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Mathematical modeling of gas release through holes in pipelines

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Abstract

A mathematical model of accidental gas release in long transmission pipelines is given by using of computational fluid mechanics. It was found that the existing hole model is suitable for predicting gas release through a small punctured hole while the other existing pipe model is suitable to predict the gas release through a complete break in the pipe. In this paper, a new model was proposed for the hole that lies between the above two situations. The results of an example show that when the initial inside pipe pressure is higher than 1.5 MPa, the mass of gas released during sonic flow is more than 90% of the total mass of gas released. The total average release rate could be substituted by 30% of initial release rate. This approximation would become more accurate with increasing of the initial inside pipe pressure.

Keywords: Release rate; Sonic flow; Unsteady state; Compressible fluid; Modeling

1. Introduction

The pipeline has proven to be one of the easiest and safest ways to transport fluid fuels such as natural gas. However, it is often subjected to interference from third parties, corrosion, accidents and human error, etc. during operation. When it is damaged, the natural gas will be released through any break in the pipeline, resulting in a hazardous situation developing from possible explosion, fire, injury and damage. The safe operation of fuel pipelines is an important issue for operators worldwide. Therefore, it is necessary to evaluate the safety of pipelines and conduct quantitative risk assessment of their operation. Thus, pipeline managers can improve the overall safety of pipeline operation and decrease the risk.

The quantitative risk assessment of the pipelines consist of estimation of failure probability and failure consequences. In order to estimate the failure consequences, gas release rate through the damaged pipeline must be known in advance. There are two common models to calculate gas release rate [1–3]. One is a hole model [1], where gas releases through a small hole and the pipeline is considered as a tank; the other is a pipe model [1], where gas releases through a hole corresponding to a complete break in the cross-section of the pipeline. There is an obvious gap in knowledge between the above models as no suitable model exists for the hole size ranging from a relatively small hole up to a large orifice with a diameter approaching to that of the pipeline diameter. Although there have been a number of studies on modeling intermediate leaks of a perfect gas within the last years [4–6], there is still no clear understanding of flowing natural gas at high pressure and unsteady state release. This paper investigated the gas release rate at a variety of hole sizes as well as the average release rate for unsteady state of gas.

2. Release model

Flowing gas in the pipeline can be considered as a compressible fluid with significant changes of density. Therefore, the analysis of such systems involves several equations, such as the energy conservation equation, the momentum conservation equation and the continuity equation. In general, the flow of fluid can be described either as an isothermal process or as an adiabatic process. The isothermal process involves fluid flowing through long, uninsulated pipelines while the adiabatic process is appropriate to short, insulated lines. Most of real processes behave between these two extremes. However, in many cases, especially long pipelines, the two processes provide rather similar result [7].

According to computational fluid mechanics, flowing gas is considered as a reversible, adiabatic process of a perfect gas, complying with the state equation and Poisson equation. However, the state equation of perfect gas can only be used for real gas at high temperature or low pressure, when pressure is high, though, use of this equation will result in

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Nomenclature

a	sonic speed of gas ($\mathrm{m\,s^{-1}}$)
A_{or}	area of hole ($\mathrm{m^2}$)
A	area of cross-section of pipeline ($\mathrm{m^2}$)
C_0	empirical discharge coefficient, for subsonic of Reynold number larger than 30,000, $C_0 = 0.61$, for other situations with $C_0 = 1$
d	hole diameter (m)
D	pipeline diameter (m)
f	friction factor
ΣF	friction force ($= 2fu^2 dL/D$) (N)
G	mass flux ($\mathrm{kg\,m^{-2}\,s^{-1}}$)
h	enthalpy of gas ($\mathrm{kJ\,kg^{-1}}$)
L	length from regulation valve to the hole (km)
L_{e}	equivalent length of pipeline (km)
m	gas mass (kg)
M	molecular weight ($\mathrm{kg\,kmol^{-1}}$)
Ma	Mach number
k	heat capacity ratio (c_p/c_v)
P	pressure (Pa)
Q	release rate ($\mathrm{kg\,s^{-1}}$)
R	constant of gas ($\mathrm{Pa\,m^3\,mol^{-1}\,K^{-1}}$)
t	time (s)
T	temperature (K)
u	velocity of gas ($\mathrm{m\,s^{-1}}$)
V	volume of pipeline ($\mathrm{m^3}$)
Y	defined by Eq. (7)
Z	compressibility factor

Greek symbols

α	defined by Eq. (17)
ε	roughness of pipeline
μ	viscosity of gas (Pa s)
ρ	density of gas ($\mathrm{kg\,m^{-3}}$)

Subscripts

0	steady state
1	initial point
2	point inside the pipeline
3	release point
a	point in the atmosphere
av	average
cr	critical
p	pipeline
w	whole

large error. In engineering calculation, a compressibility factor Z is introduced into the state equation of perfect gas in order to decrease deviation from real gas [8]. Accordingly, the state equation of gas is given as

$$P = \frac{\rho ZRT}{M} \tag{1}$$

here Z is assumed to be constant over the pipe length.

A schematic diagram of gas release through a hole is shown in Fig. 1 [4], indicating a hole located in the distance of L from a regulation valve of pipeline, through which the release takes place. Various points are considered: point 1 near the regulation valve; point 2 inside the pipeline and on a level with the hole; point 3 at the release point; point in the atmosphere.

Several assumptions are made: (1) the gas flows adiabatically in the pipeline and isentropically at the release point; (2) the model of flow is considered as a one-dimension model.

In order to delineate adiabatic flow of the gas in the pipeline quantitatively, an equation can be obtained by using of energy and momentum balance equations

$$\frac{k+1}{k}\ln\left(\frac{P_1 T_2}{P_2 T_1}\right) + \frac{M}{RG^2}\left(\frac{P_2^2}{T_2} - \frac{P_1^2}{T_1}\right) + \frac{4fL_{\mathrm{e}}}{D} = 0 \tag{2}$$

where L_{e} is the equivalent length of the pipeline, and the friction factor f is a function of roughness of pipe (ε) and Reynolds number (Re).

The expression of gas release rate can be obtained by substituting Eq. (1), Poisson equation and continuity equation into Eq. (2)

$$Q = C_0 A_{\mathrm{or}} P_2 \sqrt{\frac{2M}{ZRT_2}\frac{k}{k-1}\left[\left(\frac{P_{\mathrm{a}}}{P_2}\right)^{2/k} - \left(\frac{P_{\mathrm{a}}}{P_2}\right)^{(k+1)/k}\right]} \tag{3}$$

The value of the release rate at the orifice depends on whether gas flow is sonic or subsonic. This will be decided by the critical pressure ratio (CPR),

$$\mathrm{CPR} = \frac{P_{\mathrm{a}}}{P_{2\mathrm{cr}}} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \tag{4}$$

where $P_{2\mathrm{cr}}$ is critical pressure of point 2. When $P_2 > P_{2\mathrm{cr}}$, the gas release is sonic flow at the orifice. The release rate is described by the following equation that can be got by substituting Eq. (4) into Eq. (3)

$$Q = A_{\mathrm{or}} P_2 \sqrt{\frac{M}{ZRT_2} k \left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} \tag{5}$$

When $P_2 < P_{2\mathrm{cr}}$, the gas release is subsonic, and the release rate is described by Eq. (3).

In what follows, several gas release models will be described.

2.1. Hole model

When the hole diameter is relatively small, the pipeline is considered as a tank. Gas release rate would be calculated by the hole model. Assumptions made are: pressure inside the pipeline will not be affected by gas release; gas expansion is isentropic. Therefore, the gas release rate is constant and equal to the initial maximum release rate.

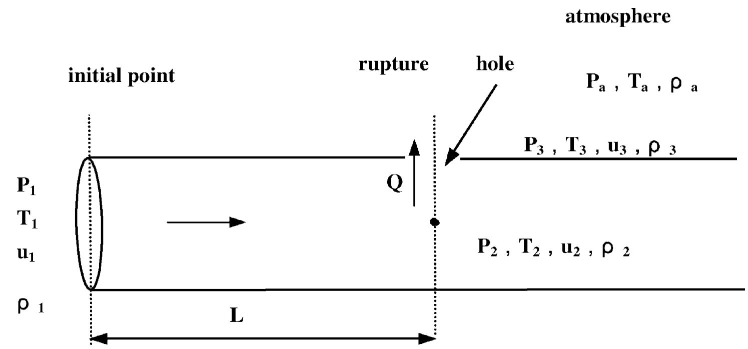

Fig. 1. Schematic diagram of gas release.

When the ratio of atmospheric pressure to pressure at point 2(Pa/P_2) is less than CPR, the gas release is sonic at the hole and the initial maximum release rate is presented by Eq. (5). Otherwise, when the ratio of atmospheric pressure to pressure at point 2(Pa/P_2) is larger than CPR, the initial maximum release rate is calculated by Eq. (3).

2.2. Pipe model

This model is suitable for a complete break of the pipeline. In this condition, the state of point 2 is similar to that of point 3, and isentropic expansion does not exist (therefore, $P_2 = P_3 = P_a$). The adiabatic flow is described by the mechanical energy conservation equation and the energy conservation equation that followed.

$$\begin{cases} u\,\mathrm{d}u + \dfrac{\mathrm{d}P}{\rho} + \sum F = 0 \\ \mathrm{d}h + u\,\mathrm{d}u = 0 \end{cases} \tag{6}$$

Assuming that friction factor is same along the pipeline, Octave [1] gave an analytical expression of Eq. (6), that is

$$\frac{T_2}{T_1} = \frac{Y_1}{Y_2}, \qquad \frac{P_2}{P_1} = \frac{Ma_1}{Ma_2}\sqrt{\frac{Y_1}{Y_2}}, \qquad \frac{\rho_2}{\rho_1} = \frac{Ma_1}{Ma_2}\sqrt{\frac{Y_2}{Y_1}},$$
$$Y_i = 1 + \frac{k-1}{2}Ma_i^2, \qquad Ma = \frac{u}{a}, \qquad a = \sqrt{\frac{kZRT}{M}} \tag{7}$$

$$G = Ma_1 P_1\sqrt{\frac{kM}{ZRT_1}} = Ma_2 P_2\sqrt{\frac{kM}{ZRT_2}} = \sqrt{\frac{2M}{ZR}\,\frac{k}{k-1}\,\frac{T_2 - T_1}{(T_1/P_1)^2 - (T_2/P_2)^2}} \tag{8}$$

Replacing P_1, P_2, T_1, T_2 and G with Eqs. (7) and (8), Eq. (2) is converted to

$$\frac{k+1}{2}\ln\left(\frac{Ma_2^2 Y_1}{Ma_1^2 Y_2}\right) - Z\left(\frac{1}{Ma_1^2} - \frac{1}{Ma_2^2}\right) + \frac{4kfL_e}{D} = 0 \tag{9}$$

When the pipeline is long enough, and pressure inside the pipeline and outside the pipeline changes greatly, gas release velocity at the outlet will approximate to sonic speed. In this condition, Ma_2 is equal to 1, causing the Eqs. (7), (8) and (9) to change accordingly.

2.3. Proposed model

Neither the hole model nor the pipe model would be suitable to be used when the orifice diameter is larger than a small hole diameter but smaller than the pipeline diameter. This condition can often exist in reality. There are several situations about the gas release from the pipeline. If gas flow is subsonic in the pipeline, gas release at the hole can either be sonic or subsonic, depending on the ratio of hole diameter to pipeline diameter.

2.3.1. Subsonic flow in the pipeline, sonic flow at the hole

When the following conditions are met

$$\begin{cases} P_2 > P_1 Ma_1\sqrt{\dfrac{2Y_1}{k+1}} \\ \dfrac{P_a}{P_2} < \mathrm{CPR} \end{cases} \tag{10}$$

gas flows subsonically in the pipeline and sonically at the hole. Therefore, gas release rate can be calculated by Eq. (5). According to the continuity equation of gas flow, mass flux of the gas flowing through any cross-section of the pipeline is equal. So

$$G = \frac{A_{\mathrm{or}}}{A}P_2\sqrt{\frac{kM}{ZRT_2}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} = Ma_1 P_1\sqrt{\frac{kM}{ZRT_1}} = Ma_2 P_2\sqrt{\frac{kM}{ZRT_2}} \tag{11}$$

The state equation of gas flow is the same as Eq. (9). The parameters relationship between point 1 and point 2 is referred to Eq. (7). And the parameters of point 3 are

$$P_3 = \left(\frac{2}{k+1}\right)^{k/(k-1)} P_2, = T_3 = \left(\frac{2}{k+1}\right) T_2,$$
$$\rho_3 = \left(\frac{2}{k+1}\right)^{1/(k-1)} \rho_2 \tag{12}$$

2.3.2. Subsonic flow both in the pipeline and at the hole

When the following conditions are met

$$\begin{cases} P_2 > P_1 Ma_1 \sqrt{\dfrac{2Y_1}{k+1}} \\ \dfrac{P_a}{P_2} \geq \text{CPR} \end{cases} \tag{13}$$

gas flows subsonically both in the pipeline and at the orifice. The release rate is expressed by Eq. (3). Mass flux through any cross-section of pipeline is

$$G = \frac{A_{or}}{A} P_2 \sqrt{\frac{M}{ZRT_2} \frac{2k}{k-1} \left[\left(\frac{P_a}{P_2}\right)^{2/k} - \left(\frac{P_a}{P_2}\right)^{(k+1)/k} \right]} = Ma_1 P_1 \sqrt{\frac{kM}{ZRT_1}} = Ma_2 P_2 \sqrt{\frac{kM}{ZRT_2}} \tag{14}$$

Eq. (9) is used to express the state equation of gas flow. Due to subsonic release of gas at the orifice, point 3 is at the same state as atmosphere ($P_3 = P_a$), then

$$T_3 = \left(\frac{P_a}{P_2}\right)^{(k-1)/k} T_2, \qquad \rho_3 = \left(\frac{P_a}{P_2}\right)^{1/k} \rho_2 \tag{15}$$

Newton iteration method is used to solve Eq. (9). Because of complexity, its implementation needs to be done on a computer.

3. Average release rate at unsteady state

The above models are suitable for estimation of the gas release rate at steady state, where the release rate is a constant. Actually, if hole diameter is small, gas release has little effect on the pressure inside the pipeline and release rate can be considered to be constant with time. However, when the hole diameter is large or even equal to the pipeline diameter, the upstream regulation valve will be closed automatically, so the pressure inside the pipeline will decrease gradually, and the release rate is a function of time.

$$Q(t) = -V_p \frac{d\rho(t)}{dt} \tag{16}$$

Even if the initial gas release at the hole is sonic before closing the valves, the release will gradually become subsonic because of the pressure decreasing in the pipeline. When the upstream valve is closed, the transition from sonic to subsonic will take place if the condition $P_a/P = \text{CPR}$ is met. The time from closing of the valves to the occurring of the transition is defined a critical time (t_{cr}).

For the state of sonic flow, the parameters at any period of unsteady state can be achieved by substituting Eq. (5) into Eq. (16)

$$\begin{aligned} &\frac{P(t)}{P_0} = [g(t)]^{2k/(k-1)}, \qquad \frac{T(t)}{T_0} = [g(t)]^2, \\ &\frac{\rho(t)}{\rho_0} = [g(t)]^{2/(k-1)}, \qquad Q(t) = Q_0[g(t)]^{(k+1)/(k-1)}, \\ &m(t) = m_0[1 - g(t)^{2/(k-1)}], \qquad g(t) = (1+\alpha t)^{-1}, \\ &\alpha = \frac{Q_0(k-1)}{2m_0} \end{aligned} \tag{17}$$

Thus, the critical time is

$$t_{cr} = \frac{1}{\alpha} \left[\frac{1}{[(k+1)/2]^{1/2} (P_a/P_0)^{(k-1)/2k}} - 1 \right] \tag{18}$$

As for the subsonic flow, a differential equation of pressure and time can be got by substituting Eq. (3) into Eq. (16)

$$\frac{dP}{dt} = -\frac{kQ_0}{m_0} P_0{}^{(3-k)/2k} \frac{[P^{(k-1)/k} - P_a^{(k-1)/k}]^{1/2} P^{(k-1)/k}}{[1 - (P_a/P_0)^{(k-1)/k}]^{1/2}} \tag{19}$$

This equation cannot be solved analytically. The fourth order Runge–Kutta method can be used to get pressure, temperature and density of gas at any moment of subsonic flow [9].

4. Example

In order to verify the validity of above models, an example is given. Suppose that a gas pipeline has an inter diameter of 0.66 m. Gas pressure at initial point of the pipeline (P_1) is 5 MPa, gas temperature (T_1) is 293 K, and molecular weight of the gas is 17.1 kg kmol^{-1}, viscosity of the gas (μ) is 1.01×10^{-5} Pa s, compressibility factor Z is 0.9. There is a hole at the equivalent distance of 126 km from the initial point. It is considered that gas release rate is always smaller than maximum release rate allowed in the pipeline. Therefore, the release rate at different holes can be calculated using the above relevant equations.

The results are shown in Fig. 2. It can be seen that when the hole is relatively small, the release rate calculated by the

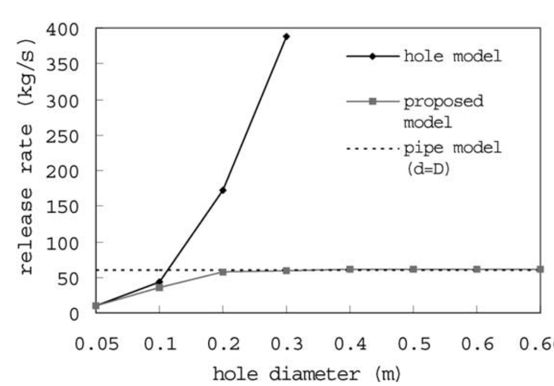

Fig. 2. Relationship between hole diameter and release rate.

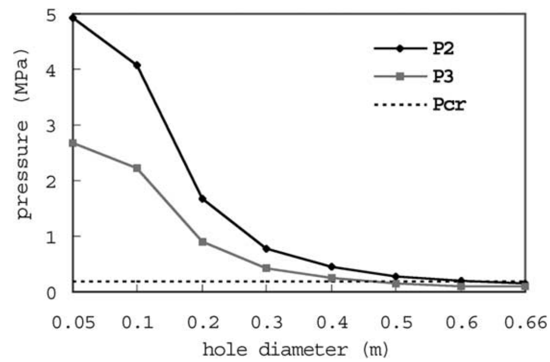

Fig. 3. Relationship between hole diameter and pressure of gas.

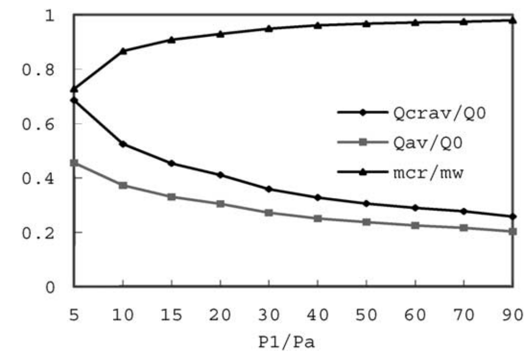

Fig. 5. Various ratios at different ratio of initial pressure to atmospheric pressure.

hole model is similar to that by the model proposed here. As the hole diameter increases, the hole model overestimates the release rate. When the hole diameter approximates to the pipeline diameter, the model proposed here gives the same result as that of the pipe model. In addition, the relationship between pressure and the temperature at point 2 and 3 and the hole diameter are shown in Figs. 3 and 4. It can be seen that the hole diameter of 0.62 m is a transition point between sonic and subsonic flow.

If it is assumed that the hole diameter is 0.1 m. It can be assumed that the gas flows subsonically in the pipeline and sonically at the orifice. Combining Eqs. (7), (9), (11), (12), (17), and (18), the critical time (t_{cr}), the critical average release rate ($Q_{cr\,av}$) and the mass of gas released (m_{cr}) at the sonic period, as well as the total mass of gas released (m_w) and average release rate (Q_{av}) of the total release process can be calculated.

At different ratios of initial pressure to atmospheric pressure (P_1/Pa), the ratio of $Q_{cr\,av}$, Q_{av} to the steady state release rate Q_0, i.e. $Q_{cr\,av}/Q_0$, Q_{av}/Q_0, as well as m_{cr}/m_w are given in Fig. 5.

From the Fig. 5, it can be seen that when the initial pressure (P_1) is larger than 1.5 MPa ($P_1/P_a > 15$), the m_{cr}/m_w is more than 90%. Therefore, it can be considered that gas release almost finishes as soon as sonic flow terminates. When the initial pressure (P_1) is less than 1.5 MPa ($P_1/P_a < 15$), the total release time and the average release rate Q_{av} must be evaluated. Moreover, it can be seen from the Fig. 5 that Q_{av}/Q_0 lies between 20 and 40% when the initial pressure is larger than 1.5 MPa. Therefore, Q_{av} can be approximately substituted by 30% of the Q_0.

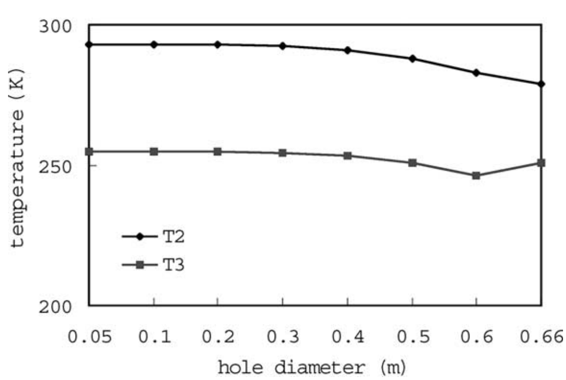

Fig. 4. Relationship between hole diameter and temperature of gas.

5. Conclusions

Several conclusions can be drawn from the analysis.

(1) When the hole is small, gas release rate can be predicted by the hole model; when the hole diameter approximates to the pipeline diameter, the pipe model can be used; while the hole lies between the above two situations, the model proposed in this paper can be used.
(2) When the initial pressure inside the pipeline is higher than 1.5 MPa ($P_1/P_a > 15$), the total average release rate (Q_{av}) can be substituted by the average release rate of the sonic period ($Q_{cr\,av}$). The Q_{av} can also be substituted by 30% of the release rate at steady state (Q_0).
(3) Gas release at a small hole can be considered as a steady state process.

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