Electric Field Calculation of High Voltage Transmission Line

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Abstract— Electric field around the overhead transmission line is researched in this paper. Surface charges method is introduced to calculate the surface charges density of every splitting conductor, the surface of every splitting conductor is divided into many arc segments, and linear interpolation is used to express the charges density of every arc segment. In the meantime, partial capacitors between two phases have been studied. An example is presented to illustrate the advantage and the precision of the new given method.

1. INTRODUCTION

Electric field around the overhead transmission line is needed to calculate accurately for design and environment evaluation. Now, three are two calculation methods as following. (1) Equivalent charges method. This method assumes that the charge of transmission line is concentrating on the centre line of the splitting conductors, and can not give the maximums of surface charges density and electric field intensity. (2) Finite element method. This method is not suit for infinite field firstly, then it use potential as solving variable, the quantities such as charges density and electric field intensity need to differentiate with respect to the potential, and will lead to more error. Lastly, surface of the splitting conductors is very difficult to dissect into small triangle element, more triangle elements and calculation time must be need.

Surface charges method for calculating electrostatic field around the double circuit compact overhead transmission line on same tower is given in this paper. This method supposes that the charges of transmission line distribute continuously on the surface of the splitting conductors, and the surface of every splitting conductor is divided into many arc segments, linear interpolation is used to express the charges density of every arc segment. The surface charges density of the splitting conductors is calculated, and maximums of surface charges density and electric field intensity can be determined. Example shows the advantage and the precision of this method.

In the present time, the concept of phase operating capacitor is used to evaluate the imbalance of three phases in power transmission design widely. This concept may have some mistake theoretically, and partial capacitors between two phases are introduced to substitute this concept.

2. SURFACE CHARGES METHOD

Every phase conductor of $500\,\text{kV}$ compact overhead transmission line has six splitting conductors which position on a circle with same interval, as shown in Fig. 1.
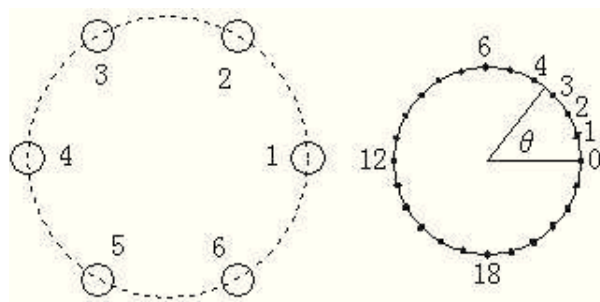

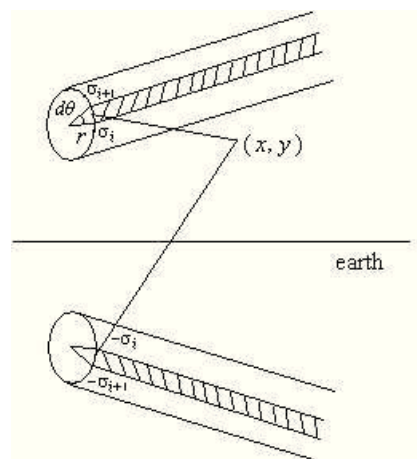Figure 1: Splitting conductor arrangement and dissection of splitting conductor surface.

Figure 2: Infinite length line charges and its mirror.

Because power frequence is very low, the electric field around the transmission line can be treated as electrostatic field, and the transmission line is supposed as infinite length linearly, the field can be simplified as a parallel plane field.

Separate the surface of every splitting conductor into 24 arc segments with same length, and number all nodes from 0 to 23, as shown in Fig. 1. Set the surface charges density of every node be σ_i, $i = 0, 1, 2, \ldots$, then the surface charges density of any point on the splitting conductor surface which have a angle θ is:

$$\sigma(\theta) = \frac{2\pi(i+1) - \theta}{24}\sigma_i + \frac{\theta - 2\pi i}{24}\sigma_{i+1} \text{ if } \frac{2\pi}{24}i < \theta < \frac{2\pi}{24}(i+1) \tag{1}$$

Thus, the surface charges on splitting conductor can be regarded as many infinite length line charges with a line density $r\sigma(\theta)d\theta$, as shown in Fig. 2.

According to electrostatic field theory, the potential of any point (x, y) produced by infinite length surface charges with an arc segment width and its mirror to the ground is:

$$d\varphi(x, y) = \frac{1}{2\pi\varepsilon_0} \int_{\frac{2\pi}{24}i}^{\frac{2\pi}{24}(i+1)} r\sigma(\theta) \ln\frac{R_2}{R_1} d\theta = c_i\sigma_i + c_{i+1}\sigma_{i+1} \tag{2}$$

$$R_1 = \sqrt{(x_0 + r\cos(\theta) - x)^2 + (y_0 + \sin(\theta) - y)^2} \tag{3}$$

$$R_2 = \sqrt{(x_0 + r\cos(\theta) - x)^2 + (y_0 + \sin(\theta) + y)^2} \tag{4}$$

Here: (x_0, y_0) is the coordinates of splitting conductor circle centre, r is radius of splitting conductor, c_i and c_{i+1} are coefficient that can be calculated by actual coordinates.

Double circuits have 36 splitting conductors and two ground lines, there are $N = 38 \times 24 = 912$ nodes and arc segments. The potential of nodes on the ground lines are zero, that is $u_E = 0$, and the potential of nodes on the conductors may be as following according its phase sequence.

$$\begin{cases} u_A = 500\cos(\beta)/\sqrt{3} \\ u_B = 500\cos(\beta - 2\pi/3)/\sqrt{3} \\ u_C = 500\cos(\beta + 2\pi/3)/\sqrt{3} \end{cases} \tag{5}$$

Here β is original phase angle.

Calculate the coefficient of every arc segment to every node using (2), then add up, then we get a linear equations:

$$u = \mathbf{A}\sigma \tag{6}$$

Here: u is the vector of nodes potential, σ is the vector of nodes surface charges density, $\mathbf{A}$ is an $N \times N$ matrix.

The surface charges density of every node can be obtained by solving (6), then the unit length charges of every splitting conductor is:

$$q_S = \sum_{i=0}^{23} \int_{\frac{2\pi}{24}i}^{\frac{2\pi}{24}(i+1)} r\sigma(\theta)d\theta \tag{7}$$

The total charges of each phase per unit length is the sum of charges of six splitting conductor. The potential and electric field intensity of any point can be calculated also.

3. PARTIAL CAPACITORS

In the engineering, operating capacitor per unit length of each phase is used to evaluate the imbalance of the charges on each phase. At the present time, the operating capacitor per unit length of each phase is defined as the ratio of its charges of unit length and potential. Because three phases is not symmetrical geometrically, the partial capacitors between two phases are not equivalent, and this definition of the operating capacitor is not correct theoretically.

Partial capacitors of the overhead transmission line are illustrated in Fig. 3.

Take A phase for example. The Charges of A phase is:

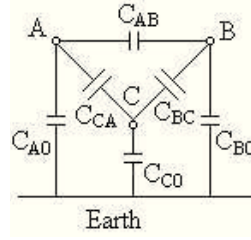$$q_A = C_{A0}u_A + C_{AB}(u_A - u_B) + C_{CA}(u_A - u_C) \tag{8}$$

Figure 3: Partial capacitor.

If $C_{AB} = C_{BC} = C_{CA}$, then we get:

$$q_A = (C_{A0} + 3C_{AB})u_A \tag{9}$$

Thus, phase charge is proportionate to phase potential, so we can define the operating capacitor as the ratio of phase charges and phase potential.

But if C_{AB}, C_{BC} and C_{CA} are not equivalent, the charges of any phase is not proportionate to its phase potential, the ratio of phase charges and phase potential will vary along with the original phase angle β, so this definition of the operating capacitor has no meaning.

Partial capacitors are more useful than the operating capacitor. The imbalance of the charges on three phases can be evaluated by the imbalance of partial capacitors.

Partial capacitor can be calculated repeatedly as following by numerical method proposed above:

(1) Set $u_A = u$ and $u_B = u_C = 0$, calculate $C_1 = q_A/u$. (2) Set $u_B = u$ and $u_A = u_C = 0$, calculate $C_2 = q_B/u$. (3) Set $u_C = u$ and $u_A = u_B = 0$, calculate $C_3 = q_C/u$. (4) Set $u_A = u_B = u$ and $u_C = 0$, calculate $C_4 = (q_A + q_B)/u$. (5) Set $u_B = u_C = u$ and $u_A = 0$, calculate $C_5 = (q_B + q_C)/u$. (6) Set $u_A = u_C = u$ and $u_B = 0$, calculate $C_6 = (q_A + q_C)/u$. Here u is an arbitrary number.

These capacitors have a relationship with partial capacitors as:

$$
\begin{cases}
C_{A0} + C_{AB} + C_{CA} = C_1 \\
C_{B0} + C_{AB} + C_{BC} = C_2 \\
C_{C0} + C_{BC} + C_{CA} = C_3 \\
C_{A0} + C_{BC} + C_{CA} + C_{B0} = C_4 \\
C_{B0} + C_{AB} + C_{CA} + C_{C0} = C_5 \\
C_{A0} + C_{BC} + C_{AB} + C_{C0} = C_6
\end{cases}
\Rightarrow
\begin{cases}
C_{A0} = (C_4 + C_6 - C_2 - C_3)/2 \\
C_{B0} = (C_4 + C_5 - C_1 - C_3)/2 \\
C_{C0} = (C_5 + C_6 - C_1 - C_2)/2 \\
C_{AB} = (C_1 + C_2 - C_4)/2 \\
C_{BC} = (C_2 + C_3 - C_5)/2 \\
C_{CA} = (C_1 + C_3 - C_6)/2
\end{cases}
\tag{10}
$$

Partial capacitors are six unaltered constants, they are determined only by medium and geometrical parameters of the transmission line, and have no relationship with phase potential.

4. CALCULATING RESULT

Surface charges density on every splitting conductor of A phase is shown in Fig. 4 when the original phase angle β is zero. Surface charges density of the No. 1 and No. 6 splitting conductor of A phase are biggest, then the No. 2 and No. 5 secondly, that of the No. 3 and No. 4 splitting conductor are smallest.

Figure 5 shows the Equipotential line distribution.

Table 1: Partial capacitors (pF/m).

C_1	C_2	C_3	C_4	C_5	C_6
14.0997	14.969	13.9402	21.1835	20.7510	20.7556
C_{A0}	C_{B0}	C_{C0}	C_{AB}	C_{BC}	C_{CA}
6.9512	6.9473	6.6551	7.0131	7.3861	7.3843

Table 1 shows the values of partial capacitors. Due to the shield effect of A phase and C phase, it can be seem that the partial capacitor of C phase to earth is smaller than the other two.
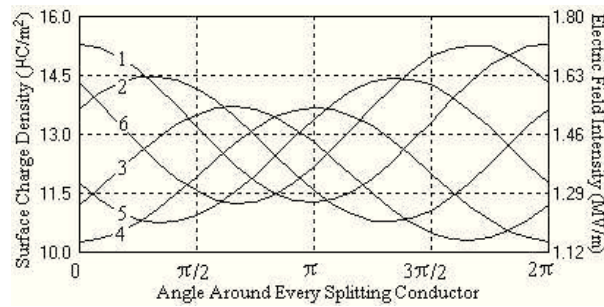
Figure 4: Surface charges density of the splitting conductors.
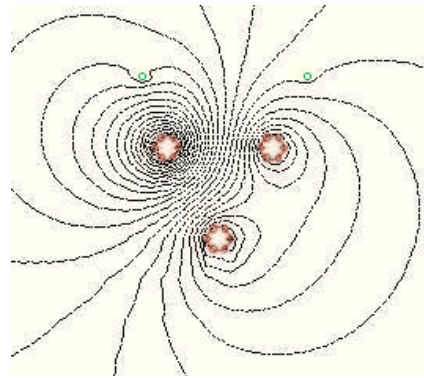

Figure 5: Equipotential line distribution.

5. CONCLUSION

Surface charges method base on strictly that the charges distribute continuously on the surface of every splitting conductor, and use linear interpolation to express the surface charges density of every arc segment. This new method is more adaptive to calculate the charges distribution on the surface of conductors.

The current concept of operating capacitors has some mistake theoretically, and partial capacitors are introduced to instead of them. Partial capacitors are more useful than operating capacitor in engineering application.

REFERENCES

1. Lee, B. Y. and J. K. Park, "An effective modelling method to analyze electric field around transmission Lines and substations using a generalized finite lines charge," *IEEE on PD*, Vol. 12, No. 3, 1143, 1997.
2. Florkowska, B., A. Jaekowicz-Korczynski, and M. Timler, "Analysis of electric field distribution around the high-voltage overhead transmission lines with an ADSS fiber-optic cable," *IEEE Tran. on PD*, Vol. 19, No. 3, 1183–1189, 2004.
3. Zhang, Q. C., J. J. Ruan, and J. H. Yu, "Mathematical models for electric field under high voltage overhead line," *High Voltage Engineering*, Vol. 26, No. 1, 19–21, 2000.