On the Impact of Analog-to-Digital Conversion on Blind I/Q Imbalance Parameter Estimation in Low-IF Receivers

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***Abstract*— The low-IF receiver is very attractive, because the need for an analog image rejection filter is avoided. Instead, the image rejection is realized by I/Q signal processing. However, unavoidable imbalances between the I- and Q-branch in the analog part of the receiver result in a limited image attenuation.**

In previous work a blind I/Q imbalance compensation scheme for low-IF receivers was presented and its performance was analyzed. However, the impact of the analog-to-digital conversion (ADC) was not taken into consideration. In particular, the quantization noise, which is unavoidable in practical implementations, has been neglected. In this paper this blind I/Q imbalance compensation scheme is analyzed regarding to its sensitivity to an imperfect ADC. It can be shown by means of analysis as well as simulation, that the compensation scheme is robust against ADC effects.

I. Introduction

A perfect low-IF receiver enables infinite image rejection due to its I/Q signal processing [1], [2]. However, unavoidable impairments of the analog complex-valued I/Q down-conversion lead to a limited attenuation of the image signal. Considering that the image signal may originate from a different carrier frequency, it may be originally more than 50 dB stronger than the desired signal. With today's technologies an amplitude imbalance of 1-2$\%$ and a phase imbalance of 1-2° are realistic, which lead to an image attenuation of only 30-40 dB [1]. This insufficient attenuation requires an additional digital I/Q imbalance compensation.

Several digital compensation schemes have been presented in the literature. Conventional approaches include the off-line calibration with analog test signals [3], and the application of signal separation algorithms, such as interference cancellation and blind source separation [4]. Although leading to significant improvements under specific receiving conditions, these methods come with inherent drawbacks [4], [5].

A novel approach for the digital I/Q imbalance compensation, which is based on a blind I/Q imbalance parameter estimation, has been proposed in [5]. However, in the derivation of the algorithm an ideal analog-to digital conversion (ADC) was assumed. The impact of a non-ideal ADC on the performance of the digital I/Q imbalance compensation will be analyzed in this paper.

The paper is outlined as follows: In section II we will describe the problem of I/Q imbalance in low-IF receivers. Section III briefly introduces the analyzed compensation scheme.

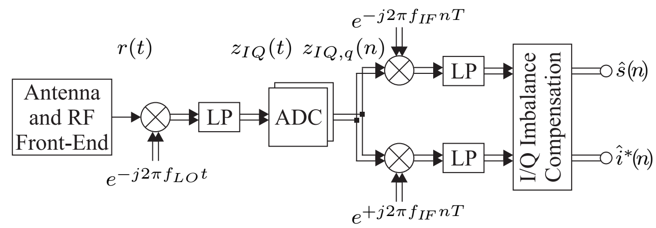

Fig. 1. Architecture of a low-IF receiver with I/Q Imbalance Compensation

The analysis of the ADC is done in the following section IV. The performance analysis is presented in section V. The final section VI concludes the paper.

II. I/Q Imbalance in Low-IF Receivers

The low-IF receiver is well known in the literature [1], [2]. The architecture of a low-IF receiver with I/Q imbalance compensation is explained in the following. The received radio frequency (RF) signal is down-converted to the intermediate frequency (IF). The resulting complex-valued IF signal is low-pass-filtered and digitized afterwards. This digital IF signal is down-converted to the baseband (BB). In order to perform the digital I/Q imbalance compensation, a second digital down-conversion branch is introduced, as shown in Figure 1.

The ideal IF signal $z(t)$, having no I/Q imbalance, can be written as:

$$z(t) = s(t)e^{j2\pi f_{IF}t} + i(t)e^{-j2\pi f_{IF}t}, \quad (1)$$

where f_{IF} denotes the IF and $s(t)$ denotes the desired signal. $i(t)$ denotes the baseband equivalent of the so-called image signal, which is separated by $2f_{IF}$ from the desired signal. With this notation, the real-valued RF signal $r(t)$ can be written as:

$$r(t) = z(t)e^{j2\pi f_{LO}t} + z^*(t)e^{-j2\pi f_{LO}t}, \quad (2)$$

where f_{LO} denotes the frequency of the complex-valued local oscillator (LO) and the asterisk denotes complex conjugation.

The I- and Q-branch of an ideal complex-valued LO have equal amplitudes and a phase difference of exactly 90°. However, analog components always have impairments. The mismatch of the amplitudes of the branches is commonly described as amplitude imbalance and the mismatch of the phases is commonly described as phase imbalance. Applying the real-valued parameters amplitude imbalance g and the phase imbalance ϕ, the time function of the imperfect complex-valued LO can be written as:

$$\begin{aligned} x_{LO}(t) &= cos(2\pi f_{LO}t) - j\, g\, sin(j2\pi f_{LO}t + \phi) \\ &= K_1 e^{-j2\pi f_{LO}t} + K_2 e^{j2\pi f_{LO}t}, \end{aligned} \tag{3}$$

where K_1 and K_2 are complex-valued imbalance parameters:

$$K_1 = \frac{1 + ge^{-j\phi}}{2}, \quad K_2 = \frac{1 - ge^{+j\phi}}{2}. \tag{4}$$

The ideal LO is a special case of equation (3) with $g = 1$ and $\phi = 0°$ ($K_1 = 1$ and $K_2 = 0$). The IF signal with I/Q imbalance $z_{IQ}(t)$ results in:

$$z_{IQ}(t) = LP\{r(t)x_{LO}(t)\} = K_1 z(t) + K_2 z^*(t), \tag{5}$$

where $LP\{\cdot\}$ denotes low-pass filtering. This IF signal has to be converted from the analog to the digital domain. Therefore the time is discretized:

$$z_{IQ}(n) = K_1 z(n) + K_2 z^*(n), \tag{6}$$

and the amplitudes are quantized. The resulting digital IF signal $z_{IQ,q}$ is as follows:

$$z_{IQ,q}(n) = z_{IQ}(n) + e_{IF}(n), \tag{7}$$

where $e_{IF}(n)$ denotes the quantization error at the IF. Assuming the low-pass filter is linear in the pass-band, the resulting BB signal is:

$$\begin{aligned} d(n) &= LP\{z_{IQ,q}(n)e^{-j2\pi f_{IF}nT}\} \\ &= K_1 s(n) + K_2 i^*(n) + \underbrace{LP\{e_{IF}(n)e^{-j2\pi f_{IF}nT}\}}_{e_d(n)}. \end{aligned} \tag{8}$$

The BB signal is a superimposition of the desired signal, the image signal and a resulting BB quantization error. For clearness the abbreviation $e_d(n)$ is applied for the resulting BB quantization error of $d(n)$ in the following. Exemplary spectra of the RF signal, IF signals and the resulting BB signal $d(n)$ are shown in Figure 2.

III. I/Q Imbalance Compensation

The analyzed compensation algorithm shall only be briefly introduced. A comprehensive presentation can be found in [5] and [6]. The algorithm aims on a compensation of the impact of the image signal on the desired signal. The approach is to evaluate an additional BB signal (see Figure 2):

$$\begin{aligned} v(n) &= LP\{\tilde{r}_{IF}(n)e^{+j2\pi f_{IF}nT}\} \\ &= K_1 i(n) + K_2 s^*(n) + \underbrace{LP\{e_{IF}(n)e^{+j2\pi f_{IF}nT}\}}_{e_v(n)}. \end{aligned} \tag{9}$$

For clearness the abbreviation $e_v(n)$ is applied for the resulting BB quantization error of $v(n)$ in the following. The relation between the BB signals can be written using matrix notation:

$$\begin{bmatrix} d(n) \\ v^*(n) \end{bmatrix} = \underbrace{\begin{bmatrix} K_1 & K_2 \\ K_2^* & K_1^* \end{bmatrix}}_{\boldsymbol{K}} \begin{bmatrix} s(n) \\ i^*(n) \end{bmatrix} + \begin{bmatrix} e_d(n) \\ e_v^*(n) \end{bmatrix} \tag{10}$$

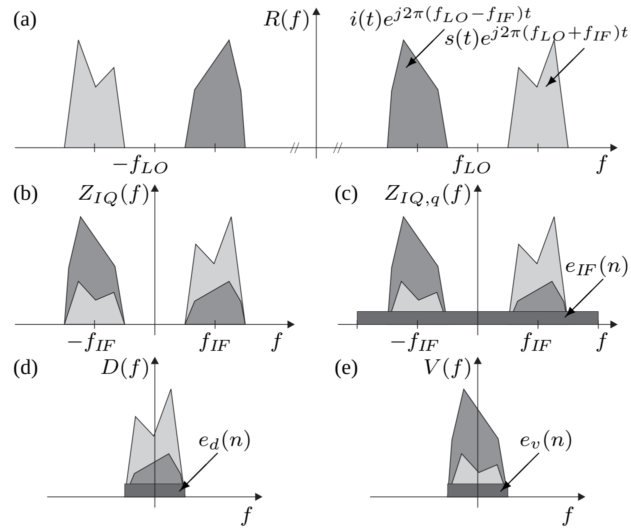

Fig. 2. Representative spectra of the analog RF signal (a), the analog IF signal (b), the digital IF signal (c) and the resulting digital BB signals d(n) (d) and v(n) (e), respectively.

The imbalance matrix $\boldsymbol{K}$ describes the mutual interference of the desired signal and the image signal. This mutual interference can be removed by applying the inverse of $\boldsymbol{K}$. However, $\boldsymbol{K}^{-1}$ is unknown in practice. The principle of the analyzed compensation scheme is to blindly gain an estimate $\hat{\boldsymbol{K}}^{-1}$. In a first step, an estimate of the product K_1K_2 is calculated by evaluating a block of N consecutive samples of the BB observations, the product $\widehat{K_1K_2}$ yields:

$$\widehat{K_1K_2} = \frac{\frac{1}{N}\sum_{n=1}^{N} d(n)v(n)}{\frac{1}{N}\sum_{n=1}^{N} |d(n) + v^*(n)|^2}. \tag{11}$$

Using the estimated product, the estimation of the real-valued imbalance parameters is possible:

$$\hat{g} = \sqrt{1 - 4Re\{\widehat{K_1K_2}\}} \qquad \hat{\phi} = \arcsin\left(-\frac{\hat{g}}{2} Im\{\widehat{K_1K_2}\}\right) \tag{12}$$

By adapting definition (4) to the complex-valued estimates $\hat{K}_1$ and $\hat{K}_2$, an estimate of $\hat{\boldsymbol{K}}^{-1}$ can be determined as follows:

$$\hat{\boldsymbol{K}}^{-1} = \frac{1}{|\hat{K}_1|^2 - |\hat{K}_2|^2} \begin{bmatrix} \hat{K}_1^* & -\hat{K}_2 \\ -\hat{K}_2^* & \hat{K}_1 \end{bmatrix}. \tag{13}$$

The multiplication of the disturbed BB signals with $\hat{\boldsymbol{K}}^{-1}$ results in the estimated BB signals $\hat{s}(n)$ and $\hat{i}(n)$:

$$\begin{bmatrix} \hat{s}(n) \\ \hat{i}^*(n) \end{bmatrix} = \hat{\boldsymbol{K}}^{-1} \cdot \begin{bmatrix} d(n) \\ v^*(n) \end{bmatrix}. \tag{14}$$

IV. Model of the Analog-to-Digital Conversion

The I/Q imbalance compensation takes place in the BB. In order to determine the performance of the compensation scheme, the impact of the ADC has to be analyzed. However, the ADC takes place at the IF. The relation between the ADC at the IF and the resulting impact on the BB is analyzed in this section.

An important issue for the effective resolution of an ADC is its saturation level. While too high signal amplitudes lead to clipping, too low amplitudes result in a decrease of the effective resolution. These effects can be modeled by defining an input power backoff (IBO) as well as a peak-to-average power ratio (PAPR).

The IBO describes the relation between the maximum possible output power $\sigma^2_{out,max}$ of the ADC and the variance σ^2_x of the input signal:

$$\text{IBO} = \frac{\sigma^2_{out,max}}{\sigma^2_x} \tag{15}$$

The statistics of the input signal are considered using the PAPR:

$$\text{PAPR} = \frac{\sigma^2_{x,max}}{\sigma^2_x} \tag{16}$$

where $\sigma^2_{x,max}$ denotes the maximum signal power. Under the condition IBO $\geq$ PAPR clipping is avoided. The quantization error of the ADC is considered as independent, uniformly distributed noise, known as the pseudo quantization noise model [7]. Under this assumption the signal-to-noise ratio (SNR) for a quantization with b bits results in:

$$\text{SNR} = 3 \cdot 4^b / \,\text{IBO}, \tag{17}$$

which can be equivalently written in dB-scale as:

$$\text{SNR}_{[\text{dB}]} = 4.77\text{dB} + b \cdot 6.02\text{dB} - \text{IBO}_{[\text{dB}]}. \tag{18}$$

In order to describe a real ADC, more errors than the quantization errors have to be considered. The signal is additionally distorted by aperture jitter, ambiguity of the comparator, nonlinearities of the characteristic and several noise processes. An excellent analysis can be found in [8]. These effects can be modeled using an effective number of bits (ENOB) instead of the nominal number of bits b. Hence the SNR is as follows:

$$\text{SNR} = 3 \cdot 4^{\text{ENOB}} / \,\text{IBO}. \tag{19}$$

This formula models an ADC considering signal statistics as well as the impact of distorting effects of a real ADC. In a low-IF receiver, the ADC takes place at the IF. Therefore the SNR of the quantized IF signal $z_{IQ}(n)$ is defined as:

$$\text{SNR} = \frac{P_{z_{IQ}}}{P_{e_{IF}}}, \tag{20}$$

where $P_{z_{IQ}}$ denotes the power of $z_{IQ}(n)$ and $P_{e_{IF}}$ denotes the power of the IF quantization error $e_{IF}(n)$, as defined by (7). Applying (6) and assuming that the desired signal and the image signal are uncorrelated, $P_{z_{IQ}}$ results in:

$$P_{z_{IQ}} = \left(|K_1|^2 + |K_2|^2\right)(P_s + P_i), \tag{21}$$

where P_s and P_i denote the power of the desired signal and the image signal, respectively.

For the analysis of the BB signals, the power of the IF quantization error $P_{e_{IF}}$ is not a useful measure. Instead, only those spectral components, which superimpose the IF signals of interest are critical, as depicted in Figure 2. In general, the power of these spectral components can be adjusted by noise-shaping techniques. However, it is reasonable to consider the special case of a uniformly distributed quantization noise. In this case, the portion of the quantization noise corresponding to the bandwidth of the BB signals is:

$$P_{e_{BB}} = \frac{P_{e_{IF}}}{\text{OSR}}, \tag{22}$$

where OSR denotes an oversampling ratio, which is determined by the ratio of the BB sampling frequency to the IF sampling frequency. By merging (19) - (22), the power of the equivalent baseband quantization noise can be calculated as follows:

$$P_{e_{BB}} = \frac{\left(|K_1|^2 + |K_2|^2\right)(P_s + P_i)}{\text{SNR}_{\text{eff}}}. \tag{23}$$

SNR_{eff} is the effective SNR, which models all effects related to the resampling and quantization process:

$$\begin{aligned}\text{SNR}_{\text{eff}} &= \frac{P_{z_{IQ}}}{P_{e_{BB}}} = \text{SNR} \cdot \text{OSR} \\ &= 3 \cdot \text{OSR} \cdot 4^{\text{ENOB}} / \,\text{IBO}\end{aligned} \tag{24}$$

As expected, the SNR_{eff} depends on the properties of the ADC. The higher ENOB or OSR are, the higher is the SNR_{eff}. A large IBO leads to a degradation.

V. Performance Analysis

A. Theoretical Analysis

1) Performance without Digital Compensation: In order to evaluate the performance of the compensation scheme, we need an appropriate reference. As reference we consider the BB signal without digital compensation. Equation (10) leads to:

$$d(n) = K_1 s(n) + K_2 i^*(n) + e_d(n). \tag{25}$$

Based on equation (25) we define an image-and-noise-to-signal ratio (INSR):

$$\text{INSR}_0 = \frac{|K_2|^2 P_i + P_{e_d}}{|K_1|^2 P_s} = \underbrace{\left|\frac{K_2}{K_1}\right|^2 \frac{P_i}{P_s}}_{\text{ISR}_0} + \underbrace{\frac{P_{e_d}}{|K_1|^2 P_s}}_{\text{NSR}_0}, \tag{26}$$

where P_{e_d} denotes the power of the BB quantization error $e_d(n)$. The reference INSR_0 is composed of two terms: an image-to-signal ratio (ISR_0) representing the impact of the I/Q imbalance and a noise-to-signal ratio (NSR_0). The NSR_0 is the performance of a reception without any I/Q imbalance. This limit holds also, if a digital compensation of I/Q imbalance is applied.

Now, we consider the special case of a uniformly distributed quantization noise ($P_{e_d} = P_{e_{BB}}$). The NSR_0 as a function of the power ratio of the desired signal and the image signal P_s/P_i is depicted in Figure 3. For a dominating image signal ($P_i \gg P_s$) the ADC is saturated by the image signal, hence the NSR_0 is increased. For ($P_s \gg P_i$) the NSR_0 is determined by the SNR_{eff}. Figure 4 shows the composition of the INSR_0,

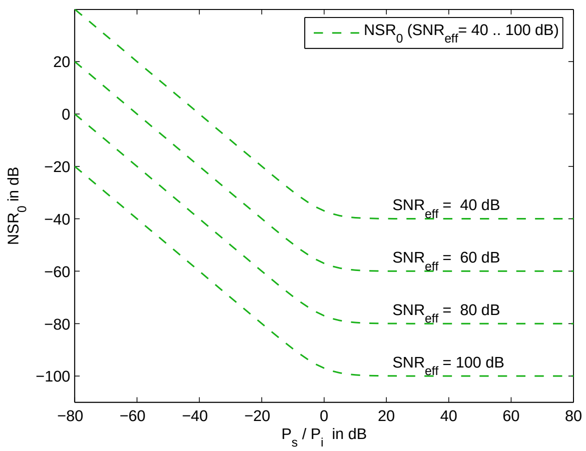

Fig. 3. Noise-to-signal ratio due to quantization without I/Q imbalance

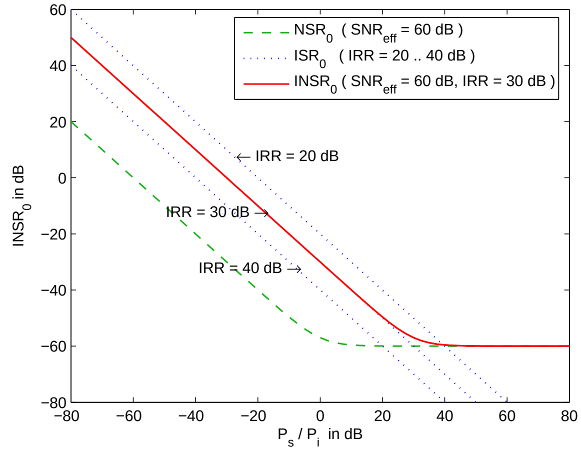

Fig. 4. Image-and-noise-to-signal ratio (no compensation of I/Q Imbalance)

based on NSR$_0$ and ISR$_0$. The plots are parameterized by the image rejection ratio (IRR):

$$\text{IRR} = \left|\frac{K_1}{K_2}\right|^2, \tag{27}$$

which describes the finite image suppression due to the I/Q imbalance. Figure 4 shows, that the INSR$_0$ is significantly degraded by the finite IRR.

2) Performance with Digital Compensation: By merging (10) and (14), the estimated BB signals $\hat{s}(n)$ and $\hat{i}(n)$ after the digital compensation can be expressed as:

$$\begin{bmatrix} \hat{s}(n) \\ \hat{i}^*(n) \end{bmatrix} = \hat{\boldsymbol{K}}^{-1} \cdot \boldsymbol{K} \cdot \begin{bmatrix} s(n) \\ i^*(n) \end{bmatrix} + \hat{\boldsymbol{K}}^{-1} \cdot \begin{bmatrix} e_d(n) \\ e_v^*(n) \end{bmatrix}. \tag{28}$$

Applying the abbreviations $a_{11} = K_1\hat{K}_1^* - K_2^*\hat{K}_2$ and $a_{12} = -K_1^*\hat{K}_2 + K_2\hat{K}_1^*$, the product $\hat{\boldsymbol{K}}^{-1}\boldsymbol{K}$ can be written as:

$$\hat{\boldsymbol{K}}^{-1} \cdot \boldsymbol{K} = \frac{1}{|\hat{K}_1|^2 - |\hat{K}_2|^2} \begin{bmatrix} a_{11} & a_{12} \\ a_{12}^* & a_{11}^* \end{bmatrix}. \tag{29}$$

Hence, the estimated desired BB signal $\hat{s}(n)$ can be written as:

$$\hat{s}(n) = \frac{a_{11}s(n) + a_{12}i^*(n) + \hat{K}_1^* e_d(n) - \hat{K}_2 e_v^*(n)}{|\hat{K}_1|^2 - |\hat{K}_2|^2}. \tag{30}$$

In the case of a perfect estimation ($\hat{\boldsymbol{K}} = \boldsymbol{K}$), the term a_{12} will be zero. However, due to the finite accuracy of the estimation, a residual interference of the image signal will persist. Hence, we define the INSR after the digital compensation:

$$\begin{aligned} \text{INSR}_c &= \frac{|a_{12}|^2 P_i + |\hat{K}_1|^2 P_{e_d} + |\hat{K}_2|^2 P_{e_v}}{|a_{11}|^2 P_s} \\ &= \underbrace{\left|\frac{a_{12}}{a_{11}}\right|^2 \frac{P_i}{P_s}}_{\text{ISR}_c} + \underbrace{\frac{|\hat{K}_1|^2 P_{e_d} + |\hat{K}_2|^2 P_{e_v}}{|a_{11}|^2 P_s}}_{\text{NSR}_c}, \end{aligned} \tag{31}$$

where P_{e_v} denotes the power of the BB quantization error $e_v(n)$. In analogy to the INSR$_0$ (no compensation), the INSR$_c$ is determined by the imperfect compensation (ISR$_c$) and the impact of the ADC (NSR$_c$).

In the following we will derive analytic expressions of ISR$_c$ and NSR$_c$. In order to simplify our analysis we incorporate the following useful properties: For realistic I/Q imbalances definition (4) yields $K_1 \approx 1$, $K_2 \approx 0$, hence $|K_1| \gg |K_2|$. Furthermore, we assume the estimates to be close to, but not necessarily identical to the desired values, i.e. $\hat{K}_1 \approx K_1$ and $\hat{K}_2 \approx K_2$.

First we analyze the term NSR$_c$. The additional assumption of similar noise powers $P_{e_d} \approx P_{e_v}$ leads to:

$$\text{NSR}_c \approx \frac{|K_1|^2 P_{e_d}}{|K_1 - K_2|^2 P_s} \approx \frac{P_{e_d}}{P_s}. \tag{32}$$

In contrast, the ISR$_c$ is more complicated. The results of the estimation depend on the certain realization and are therefore not deterministic. However, the performance can be evaluated in a statistical sense. It has been shown in [6], that the expectation of ISR$_c$ can be approximated by:

$$E\{\text{ISR}_c\} \approx \left|\frac{K_2}{K_1}\right|^2 E\left\{\left|\frac{\Delta_{K_1K_2}}{K_1K_2}\right|^2\right\} \frac{P_i}{P_s}, \tag{33}$$

where $\Delta_{K_1K_2} = \hat{K}_1\hat{K}_2 - K_1K_2$ denotes the absolute estimation error of the product K_1K_2 and $E\{\cdot\}$ denotes expectation.

Our next goal is to analyze the expectation of the squared magnitude of the relative estimation error on the right hand side of (33). Therefore, the numerator and denominator of (11) have to be analyzed using (8) and (9). For clearness, we introduce the following abbreviations with the representative complex-valued signals $x(n)$ and $y(n)$:

$$\hat{P}_x = \frac{1}{N}\sum_{n=1}^{N} |x(n)|^2 \qquad \hat{R}_{xy} = \frac{1}{N}\sum_{n=1}^{N} |x(n)y^*(n)|^2 \tag{34}$$

With this notation the numerator of (11) can be written as:

$$\begin{aligned} \frac{1}{N}\sum_{n=1}^{N} d(n)v(n) = &K_1^2\hat{R}_{si^*} + K_2^2\hat{R}_{si^*}^* + K_1(\hat{R}_{se_v^*} + \hat{R}_{ie_d^*}) \\ &+ K_1K_2(\hat{P}_s + \hat{P}_i) + K_2(\hat{R}_{ie_v}^* + \hat{R}_{se_d}^*) + \hat{R}_{e_de_v^*}. \end{aligned}$$

Assuming $|K_2(\hat{R}^*_{ie_v}+\hat{R}^*_{se_d})| \ll |K_1(\hat{R}_{se_v^*}+\hat{R}_{ie_d^*})|$ yields:

$$\frac{1}{N}\sum_{n=1}^{N} d(n)v(n) \approx K_1^2\hat{R}_{si^*} + K_1(\hat{R}_{se_v^*}+\hat{R}_{ie_d^*}) + K_1K_2(\hat{P}_s+\hat{P}_i)+\hat{R}_{e_de_v^*}. \quad (35)$$

The denominator of equation (11) can be written as:

$$\frac{1}{N}\sum_{n=1}^{N}|d(n)+v^*(n)|^2 = \hat{P}_s+\hat{P}_i+\hat{P}_{e_d}+\hat{P}_{e_v}+2Re\{\hat{R}_{si^*}+\hat{R}_{se_d} +\hat{R}_{se_v}+\hat{R}_{ie_d^*}+\hat{R}_{ie_v}+\hat{R}_{e_de_v^*}\}.$$

This term can be simplified with the following assumptions: We assume that the sum of the cross-correlations of a BB signal with the quantization errors is smaller than the corresponding signal power: $|\hat{R}_{se_d}|+|\hat{R}_{se_v}| \ll \hat{P}_s$ and $|\hat{R}_{ie_d^*}|+|\hat{R}_{ie_v}| \ll \hat{P}_i$. The additional assumption, that the cross-correlation of two signals is smaller than the sum of the corresponding signal powers: $|\hat{R}_{si^*}| \ll \hat{P}_s+\hat{P}_i$ and $|\hat{R}_{e_de_v^*}| \ll \hat{P}_{e_d}+\hat{P}_{e_v}$ yields for the denominator of (11):

$$\frac{1}{N}\sum_{n=1}^{N}|d(n)+v^*(n)|^2 \approx \hat{P}_s+\hat{P}_i+\hat{P}_{e_d}+\hat{P}_{e_v}. \quad (36)$$

Applying (35) and (36), the expectation of the squared magnitude of the relative estimation error can be written as:

$$E\left\{\left|\frac{\Delta_{K_1K_2}}{K_1K_2}\right|^2\right\} = E\left\{\left|\frac{\hat{K}_1\hat{K}_2-K_1K_2}{K_1K_2}\right|^2\right\} \quad (37)$$
$$= E\left\{\left|\frac{K_1^2\hat{R}_{si^*}+K_1(\hat{R}_{se_v^*}+\hat{R}_{ie_d^*})+\hat{R}_{e_de_v^*}-K_1K_2(\hat{P}_{e_d}+\hat{P}_{e_v})}{K_1K_2(\hat{P}_s+\hat{P}_i+\hat{P}_{e_d}+\hat{P}_{e_v})}\right|^2\right\}$$

When splitting the expectation term in equation (37), several products of cross-correlations arise. In order to simplify the terms, we assume a sufficiently large block size N. The estimates of the cross-correlations converge under this condition to their expectations. Furthermore, we assume independent correlation functions and uncorrelated signals. Under these conditions the following equation with the representative signals x, y, a and b holds:

$$E\{\hat{R}_{xy^*}\hat{R}_{ab^*}\} = E\{\hat{R}_{xy^*}\}E\{\hat{R}_{ab^*}\} = R_{xy^*}R_{ab^*} = 0. \quad (38)$$

The signal powers converge for a sufficiently large block size N as well to their expectations $\hat{P}_x \rightarrow E\{\hat{P}_x\} = P_x$. With these assumptions and the additional reasonable assumption $P_{e_d}+P_{e_v} \ll P_s+P_i$, the expectation of the squared magnitude of the relative estimation error results in:

$$E\left\{\left|\frac{\Delta_{K_1K_2}}{K_1K_2}\right|^2\right\} \approx \frac{1}{(P_s+P_i)^2}\left[\left|\frac{K_1}{K_2}\right|^2 E\{|\hat{R}_{si^*}|^2\} + \frac{1}{|K_2|^2}\left(E\{|\hat{R}_{se_v^*}|^2\} + E\{|\hat{R}_{ie_d^*}|^2\}\right) + \frac{E\{|\hat{R}_{e_de_v^*}|^2\}}{|K_1K_2|^2} + (P_{e_d}+P_{e_v})^2\right]$$

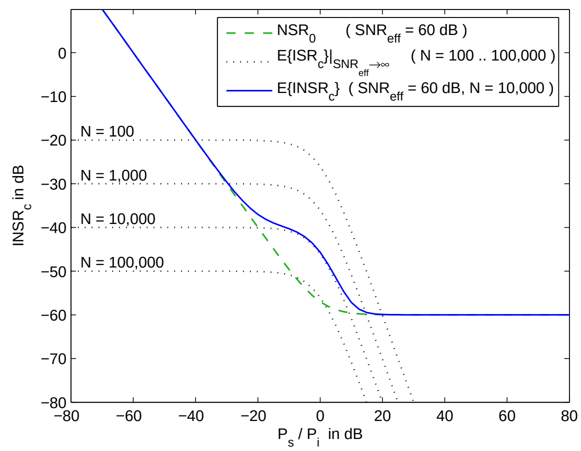

Fig. 5. Composition of the image-and-noise-to-signal ratio with I/Q imbalance compensation

As derived in [6], the variance of correlation functions can be written as: $E\{|\hat{R}_{xy^*}|^2\} = \frac{1}{N}P_xP_y$. Hence the $E\{\text{ISR}_c\}$ yields:

$$E\{\text{ISR}_c\} \approx \frac{1}{N}\frac{1}{\left(1+\frac{P_s}{P_i}\right)^2} + \left|\frac{K_2}{K_1}\right|^2\frac{P_i}{P_s}\frac{(P_{e_d}+P_{e_v})^2}{(P_s+P_i)^2} + \frac{1}{N|K_1|^2}\frac{P_i}{P_s}\frac{1}{(P_s+P_i)^2}\left[P_sP_{e_v}+P_iP_{e_d}+\frac{P_{e_d}P_{e_v}}{|K_1|^2}\right]. \quad (39)$$

This general result simplifies for the special case of a uniformly distributed quantization noise $P_{e_d}=P_{e_v}=P_{e_{BB}}$. By merging (23) into (39) we get:

$$E\{\text{ISR}_c\} \approx E\{\text{ISR}_c\}\Big|_{\text{SNR}_{\text{eff}}\rightarrow\infty} + \gamma, \quad (40)$$

where

$$E\{\text{ISR}_c\}\Big|_{\text{SNR}_{\text{eff}}\rightarrow\infty} = \frac{1}{N}\frac{1}{\left(1+\frac{P_s}{P_i}\right)^2} \quad (41a)$$

$$\gamma = \frac{1}{\text{SNR}_{\text{eff}}}\frac{P_i}{P_s}\left[\frac{1}{N}+\frac{1}{N\text{SNR}_{\text{eff}}}+\frac{4}{\text{SNR}_{\text{eff}}}\left|\frac{K_2}{K_1}\right|^2\right]. \quad (41b)$$

The $E\{\text{ISR}_c\}$ is composed of two terms: The term $E\{\text{ISR}_c\}\Big|_{\text{SNR}_{\text{eff}}\rightarrow\infty}$ represents the impact of the imperfect digital image suppression due to finite estimator block length N in the absence of any quantization noise. This result corresponds to the results of [6]. The second term γ represents the degradation of the estimation of the I/Q imbalance parameters due to the impact of the ADC.

Similarly, by merging (23) into (32), the NSR_c yields:

$$\text{NSR}_c \approx \text{NSR}_0 \approx \frac{1}{\text{SNR}_{\text{eff}}}\left(1+\frac{P_i}{P_s}\right) \quad (42)$$

The application of (40) and (42) to the expectation of the INSR_c of equation (31) results in:

$$E\{\text{INSR}_c\} \approx E\{\text{ISR}_c\}\Big|_{\text{SNR}_{\text{eff}}\rightarrow\infty} + \text{NSR}_0 + \gamma \quad (43)$$

The performance $E\{\text{INSR}_c\}$ after the digital compensation is composed of three terms. The first term represents the

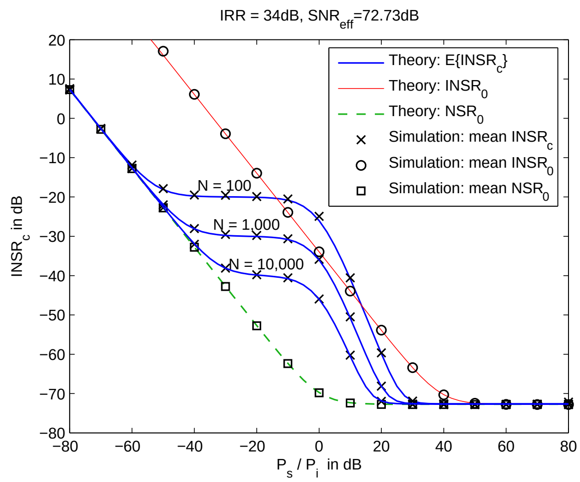

Fig. 6. Comparison of NSR_0, INSR_0 and $E\{\mathrm{INSR}_c\}$ applying theoretical analysis and simulation results

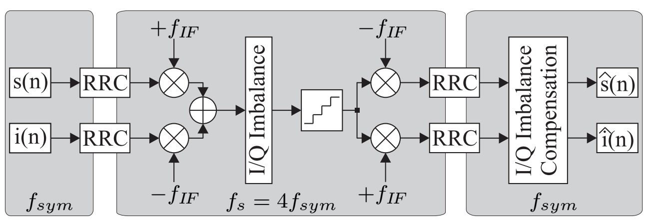

Fig. 7. Overview of the simulation

performance of the digital compensation scheme with a finite block size N but no limitations due to the ADC. The second term NSR_0 represents the impact of the ADC on the desired signal. The third term γ represents the impact of the ADC on the estimation of the I/Q imbalance parameters.

A comparison of (41b) and (42) yields $\gamma \ll \mathrm{NSR}_0$ for realistic values of the $\mathrm{SNR}_{\mathrm{eff}}$ and the block size N. Hence γ can be neglected in (43):

$$E\{\mathrm{INSR}_c\} \approx E\{\mathrm{ISR}_c\}\Big|_{\mathrm{SNR}_{\mathrm{eff}}\to\infty} + \mathrm{NSR}_0. \quad (44)$$

The composition of the $E\{\mathrm{INSR}_c\}$ is depicted in Figure 5.

B. Simulation

The theoretical results were verified by the results of MATLAB simulations. For an overview see Figure 7. As source signals, we used randomly generated QPSK signals with a symbol rate of $f_{sym} = 3.84$ MHz. These signals were upsampled using a root-raised cosine (RRC) filter to a sample rate of $f_s = 4f_{sym}$, hence the oversampling ratio was OSR=4. Afterwards the BB signals were upconverted to the IF $f_{IF} = \pm f_{sym}$, respectively. The I/Q imbalance was modeled at the IF according to equation (6). The IF signal with I/Q imbalance was quantized with an uniform mid-rise quantizer. Based on the quantized IF signal the observations $d(n)$ and $v(n)$ were generated by an appropriate down-conversion, RRC filtering and decimation. The proposed compensation scheme was applied in order to reconstruct the desired signal $\hat{s}(n)$.

Figure 6 shows the results of the theoretical analysis in comparison to results of the simulation for the exemplary I/Q imbalance parameters $g = 1.02$ and $\phi = 2°$ (IRR=34 dB) and the exemplary ADC parameters ENOB=12 Bit and IBO=10 dB ($\mathrm{SNR}_{\mathrm{eff}}$=72.73 dB). It can be easily seen, that the simulation results confirm the results of the theoretical analysis.

VI. Conclusion

The impact of a non-ideal analog-to-digital conversion (ADC) on the performance of a novel blind I/Q imbalance compensation has been analyzed in this paper. We developed a set of equations for the theoretical performance evaluation with and without digital I/Q imbalance compensation. The validity of these equations has been confirmed by computer simulations.

The quantization noise caused by the finite number of discrete levels of the ADC has been identified as one of the key problems in the practical implementation of the low-IF receiver. Even in the presence of no I/Q imbalance, a powerful image signal can saturate the ADC, such that the signal quality of the desired signal is impaired. However, the situation is significantly aggravated in the presence of I/Q imbalance. In this case, interference by the insufficiently attenuated image signal is the dominant source of signal degradation.

Fortunately, the interference by the image signal can be removed by a digital post-correction of the I/Q imbalance. Our analysis shows, that the analyzed blind I/Q imbalance parameter estimation algorithm is insensitive to the presence of quantization errors. Depending on the requirements of the communication standard, the compensation algorithm can be dimensioned, such that the performance with digital compensation is arbitrarily close to the reference case of a low-IF receiver with ADC errors, but without any I/Q imbalance. Hence, the demands to the image rejection capabilities of the analog front-end are significantly reduced due to the digital I/Q imbalance compensation.

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