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Stability criteria for large-scale time-delay systems: the LMI approach and the Genetic Algorithms

by

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Abstract: This paper addresses the asymptotic stability analysis problem for a class of linear large-scale systems with time delay in the state of each subsystem as well as in the interconnections. Based on the Lyapunov stability theory, a delay-dependent criterion for stability analysis of the systems is derived in terms of a linear matrix inequality (LMI). Finally, a numerical example is given to demonstrate the validity of the proposed result.

Keywords: delay-dependent criterion, large-scale systems, linear matrix inequality, genetic algorithms.

1. Introduction

Time delay naturally arises in the processing of information transmission between subsystems and its existence is frequently the main source of instability and oscillation in many important systems. Many practical control applications are encountered, involving the phenomena in question, including electrical power systems, nuclear reactors, chemical process control systems, transportation systems, computer communication, economic systems, etc. Hence, the stability analysis problem for time-delay systems has received considerable attention (Kim, 2001; Kolmanovskii and Myshkis, 1992; Kwon and Park, 2004; Li and De Souza, 1997; Niculescu, 2001; Park, 1999; Richard, 2003; Su et al., 2001). In the recent years, the problem of stability analysis for large-scale systems with or without time delay has been extensively studied by a number of authors. Moreover, depending on whether the stability criterion itself contains the delay argument as a parameter, stability criteria for systems can be usually classified into two categories, namely the delay-independent criteria (Hmamed, 1986; Huang et al., 1995; Lyou et al., 1984; Michel and Miller, 1977; Schoen and Geering, 1995; Siljak, 1978; Wang et al., 1991) and the delay-dependent criterion (Tsay et al., 1996). In general, the latter ones are less conservative than the former ones, but the former ones are also important when the effect of time delay is small. However, there are few papers to derive the delay-dependent and delay-independent stability criteria for a class of large-scale systems with time delays, to my knowledge. This has motivated my study.

In this paper, delay-independent and delay-dependent criteria for such systems can be derived to guarantee the asymptotic stability for large-scale systems with time-delays in the state of each subsystem as well as in the interconnections. Appropriate model transformation of original systems is useful for the stability analysis of such systems, and some tuning parameters, which satisfy the constraint on an LMI can be easily obtained by the GAs technique. A numerical example is given to illustrate that the proposed result is useful.

The notation used throughout this paper is as follows. We denote the set of all nonnegative real numbers by $\Re_+$, the set of all continuous functions from $[-H,\, 0]$ to $\Re^n$ by C_0, the transpose of matrix A (respectively, vector x) by A^T (respectively x^T), the symmetric positive (respectively, negative) definite by $A > 0$ (respectively $A < 0$). We denote identity matrix by I and the set $\{1, 2, ..., N\}$ by $\bar{N}$.

2. Main results

Consider the following large-scale time-delay systems, which is composed of N interconnected subsystems $S_i,\ i \in \bar{N}$. Each subsystem $S_i,\ i \in \bar{N}$ is described as

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{j=1}^{N} A_{ij} x_j(t - h_{ij}),\ t \geq 0, \tag{1a}$$

where $x_i(t)$ is the state of each subsystem $S_i,\ i \in \bar{N}$, then, $A_i,\ A_{ij},\ i,j \in \bar{N}$ are known constant matrices of appropriate dimensions, and $h_{ij},\ i,j \in \bar{N}$, are nonnegative time delays in the state of each subsystems as well as in the interconnections. The initial condition for each subsystem is given by

$$x_i(t) = \theta_i(t),\ t \in [-H, 0],\ H = \max_{i,j}\{h_{ij}\}, \qquad i,j \in \bar{N}, \tag{1b}$$

where $\theta_i(t),\ i \in \bar{N}$ is a continuous function on $[-H,\, 0]$.

A model transformation is constructed for the large-scale time-delay systems of the form:

$$\begin{aligned} \frac{d}{dt}\Big[x_i(t) + \sum_{j=1}^{N} B_{ij} \int_{t-h_{ij}}^{t} x_j(s)ds\Big] = A_i x_i(t) + \sum_{j=1}^{N} \Big[&B_{ij} x_j(t) \\ &+(A_{ij} - B_{ij}) x_j(t - h_{ij})\Big], \end{aligned} \tag{2}$$

where $B_{ij} \in \Re^{n_i \times n_j},\ i,j \in \bar{N}$ are some matrices that can be chosen by GAs such that the matrix $\bar{A}_i = A_i + B_{ij},\ i,j \in \bar{H}$ is Hurwitz.

Now, we present a delay-dependent criterion for asymptotic stability of system (1).

THEOREM 2.1 *The system (1) is asymptotically stable for any constant time delays h_{ij} satisfying $0 \leq h_{ij} \leq \bar{h}_{ij}$, $i, j \in \bar{N}$ under the condition that $\bar{A}_i$ are Hurwitz and there exist symmetric positive definite matrices P_i, R_{ij}, T_{ij}, V_{ij}, and W_{ijk}, $i, j, k \in \bar{N}$ such that the following LMI condition holds:*

$$\breve{\Xi} = \begin{bmatrix} \breve{\Xi}_{11} & \breve{\Xi}_{12} & \breve{\Xi}_{13} & \breve{\Xi}_{14} & \breve{\Xi}_{15} \\ \breve{\Xi}_{12}^T & -\breve{\Xi}_{22} & 0 & 0 & 0 \\ \breve{\Xi}_{13}^T & 0 & -\breve{\Xi}_{33} & 0 & 0 \\ \breve{\Xi}_{14}^T & 0 & 0 & -\breve{\Xi}_{44} & 0 \\ \breve{\Xi}_{15}^T & 0 & 0 & 0 & -\breve{\Xi}_{55} \end{bmatrix} < 0 \tag{3}$$

where

$$\breve{\Xi}_{11} = \begin{bmatrix} \breve{\Psi}_1 & B_{21}^T P_2 + P_1 B_{12} & \cdots & B_{N1}^T P_N + P_1 B_{1N} \\ P_2 B_{21} + B_{12}^T P_1 & \breve{\Psi}_2 & \cdots & B_{N2}^T P_N + P_2 B_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ P_M B_{N1} + B_{1N}^T P1 & P_N B_{N2} + B_{2N}^T P_2 & \cdots & \breve{\Psi}_N \end{bmatrix},$$

$$\breve{\Psi}_i = \bar{A}_i^T P_i + P_i \bar{A}_i + \sum_{j=1}^{N} \big[\bar{h}_{ji} \cdot R_{ji} + V_{ij} + \sum_{k=1}^{N} \bar{h}_{ki} \cdot (T_{kji} + W_{kji})\big],$$

$$\breve{\Xi}_{12} = diag(\Gamma_1, \cdots, \Gamma_N), \ \Gamma_i = [\sqrt{\bar{h}_{i1}} \cdot A_i^T P_i B_{i1} \cdots \sqrt{\bar{h}_{iN}} \cdot A_i^T P_i B_{iN}],$$

$$\breve{\Xi}_{22} = diag(\hat{\Gamma}_1, \cdots, \hat{\Gamma}_N), \ \hat{\Gamma}_i = diag(R_{i1}, \cdots, R_{iN}),$$

$$\breve{\Xi}_{13} = diag(\Pi_1, \cdots, \Pi_N), \ \Pi_i = [\Pi_{i1} \cdots \Pi_{iN}],$$

$$\Pi_{ki} = [\sqrt{\bar{h}_{i1}} \cdot B_{ik}^T P_i B_{i1} \cdots \sqrt{\bar{h}_{iN}} \cdot B_{ik}^T P_i B_{iN}],$$

$$\breve{\Xi}_{33} = diag(\hat{\Pi}_1, \cdots, \hat{\Pi}_N), \ \hat{\Pi}_i = diag(\hat{\Pi}_{i1} \cdots \hat{\Pi}_{iN}),$$

$$\hat{\Pi}_{ki} = diag(T_{ik1}, \cdots, T_{ikN}),$$

$$\breve{\Xi}_{14} = diag(\Lambda_1, \cdots, \Lambda_N), \ \Lambda_i = [(A_{1i} - B_{1i})^T P_1 \cdots (A_{Ni} - B_{Ni})^T P_N],$$

$$\breve{\Xi}_{44} = diag(\hat{\Lambda}_1, \cdots, \hat{\Lambda}_N), \ \hat{\Lambda}_i = diag(V_{1i}, \cdots, V_{Ni}),$$

$$\breve{\Xi}_{15} = diag(\Omega_1, \cdots, \Omega_N), \ \Omega_i = [\Omega_{i1} \cdots \Omega_{iN}],$$

$$\Omega_{ki} = [\sqrt{\bar{h}_{i1}} \cdot (A_{ik} - B_{ik})^T P_i B_{i1} \cdots \sqrt{\bar{h}_{iN}} \cdot (A_{ik} - B_{ik})^T P_i B_{iN}],$$

$$\breve{\Xi}_{55} = diag(\hat{\Omega}_1, \cdots, \hat{\Omega}_N), \ \hat{\Omega}_i = diag(\hat{\Omega}_{i1} \cdots \hat{\Omega}_{iN}),$$

$$\hat{\Omega}_{ki} = diag(W_{ik1}, \cdots, W_{ikN}).$$

Proof. From the Schur Complements of Boyd et al. (1994), the condition (3) is equivalent to the following inequality:

$$\Xi(\bar{h},\ P_i,\ R_{ij},\ T_{ijk}, V_{ij},\ W_{ijk}) =$$
$$\begin{bmatrix} \phi_1(\bar{h}) & B_{21}^T P_2 + P_1 B_{12} & \cdots & B_{N1}^T P_N + P_1 B_{1N} \\ P_2 B_{21} + B_{12}^T P_1 & \phi_2(\bar{h}) & \cdots & B_{N2}^T P_N + P_2 B_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ P_M B_{N1} + B_{1N}^T P_1 & P_N B_{N2} + B_{2N}^T P_2 & \cdots & \phi_N(\bar{h}) \end{bmatrix} < 0 \quad (4)$$

where $\bar{h}$ denotes $\{\bar{h}_{ij}, i, j \in \bar{N}\}$, and

$$\begin{aligned} \phi_i(\bar{h}) &= \bar{A}_i^T P_i + P_i \bar{A}_i \\ &+ \sum_{j=1}^{N} \big[\bar{h}_{ij} \cdot A_i^T P_i B_{ij} R_{ij}^{-1} B_{ij}^T P_i A_i + \bar{h}_{ji} \cdot R_{ji} \\ &+ (A_{ji} - B_{ji})^T P_j V_{ji}^{-1} P_j (A_{ji} - B_{ji}) + V_{ij}\big] \\ &+ \sum_{j=1}^{N} \sum_{k=1}^{N} \big[\bar{h}_{jk} \cdot B_{jk}^T P_j B_{jk} T_{jik}^{-1} B_{jk}^T P_j B_{ji} \\ &+ \bar{h}_{ki} \cdot T_{kji} + \bar{h}_{jk} \cdot (A_{ji} - B_{ji})^T P_j B_{jk} W_{jik}^{-1} B_{jk}^T P_j (A_{ji} - B_{ji}) + \bar{h}_{ki} \cdot W_{kji}\big]. \end{aligned}$$

The Lyapunov functional is given by

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t), \tag{5a}$$

where

$$\begin{aligned} V_1(x_t) &= \sum_{i=1}^{N} L_i(x_t)^T \cdot P \cdot L_i(x_t),\ L_i(x_t) \\ &= x_i(t) + \sum_{j=1}^{N} B_{ij} \int_{t-h_{ij}}^{t} x_j(s) ds,\ i \in \bar{N} \end{aligned} \tag{5b}$$

$$V_2(x_t) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_j^T(s)[(s - t + h_{ij}) \cdot R_{ij}$$
$$+ (A_{ij} - B_{ij})^T P_i (V_{ij}^{-1} + \sum_{k=1}^{N} h_{ik} \cdot B_{ik} W_{ijk}^{-1} B_{ik}^T) P_i (A_{ij} - B_{ij})] x_j(s) ds, \tag{5c}$$

$$V_3(x_t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \int_{t-h_{ik}}^{t} (s - t + h_{ik}) \cdot x_k^T(s)(T_{ijk} + W_{ijk}) x_k(s) ds, \tag{5d}$$

are the legitimate Lyapunov functional candidates. The time derivatives of $V_i(x_t)$, $i=\overline{3}$, along the trajectories of the system (2) are given by

$$
\begin{aligned}
\dot{V}_1(x_t) &= \sum_{i=1}^{N} x_i^T(t)(A_i^T P_i + P_i A_i)x_i(t) \\
&+ 2\sum_{i=1}^{N}\sum_{j=1}^{N} x_i^T(t)A_i^T P_i B_{ij}\int_{t-h_{ij}}^{t} x_j(s)ds + 2\sum_{i=1}^{N}\sum_{j=1}^{N} x_j^T(t)B_{ij}^T P_i x_i(t) \\
&+ 2\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} x_j^T(t)B_{ij}^T P_i B_{ik}\int_{t-h_{ik}}^{t} x_k(s)ds \\
&+ 2\sum_{i=1}^{N}\sum_{j=1}^{N} x_j^T(t-h_{ij})(A_{ij}-B_{ij})^T P_i x_i(t) \\
&+ 2\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} x_j^T(t-h_{ij})(A_{ij}-B_{ij})^T P_i B_{ik}\int_{t-h_{ij}}^{t} x_k(s)ds.
\end{aligned}
$$

It is known fact that for any $x, y \in \Re^n$ and $Z \in \Re^{n\times n} > 0$, the inequality $2x^T y \le x^T Z x + y^T Z^{-1} y$ being true, we have

$$
\begin{aligned}
&\dot{V}_1(x_t) \le \sum_{i=1}^{N} x_i^T(t)(A_i^T P_i + P_i A_i)x_i(t) \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\left[h_{ij}\cdot x_i^T(t)A_i^T P_i B_{ij}R_{ij}^{-1}B_{ij}^T P_i A_i x_i(t) + \int_{t-h_{ij}}^{t} x_i^T(s)R_{ij}x_j(s)ds\right] \\
&+\sum_{i=1}^{N}\left[x_i^T(t)(B_{ii}^T P_i + P_i B_{ii})x_i(t) + 2\sum_{j=1,j\neq i}^{N} x_j^T(t)B_{ij}^T P_i x_i(t)\right] \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left[h_{ik}\cdot x_j^T(t)B_{ij}^T P_i B_{ik}T_{ijk}^{-1}B_{ik}^T P_i B_{ij}x_j(t) + \int_{t-h_{ik}}^{t} x_k^T(s)T_{ijk}x_k(s)ds\right] \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}[x_j^T(t-h_{ij})(A_{ij}-B_{ij})^T P_i V_{ij}^{-1}P_i(A_{ij}-B_{ij})x_j(t-h_{ij}) + x_i^T(t)V_{ij}x_i(t)] \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left[h_{ik}\cdot x_j^T(t-h_{ij})(A_{ij}-B_{ij})^T P_i B_{ik}W_{ijk}^{-1}B_{ik}^T P_i(A_{ij}-B_{ij})x_j(t-h_{ij})\right. \\
&\left.+\int_{t-h_{ik}}^{t} x_k^T(s)W_{ijk}x_k(s)ds\right]
\end{aligned}
$$

$$\begin{aligned}
\dot{V}_2(x_t) = &\sum_{i=1}^{N}\sum_{j=1}^{N}\big\{x_j^T(t)\big[h_{ij}\cdot R_{ij} + (A_{ij}-B_{ij})^T \\
&\times\ P_i(V_{ij}^{-1} + \sum_{k=1}^{N} h_{ik}\cdot B_{ik}W_{ijk}^{-1}B_{ik}^T)P_i(A_{ij}-B_{ij})\big]x_j(t) \\
&- \int_{t-h_{ij}}^{t} x_j^T(s)R_{ij}x_j(s)ds - x_j^T(t-h_{ij})[(A_{ij}-B_{ij})^T \\
&\times\ P_i(V_{ij}^{-1} + \sum_{k=1}^{N} h_{ik}\cdot B_{ik}W_{ijk}^{-1}B_{ik}^T)P_i(A_{ij}-B_{ij})]x_j(t-h_{ij})\big\}, \\
\dot{V}_3(x_t) = &\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\big[h_{ik}\cdot x_k^T(t)(T_{ijk}+W_{ijk})x_k(t) \\
&- \int_{t-h_{jk}}^{t} x_k^T(s)(T_{ijk}+W_{ijk})x_k(s)ds\big].
\end{aligned}$$

Now, considering that

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{N} x_j^T(t)\big[h_{ij}\cdot R_{ij} + (A_{ij}-B_{ij})^T P_i V_{ij}^{-1} P_i(A_{ij}-B_{ij}) \\
&\qquad + \sum_{k=1}^{N} h_{ik}\cdot B_{ij}^T P_i B_{ik} T_{ijk}^{-1} B_{ik}^T P_i B_{ij} \\
&\qquad + h_{ik}\cdot (A_{ij}-B_{ij})^T P_i B_{ik} W_{ijk}^{-1} B_{ik}^T P_i (A_{ij}-B_{ij})]x_j(t) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N} x_j^T(t)[h_{ji}\cdot R_{ji} + (A_{ji}-B_{ji})^T P_j V_{ji}^{-1} P_j(A_{ji}-B_{ji}) \\
&\qquad + \sum_{k=1}^{N} h_{jk}\cdot B_{ji}^T P_j B_{jk} T_{jik}^{-1} B_{jk}^T P_j B_{ji} \\
&\qquad + h_{jk}\cdot (A_{ji}-B_{ji})^T P_j B_{jk} W_{jik}^{-1} B_{jk}^T P_j (A_{ji}-B_{ji})]x_i(t) \\
&\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} H_{ik}\cdot x_k^T(t)(T_{ijk}+W_{ijk})x_k(t) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} h_{ki}\cdot x_i^T(t)(T_{kji}+W_{kji})x_i(t).
\end{aligned}$$

Then, the derivative of $V(x_t)$ satisfies

$$\begin{aligned}\dot{V}(x_t) &= \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t)\\ &\leq \sum_{i=1}^{N}[x_i^T(t)\phi_i(h)x_i(t) + 2\sum_{j=1,j\neq i}^{N} x_j^T(t)B_{ij}^T P_i x_i(t)]\\ &\leq X^T(t)\Xi(\bar{h}, P_i, R_{ij}, T_{ijk}, V_{ij}, W_{ijk})X(t), \end{aligned} \tag{6}$$

where $X(t) = [x_1^T(t) \cdots x_N^T(t)]^T$.

Hence, by Theorem 9.8.1 of Hale and Verduyn Lunel (1993) with (4)-(6), we conclude that systems (1) and (2) are both asymptotically stable for any constant time delays h_{ij} satisfying $0 \leq h_{ij} \leq \bar{h}_{ij}$, $i,j \in \bar{N}$. ■

Remark 2.1 *Notice that for fixed $\bar{h}_{ij}$, and B_{ij}, $i,j \in \bar{N}$, the entries of $\breve{\Xi}$ in (3) are affine in P_i, R_{ij}, T_{ijk}, V_{ij}, and W_{ijk}, $i,j,k \in \bar{N}$, so that the asymptotic stability problem for the large-scale time-delay systems can be converted into a strictly feasible LMI problem.*

Remark 2.2 *Notice that for any chosen matrix $B_{ij} = 0$, $i,j \in \bar{N}$ it means that the delay terms $B_{ij}\int_{t-h_{ij}}^{t} x_j(s)ds$, $i,j \in \bar{N}$ have not been converted to the left side of the system (2). By the above Theorem 2.1, the corresponding matrices R_{ij}, T_{ijk}, U_{ijk}, and W_{ijk}, $i,j,k \in \bar{N}$ could be chosen as zero matrices, and the LMI condition in (3) is reduced by it's corresponding elements.*

Letting $B_{ij} = 0$, $i,j \in \bar{N}$ in Theorem 2.1, we achieve the following result that does not depend on delay arguments.

Corollary 2.1 *The system (1) is asymptotically stable with $h_{ij} \in \Re_+$, $i,j \in \bar{N}$, provided that A_i is Hurwitz, and there exist P_i, and V_{ij}, $i,j \in \bar{N}$, such that the following LMI condition holds:*

$$\begin{bmatrix} \hat{\Xi}_{11} & \hat{\Xi}_{12} \\ \hat{\Xi}_{11}^T & -\hat{\Xi}_{22} \end{bmatrix} < 0. \tag{7}$$

where

$$\hat{\Xi}_{11} = \begin{bmatrix} \breve{\Psi}_1 & 0 & \cdots & 0 \\ 0 & \breve{\Psi}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \breve{\Psi}_N \end{bmatrix}, \ \breve{\Psi}_i = A_i^T P_i + P_i A_i + \sum_{j=1}^{N} V_{ij},$$

$$\breve{\Psi}_{12} = diag(\Lambda_1, \cdots, \Lambda_N),\ \Lambda_i = [A_{1i}^T P_1 \cdots A_{Ni}^T P_N],$$

$$\breve{\Psi}_{22} = diag(\hat{\Lambda}_1, \cdots, \hat{\Lambda}_N),\ \hat{\Lambda}_i = diag(V_{1i}, \cdots, V_{Ni}).$$

REMARK 2.3 *Suppose that for any chosen matrix $B_{ij} = A_{ij}$, $i, j \in \bar{N}$ we have that the delay term $A_{ij} \int_{t-h_{ij}}^{t} x_j(s)ds$, $i, j \in \bar{N}$, have been converted to the left side of the system (2). By the proof of Theorem 2.1, the corresponding matrices V_{ij}, and W_{ijk}, $i, j, k \in \bar{N}$, could be chosen as zero matrices, and the LMI condition in (3) is reduced by its corresponding elements.*

Choosing $B_{ij} = A_{ij}$, $i, j \in \bar{N}$ in Theorem 2.1 we achieve the following result that depends on delay arguments.

COROLLARY 2.2 *The system (1) is asymptotically stable for any constant time delays h_{ij} satisfying $0 \leq h_{ij} \leq \bar{h}_{ij}$, $i, j \in \bar{N}$, provided that $\hat{A}_i = A_i + A_{ii}$ is Hurwitz and there exist symmetric positive definite matrices P_i, R_{ij}, and T_{ij}, $i, j \in \bar{N}$ satisfying the following LMI condition:*

$$\begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12} & \bar{\Xi}_{13} \\ \bar{\Xi}_{12}^T & -\bar{\Xi}_{22} & 0 \\ \bar{\Xi}_{13}^T & 0 & -\bar{\Xi}_{33} \end{bmatrix} < 0 \tag{8}$$

where

$$\bar{\Xi}_{11} = \begin{bmatrix} \bar{\Psi}_1 & A_{21}^T P_2 + P_1 A_{12} & \cdots & A_{N1}^T P_N + P_1 A_{1N} \\ P_2 A_{21} + A_{12}^T P_1 & \bar{\Psi}_2 & \cdots & A_{N2}^T P_N + P_2 A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ P_N A_{N1} + A_{1N}^T P_1 & P_N A_{N2} + A_{2N}^T P_2 & \cdots & \bar{\Psi}_N \end{bmatrix},$$

$$\bar{\Psi}_1 = \hat{A}_i^T P_i + P_i \hat{A}_i + \sum_{j=1}^{N} (\bar{h}_{ji} \cdot R_{ji} + \sum_{k=1}^{N} \bar{h}_{ki} \cdot T_{kji}),$$

$$\bar{\Xi}_{12} = diag(\Gamma_1, \cdots, \Gamma_N),\ \Gamma_i = [\sqrt{\bar{h}_{i1}} \cdot A_i^T P_i A_{i1} \cdots \sqrt{\bar{h}_{iN}} \cdot A_i^T P_i A_{iN}],$$

$$\bar{\Xi}_{22} = diag(\hat{\Gamma}_1, \cdots, \hat{\Gamma}_N),\ \hat{\Gamma}_i = diag(R_{i1}, \cdots, R_{iN}),$$

$$\bar{\Xi}_{13} = diag(\Pi_1, \cdots, \Pi_N),\ \Pi_i = [\Pi_{i1} \cdots \Pi_{iN}],$$

$$\Pi_{ki} = [\sqrt{\bar{h}_{i1}} \cdot A_{ik}^T P_i A_{i1} \cdots \sqrt{\bar{h}_{iN}} \cdot A_{ik}^T P_i A_{iN}],$$

$$\bar{\Xi}_{33} = diag(\hat{\Pi}_1, \cdots, \hat{\Pi}_N),\ \hat{\Pi}_i = diag(\hat{\Pi}_{i1} \cdots \hat{\Pi}_{iN}),$$

$$\hat{\Pi}_{ki} = diag(T_{ik1}, \cdots, T_{ikN}).$$

Now, we provide a procedure for testing the asymptotic stability of the system (1).

Step 1. Choosing all $B_{ij} = 0$, $i, j \in \bar{N}$ the delay-independent criterion in Corollary 2.1 is applied to test the stability of the system (1).

Step 2. If the condition in Corollary 2.1 cannot be satisfied, we choose all $B_{ij} = A_{ij}$, $i, j \in \bar{N}$ and the delay-dependent criterion in Corollary 2.2 is applied to test the stability of the system (1).

Step 3. By employing the method of GAs (Chen, 1998; Eshelman and Schaffer, 1993; Michalewicz, 1994; Gen and Cheng, 1997), the matrix parameters B_{ij}, $i, j \in \bar{N}$ that maximize the fitness function $\bar{h}_{ij}$, $i, j \in \bar{N}$ and Theorem 2.1 will be search in this algorithm. Continue this algorithm until finding optimal matrix parameters $B_{ij} = 0$, $i, j \in \bar{N}$ such that the system (1) and (2) are both asymptotically stable for any constant time-delays h_{ij} satisfying $0 \le h_{ij} \le \bar{h}_{ij}$, $i, j \in \bar{N}$.

3. Example

EXAMPLE 3.1 Large-scale systems with time delays:

Subsystem 1:

$$\dot{x}_1(t) = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} x_1(t) + \begin{bmatrix} -1 & -0.2 \\ 0 & -1 \end{bmatrix} x_1(t - h_{11}) + \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix} x_2(t - h_{12}),$$

Subsystem 2:

$$\dot{x}_2(t) = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} x_2(t) + \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix} x_1(t - h_{21}) + \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix} x_2(t - h_{22}). \quad (9)$$

Comparison of system (1) with system (9), we have $N = 2$. By Theorem 2.1 and GAs with $\bar{h}_{11} = 2.1143$, $B_{11} = \begin{bmatrix} -0.0645 & 0.1668 \\ -0.1179 & -0.1064 \end{bmatrix}$, and $B_{12} = B_{21} = B_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, the LMI (3) is satisfied with $P_1 = \begin{bmatrix} 460.4918 & 71.8576 \\ 71.8576 & 648.4622 \end{bmatrix}$, $P_{12} = \begin{bmatrix} 0.6084 & 0.0044 \\ 0.0044 & 0.3914 \end{bmatrix}$, $R_{11} = \begin{bmatrix} 389.6232 & 48.6947 \\ 48.6947 & 528.4440 \end{bmatrix}$, $V_{11} = \begin{bmatrix} 451.9861 & 112.1321 \\ 121.1321 & 598.9035 \end{bmatrix}$, $V_{21} = \begin{bmatrix} 0.4502 & 0.0170 \\ 0.0170 & 0.1432 \end{bmatrix}$, $V_{12} = \begin{bmatrix} 334.8959 & 52.2607 \\ 52.2607 & 15762 \end{bmatrix}$, $V_{22} = \begin{bmatrix} 1.7714 & -0.0122 \\ -0.0122 & 0.5554 \end{bmatrix}$, $T_{111} = \begin{bmatrix} 12.0867 & 2.2675 \\ 2.2675 & 17.6433 \end{bmatrix}$, $W_{111} = \begin{bmatrix} 76.8247 & 1.9405 \\ 1.9405 & 89.1764 \end{bmatrix}$, $W_{121} = \begin{bmatrix} 0.0175 & 0.0010 \\ 0.0010 & 0238 \end{bmatrix}$.

Hence, we conclude that system (9) is asymptotically stable for $0 \le h_{11} \le 2.1143$, and $h_{12}, h_{21}, h_{22} \in \Re_+$. The delay-independent criteria in Hmamed (1986), Huang et al. (1995), Lyou et al. (1984), Schoen and Geering (1995), Wang et al. (1991) cannot be satisfied. The delay-dependent stability criteria of Tsay et al. (1996) cannot be applied for sufficiently large time delays, $h_{12}, h_{21}, h_{22} \in \Re_+$.

4. Conclusions

In this paper, asymptotic stability analysis problem for a class of large-scale neutral time-delay systems has been considered. A model transformation and the Lyapunov stability theorem have been applied to obtain some stability criteria for such systems. Furthermore, the computational programming LMI with GAs has been used to improve the proposed results. Finally, it has been shown using a numerical example that the result shown in this paper is flexible and sharp.

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