuit. Tolerances in the on-chip resistors and capacitors however cause imbalances resulting in a worse image rejection. Another way of generating quadrature signals is by using Poly-phase filters [CS95a], these filters however are power hungry. We therefore propose the use of a "plain RC-CR" circuit together with an imbalance compensation scheme. Compared to other compensation techniques ([Cav93, PBQ97]), our approach is able to measure phase and amplitude imbalances independently and in a fast (non-iterative) and computational inexpensive way.

The algorithm can be applied in two situations: a) During manufacturing/testing an external test-tone can be used to measure and compensate for imbalances in the whole receive chain. And b) during startup a phone can generate its own test-tone. This method however imposes some restrictions which will be explained later on.

In the following section we will first analyze the image rejection properties of a Low-IF receiver with traditional mixing. In section III we then propose the imbalance measurement scheme. The second part of that section describes how to digitally compensate for imbalances once they are known. Section IV motivates the simulation setup and presents simulation results for both the measurement and compensation stage. Finally section V summarizes important results.

II IMAGE REJECTION IN A LOW-IF RECEIVER

A schematic of a Low-IF front-end architecture [CS95b] with its relevant signals is shown in figure 1. This architecture differs from a direct-conversion approach in the sense that the RF-mixers beat the received signal down to a Low-IF frequency equal to half the channel spacing (100 kHz in GSM) to avoid common direct-conversion problems like DC-offset. The final down-conversion to DC is performed using quadrature mixing in the digital domain.

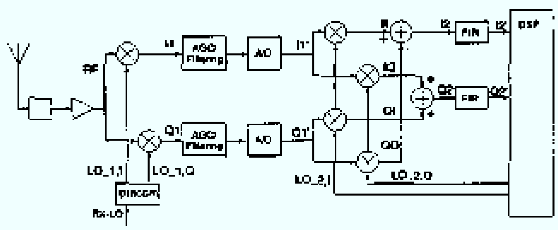

Figure 1: Low-IF Front-End Architecture

Let us write the input RF-signal as follows:

$$\mathrm{rf}(t) = A\cos((\omega_1 + \omega_d)t + \theta(t)) \tag{1}$$

With A being the amplitude, $\omega_1 + \omega_d$ the RF center-frequency where ω_1 is the LO1 frequency. $\theta(t)$ (to be referred to as θ) is the modulated data signal with constant phase-offset. The I/Q local-oscillator signals with their inaccuracies are modeled as follows:

$$LO_{1,I} = (1 + \Delta_1)\cos(\omega_1 t + \phi_1) \tag{2a}$$

$$LO_{1,Q} = (1 - \Delta_2)\sin(\omega_1 t - \phi_2) \tag{2b}$$

0-7803-4984-9/98/$10.00

RF-mixing and low-pass filtering yields (LNA and AGC gains set to unity):

$$I1' = \frac{A}{2}\,(1+\Delta_1)\,\cos(\omega_d t + \theta - \phi_1) \tag{3a}$$

$$Q1' = \frac{-A}{2}\,(1-\Delta_2)\,\sin(\omega_d t + \theta + \phi_2) \tag{3b}$$

Both $I1'$ and $Q1'$ signals are now converted into the digital domain. After that, another mixer stage takes care of the final down conversion from low-IF (ω_d) to DC. As this conversion stage takes place in the digital domain, inaccuracies will be much smaller than those caused by the front-end. Ideal mixing is therefore assumed yielding the following signals:

$$\begin{aligned} II = I1'{\cdot}LO_{2,I} &= A\,(1+\Delta_1)/4 \\ &\cdot[\cos(\theta-\phi_1) + \cos(2\,\omega_d\,t + \theta - \phi_1)] \end{aligned} \tag{4a}$$

$$\begin{aligned} IQ = I1'{\cdot}LO_{2,Q} &= -A\,(1+\Delta_1)/4 \\ &\cdot[\sin(\theta-\phi_1) - \sin(2\,\omega_d\,t + \theta - \phi_1)] \end{aligned} \tag{4b}$$

$$\begin{aligned} QI = Q1'{\cdot}LO_{2,I} &= -A\,(1-\Delta_2)/4 \\ &\cdot[\sin(\theta+\phi_2) + \sin(2\,\omega_d\,t + \theta + \phi_3)] \end{aligned} \tag{4c}$$

$$\begin{aligned} QQ = Q1'{\cdot}LO_{2,Q} &= -A\,(1-\Delta_2)/4 \\ &\cdot[\cos(\theta+\phi_2) - \cos(2\,\omega_d\,t + \theta + \phi_2)] \end{aligned} \tag{4d}$$

The wanted and image signals can be obtained in the following way:

$$I2 = II - QQ \tag{5a}$$

$$Q2 = IQ + QI \tag{5b}$$

$$I2_{\text{image}} = II + QQ \tag{5c}$$

$$Q2_{\text{image}} = IQ - QI \tag{5d}$$

After adding/subtracting, a digital FIR-filter is usually applied to further filter out adjacent channels, this filter however also suppresses the double frequency terms in (4). By assuming $(1+\Delta_1-\Delta_2)\approx 1$ and $\Delta_1\,\Delta_2\,\cos(\phi_1+\phi 2)\approx \Delta_1\,\Delta_2$ (valid for small values of ϕ and Δ), we find for the image-rejection:

$$\begin{aligned} \frac{P_{\text{image}}}{P} &\approx \sin^2\left(\frac{\phi_1+\phi_2}{2}\right) + \frac{(\Delta_1+\Delta_2)^2}{4} \\ &= \sin^2\left(\frac{\phi_{\text{total}}}{2}\right) + \frac{\Delta_{\text{total}}^2}{4} \end{aligned} \tag{6}$$

III COMPENSATION OF I/Q IMBALANCES

The imbalance compensation algorithm is based on the following two steps:

1. Obtaining an independent estimate of the amplitude and phase imbalance by evaluation of both the wanted and image response of a test-tone.
2. Compensation of the imbalances at Low-IF after A/D conversion.

A Imbalance Estimation using an External Test-tone

The estimation scheme (shown in figure 2) makes use of a test-tone introduced in the RF-chain right after the LNA. We first concentrate on the situation where an external test-tone is applied.

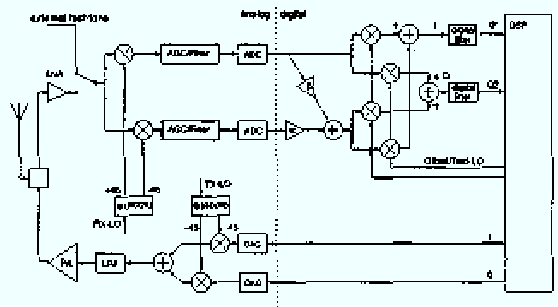

Figure 2: Measuring imbalances using an external test-tone

Once the test-tone is fed into the receiver chain this signal is demodulated in the usual way, however except for the "wanted" signal also the "image" signal is measured by switching the adding/subtracting stages after the second mixing stage. As those signals are inherently of different amplitude, the A/D converter has to cover this dynamic range. This is however no problem as an image signals only deteriorates the BER if it can be measured. α and β are set to respectively 1 and 0 for the moment.

For convenience reasons we will assume both the imbalance and phase error to be symmetric:

$$\Delta_1 = \Delta_2 = \Delta = \frac{\Delta_{\text{total}}}{2} \tag{7a}$$

$$\phi_1 = \phi_2 = \phi = \frac{\phi_{\text{total}}}{2} \tag{7b}$$

leading to:

$$\begin{aligned} I2' = u_1 &= \frac{A}{2}\,[\cos(\phi)\,\cos(\theta) + \Delta\,\sin(\phi)\,\sin(\theta)] \\ &\approx \frac{A}{2}\,\cos(\phi)\,\cos(\theta) \end{aligned} \tag{8a}$$

$$\begin{aligned} Q2' = u_2 &= \frac{A}{2}\,[\Delta\,\sin(\phi)\,\cos(\theta) - \cos(\phi)\,\sin(\theta)] \\ &\approx \frac{-A}{2}\,\cos(\phi)\,\sin(\theta) \end{aligned} \tag{8b}$$

$$I2'_{\text{image}} = u'_1 = \frac{A}{2}\,[\Delta\,\cos(\phi)\,\cos(\theta) + \sin(\phi)\,\sin(\theta)] \tag{8c}$$

$$Q2'_{\text{image}} = u'_2 = \frac{A}{2}\,[\sin(\phi)\,\cos(\theta) - \Delta\,\cos(\phi)\,\sin(\theta)] \tag{8d}$$

Yielding:

$$\frac{u_1'}{u_1} \approx \Delta + \tan(\phi)\,\tan(\theta) \tag{9a}$$

$$\frac{u_2'}{u_2} \approx \Delta - \frac{\tan(\phi)}{\tan(\theta)} \tag{9b}$$

$$\frac{v_1}{u_2} \approx \frac{-1}{\tan(\theta)} \tag{9c}$$

which can be rewritten to obtain expressions for Δ and ϕ:

$$\phi = \arctan\left(\frac{u_1\,u_2' - u_1'\,u_2}{u_1^2 + u_2^2}\right) \tag{10a}$$

$$\Delta = \frac{u_1\,u_1' + u_2\,u_2'}{u_1^2 + u_2^2} \tag{10b}$$

It is not likely that imbalance errors will rapidly change, they will however be dependent on frequency. Although the dependence is predictable, it is useful and - due to the measurement speed - possible to repeat this measurement for a number of frequencies in the receive band.

B Imbalance Estimation at Start-up

The estimation scheme (shown in figure 3) makes use of a test-tone introduced in the RF-chain right after the LNA, as shown in figure 2. At this point the dynamic range reaches (for GSM) from about -85 to -10 dBm. When we assume the reverse-gain of the LNA to be -20 dB, the test-tone is allowed to have a power of up to -16 dBm [GSM96, section 4.3] which is more than sufficient. The test-tone can be generated by the I/Q modulator already present in the transmission chain and is controlled by the DSP.

Putting the two switches of figure 3 in the states as drawn results in normal operation. By putting both switches in their opposite positions the transceiver goes into calibration mode and generates its own test-tone in the receive band. As the Rx LO is generated by mixing the offset-VCO signal and the Rx-VCO signal, no frequency-error is introduced.

Indirect up-conversion is used in this architecture to avoid the undesired effect of - during up-conversion - generating a spurious signal in the "image channel". The first conversion stage (I/Q modulation to first IF) still creates this spurious signal. However the quadrature local oscillator signals used for up-conversion are now derived from a "divide-by-four" circuit. These circuits can be made very accurate and can easily provide an image rejection of 50 dB or better. As this approach requires the generation of a signal with a frequency of four times the local oscillator frequency, this is unsuitable for direct up-conversion

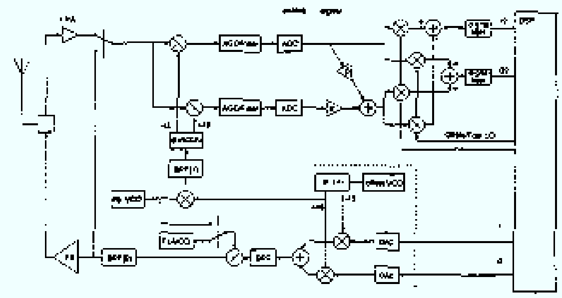

Figure 3: Measuring imbalances using a self-generated test-tone

(the transmit IF frequency is likely to be in the range 70-300 MHz). The image-rejection of this up-conversion stage determines an upper bound to the maximum image-rejection of the imbalance compensation scheme.

There is yet another point of attention: in the receive-chain (during imbalance measurement), the Rx-LO is generated by mixing the Tx-IF frequency (offset-VCO/4) with the Rx-VCO. Except for the wanted (sum) term also the unwanted (minus) term appears in the LO-signal which is partly suppressed by the following BPF(1). However, this minus-term has exactly the same frequency as the spurious which is generated in the second Tx-up-conversion stage. This spurious signal could now deteriorate the accuracy of the imbalance measurement. This the spurious response is suppressed twice: first by BPF(2) after the second mixer in the transmitter chain and secondly by BPF(1) in the Rx LO-stage. Careful design and possibly an extra filter is required to tackle this issue. The combined suppression of the two BPFs also introduces an upper-bound to the maximal image suppression of the imbalance compensation scheme.

The Imbalance Estimation itself is performed in the same way as in the case of the availability of an external test-tone.

C Compensation of Imbalance Errors

There are basically two ways to compensate for imbalance errors: at the source in the RF phase-splitter, or digitally after A/D conversion. We choose the latter approach as digital operations are generally speaking less expensive and more precise.

Figure 4 shows the operation of the imbalance compensation scheme. Note that compensation in one branch (Q) is sufficient as imbalances rather than absolute numbers are relevant. At this place we recall equations (3) with all

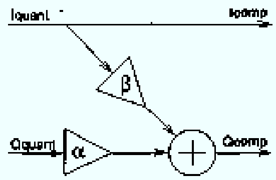

Figure 4: Imbalance compensation scheme

imbalances concentrated in the Q-branch:

$$I_{\text{quant}} = \frac{A}{2}\cos(\omega_d t + \theta) \tag{11a}$$

$$Q_{\text{quant}} = \frac{-A}{2}(1 - \Delta_{\text{total}})\sin(\omega_d t + \theta + \phi_{\text{total}}) \tag{11b}$$

Compensation yields:

$$I_{\text{comp}} = \frac{A}{2}\cos(\omega_d t + \theta) \tag{12a}$$

$$\begin{aligned} Q_{\text{comp}} = \frac{-A}{2}\,[\alpha\cos(\phi_{\text{total}})(1 - \Delta_{\text{total}})\sin(\omega_d t + \theta) \\ + \{\alpha\,[(1 - \Delta_{\text{total}}\sin(\phi)] + \beta\}\cos(\omega_d t + \theta)] \end{aligned} \tag{12b}$$

Fixing the values for α and β gives:

$$\alpha = \frac{1}{(1 - \Delta_{\text{total}})\cos(\phi_{\text{total}})} \tag{13a}$$

$$\beta = -\tan(\phi_{\text{total}}) \tag{13b}$$

which is possible as both ϕ_{total} and Δ_{total} can be measured by following the procedure in the previous section. By using those values for ϕ_{total} and Δ_{total} imbalances are completely compensated for.

D Implementation Issues

To overcome the need for trigonometric or square-root operations, we propose the following way of calculating α and β. Recall equations (10):

$$\phi_{\text{total}} = 2\arctan\left(\frac{u_1 u_2' - u_1' u_2}{u_1^2 + u_2^2}\right)$$

$$\Delta_{\text{total}} = 2\,\frac{u_1 u_1' + u_2 u_2'}{u_1^2 + u_2^2}$$

We will first calculate some intermediate variables:

$$\text{denom} = u_1^2 + u_2^2 \tag{14a}$$

$$\text{amp\_imb} = (u_1 u_1' + u_2 u_2')/\text{denom} \tag{14b}$$

$$\text{tan\_phi} = (u_1 u_2' - u_1' u_2)/\text{denom} \tag{14c}$$

now we write:

$$\text{cos\_2est} := 1 - \frac{2\,(\text{tan\_phi})^2}{1 - (\text{tan\_phi})^2} \tag{14d}$$

α and β can now be calculated (making use of the property: $\tan(2x) \approx 2\tan(x)$ for small values of x):

$$\alpha = [(1 - \text{amp\_imb}) \cdot \text{cos\_2est}]^{-1} \tag{15a}$$

$$\beta = -2\,\text{tan\_phi} \tag{15b}$$

which provides us with an inexpensive way to calculate α and β. This method is also applied in the simulations described below.

IV SIMULATION RESULTS

A Simulation Setup

The above described angle/imbalance estimation scheme is implemented in matlab, we refer to figure 2 for an outline. In GSM-like applications the bit-rate is (r_d) about 271 kbit/s. We select the test-tone to have a frequency offset of $r_d/2$ with the local oscillator. At Low-IF this translates to a CW-tone at $r_d/2 \approx 135.4$kHz. For practical reasons, simulation starts at Low-IF. However, the second-order Σ/Δ converter with a sampling frequency of $64 \cdot r_d$ is included in the simulation.

After the decimation filter ($sinc^3$), the resulting signal is down sampled to a speed of 1083 kHz (13000/12 = $4\,r_d$). At this place the I/Q compensation takes place. The second (quadrature) down conversion takes care about shifting the wanted signal to DC. The LO-frequency for these (digital) mixers is $r_d/2$. An FIR-filter usually taking care of extra adjacent channel rejection now removes the remaining 2x $r_d/2$ which interferes with the angle/imbalance measurement. After that filter the imbalance measurement is performed as described in the previous section.

A single measurement consisted of processing a time span equal to 20 symbol periods ($\approx 74\mu s$). As for the "measurement" of the values of u_1, u_2, u_1' and u_2', the group delay of the filters had to be taken into account, those values were obtained by averaging the signals (equations (8)) over the latter 8 symbol periods.

B Simulation Results

Concerning the imbalance measurement scheme, there are basically 3 variables that can be varied: ϕ, Δ and θ. The results are presented in terms of "measurement errors". Also of interest is the equivalent image rejection (EIR) that we would result after complete compensation for the measured imbalances (calculated by (6)).

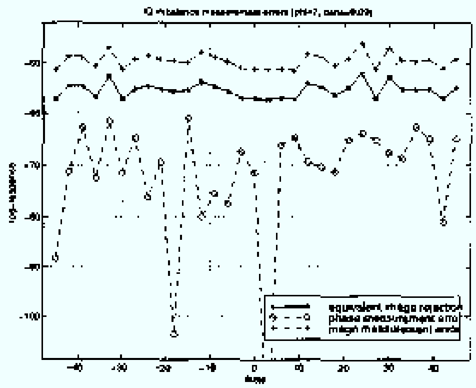

Figure 5: I/Q imbalance measurements as a function of θ

Figure 5 gives the imbalance measurement errors as a function of θ. In this simulation ϕ was equal to 7 degrees and Δ equal to 9%. The scale is logarithmic, the following equations were used for conversion:

$$\text{phase\_error} = 20\,^{10}\log[\Delta_{\text{error}}] \tag{16}$$

$$\text{magn\_error} = 20\,^{10}\log[\phi_{\text{error}}] \tag{17}$$

$$\text{EIR} = 10\,^{10}\log\left[(\sin^2(\phi_{\text{error}}/2)^2 + (\Delta_{\text{error}}/2)^2\right] \tag{18}$$

with Δ_{error} being the amplitude and ϕ_{error} the phase imbalance measurement error. We see that on logarithmic scale there the equivalent image rejection varies between 50 and 60 dB as a function of θ.

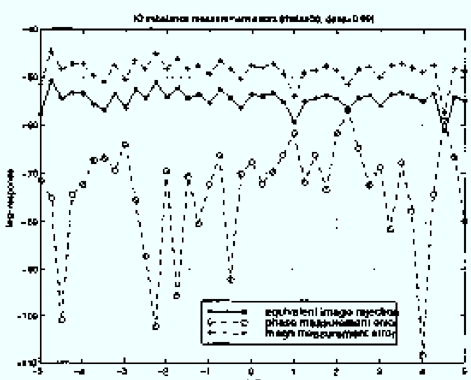

Figure 6: I/Q imbalance measurements as a function of ϕ

In figure 6 both θ (30 degrees) and Δ (7%) are constant. Again we see that the equivalent image rejection varies between 50 and 60 dB, this time when ϕ varies between -5 and +5 degrees.

We get the same results when varying Δ (figure 7). This time θ is fixed to a value of 30 degrees and ϕ is equal to 7 degrees, Δ varies between -10% and +10%.

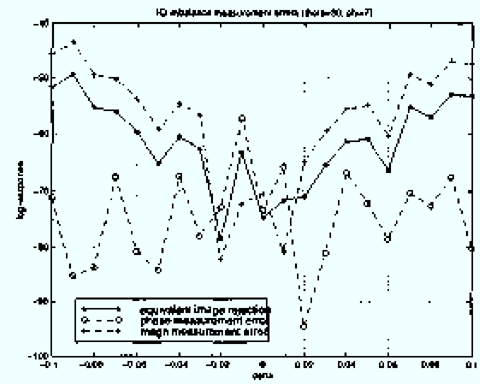

Figure 7: I/Q imbalance measurements as a function of Δ

Summarizing: the phase and amplitude imbalance measurement works over a range of 50 dB. Further experiments show that the accuracy is limited by the A/D converters. But that is not a problem as it is not necessary to measure an image signal if it is so small that its interference is not noticeable.

We will now focus on the compensation scheme. An issue here is how to determine the quality of the algorithm. We decided to perform every "simulation run" twice: the first pass calculating the image-rejection without compensation and determining the values of α and β. In the second pass these values were applied and the resulting image-rejection was measured as:

$$\text{image rej.} = 10\,^{10}\log\left[\frac{{u'_1}^2 + {u'_2}^2}{u_1^2 + u_2^2}\right] \tag{19}$$

To also incorporate the random nature of θ, this parameter was implemented as uniform distributed in $[0, 2\pi]$. Both passes of a simulation run used different (random) values of θ.

Simulation results are given in the Figures 8 and 9, for varying ϕ_1 and Δ_1 respectively. Default values were for ϕ_1 and ϕ_2, 4 and 3 degrees respectively and for Δ_1 and Δ_2, 7% and 2% respectively.

From the plot we see that the compensation scheme works as expected: the resulting image rejection after compensation is about 50 dB.

V CONCLUSION

In this document we described an angle/amplitude imbalance measurement algorithm for the I/Q paths in a Low-IF receiver. The properties can be summarized as follows:

- The algorithm enables fast en very accurate I/Q imbalance compensation at low cost. Simulation runs

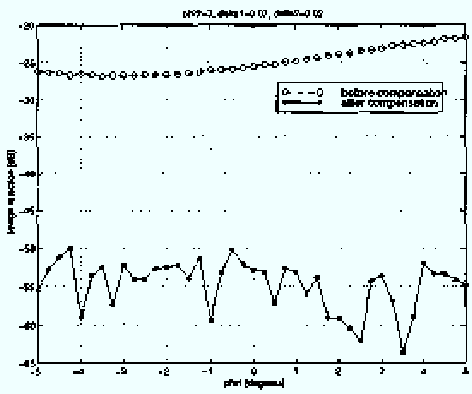

Figure 8: image rejection before and after compensation, varying ϕ_1

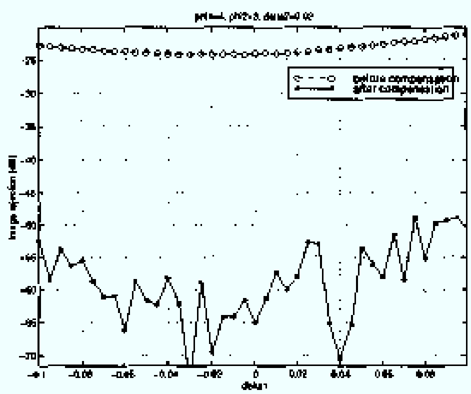

Figure 9: image rejection before and after compensation, varying Δ_1

show image rejection values of -50 dB after a compensation phase which only took a fraction of the duration of a time-slot.

- The algorithm measures and compensates for imbalances in the whole receiver chain, not only those introduced by the phase-splitter.
- Although the reachable image rejection is bounded by the the quantization noise of the A/D converters, there are no extra requirements towards them. Once the image cannot be measured, it is not relevant anymore.
- The reachable image rejection for compensation during startup is bounded by imbalances in the first upconverter in the transmit chain and the suppression of the spurious term due to mixing with the offset-VCO.
- The LNA isolation is likely to be enough to guarantee that no unwanted spurious tones are generated when applying the algorithm at start-up.
- The estimation algorithm works on "calibration"-basis either at fabrication or during start-up. We assume the phase-splitter characteristics will not change much during operation. If we cannot depend on that, calibration steps with limited performance in "empty" time slots are required.
- This article focussed on Low-IF architectures. A conceptually simular measurement algorithm can be derived for Double Low-IF architectures which apply IF-sampling.

In conclusion we can say that this scheme looks applicable for current and future GSM Mobile Stations.

VI ACKNOWLEDGMENTS

The author wishes to thank Mihai Banu, Peter Kinget and Homer Wang for the several technical discussions on this subject and for their valuable comments which were important in the development of the Imbalance Compensation Scheme.

REFERENCES

[BWS+97] Mihai Banu, Hongmo Wang, Mark Seidel, Maurice Tarsia, William Fischer, Jack P.F. Glas, Alex Dec, and Vito Boccuzzi. A bicmos double-low-if receiver for gsm. In *Proceedings of the CICC'97*, pages 521–524. IEEE, 1997.

[Cav93] James K. Cavers. Adaptive compensation for imbalance and offset losses in direct conversion transceivers. *IEEE Transactions on Vehicular Technology*, 42(4):581–588, November 1993.

[CS95a] J. Crols and M. Steyaert. An analog integrated polyphase filter for a high performance low-if receiver. In *Proc. of the VLSI Circuits Symposium*, pages 87–88, June 1995.

[CS95b] J. Crols and M. Steyaert. A single-chip 900 mhz cmos receiver front-end with a high performance low-if topology. *IEEE Journal of Solid-State Circuits*, 30(7):736–742, July 1995.

[GSM96] GSM 05.05 (ETS 300 577). European digital cellular telecommunictions system (phase 2+); radio transmission and reception, May 1996.

[PBQ97] José Páez-Borrallo and Francisco J. Casajús Quirós. Self adjusting digital image rejection receiver for mobile communications. In *Proceedings of the VTC '97*, pages 686–690, 1997.