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Observer-Based Adaptive Iterative Learning Control for Nonlinear Systems with Time-Varying Delays

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Abstract: An observer-based adaptive iterative learning control (AILC) scheme is developed for a class of nonlinear systems with unknown time-varying parameters and unknown time-varying delays. Linear matrix inequality (LMI) method is employed to design the nonlinear observer. The designed controller contains a proportional-integral-derivative (PID) feedback term in time domain. The learning law of unknown constant parameter is differential-difference-type, and the learning law of unknown time-varying parameter is difference-type. It is assumed that the unknown delay-dependent uncertainty is nonlinearly parameterized. By constructing a Lyapunov-Krasovskii-like composite energy function (CEF), we prove the boundedness of all closed-loop signals and the convergence of tracking error. A simulation example is provided to illustrate the effectiveness of control algorithm proposed in this paper.

Keywords: Adaptive iterative learning control, Nonlinearly parameterized systems, Time-varying delays, Lyapunov-Krasovskii-like composite energy function.

1 Introduction

Iterative learning control (ILC) and repetitive control (RC) have become two of the most effective control strategies in dealing with repeated tracking problem or periodic disturbance rejection problem for systems. Generally speaking, ILC deals with tracking tasks that repeat in a finite time interval and require identical initial condition (i.i.c.), whereas RC copes with periodic tracking tasks over an infinite time interval without requirement of i.i.c. Although there exist some differences between ILC and RC, they are identical in basic idea, i.e., using the information obtained from previous trial or period to improve the control for current trial or period.

In last two decades, both control strategies have made much progress [1−21,24−29]. The classical ILC and RC are usually designed based on contraction mapping theory [1−3,17], where the current control input is computed simply by adding the information of errors from preceding trial or period. Thus, on the one hand, it achieves geometric convergence speed with very little system knowledge, on the other hand, it is hard to make full use of any available system information, whether parametric or structural, which is different from adaptive control approaches. Recently, by combining adaptive control techniques into ILC and RC, a kind of so-called adaptive iterative learning control(AILC) [4−16,24−26] and the adaptive repetitive control (ARC) [18−20,27−28], have been developed, where the uncertain parameters are estimated by various ways to generate the current control input. For instance, the unknown parameters are estimated along time axis [8−9], or iteration axis [4−7,10−13], or both of them [14,22]. It should be mentioned that the composite energy function (CEF)-based AILC or ARC places an important role in dealing with periodic time-varying parameters. This control method is originally proposed in [13], and then successfully applied to periodic adaptive control [22,23], where the unknown time-varying parameters are periodic, but the reference signals are allowed to be nonperiodic.

Although so many approaches have been developed in the field of AILC and ARC, only a few of them are concerning time-delay systems [39−41], and they are investigated only within the framework of classical ILC. To the best of our knowledge, up to now no work is reported from the viewpoint of AILC or ARC to deal with nonlinear time-delay systems, especially in the case when the unknown time-varying parameters and the unknown time-varying delays appear in systems. In fact, delay often exists in various engineering systems, such as electrical networks, microwave oscillators, nuclear reactor etc. The existence of delay is a source of instability and poor performance [30]. Therefore, the controller design and stability analysis for time-delay systems, especially for uncertain nonlinear time-delay systems, have attracted a number of researchers over the past years, and some interesting results have been obtained (see [31-37, 44-45], and the references therein).

Motivated by the aforementioned discussion, instead of ARC problem for linearly parameterized systems addressed in [23], in this paper we will focus on the observer-based AILC problem for nonlinearly parameterized systems with unknown time-varying parameters and unknown time-varying delays. Our work is the further extension of the results obtained in [23]. The main design idea is partly inspired by [23], but this paper is different from [23] in the following aspects:

1). Form the viewpoint of system structure, the system studied in [23] is a special case of system addressed in this paper. For the first time, the additional mismatched nonlinear term and the matched uncertain time-varying time-delay term are considered in this paper.

2). As far as the observer design is concerned, due to the mismatched nonlinear term, we develop a nonlinear observer based on linear matrix inequality (LMI), which is

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different from [23], where the designed observer is linear.

3). As far as the control design is concerned, different from [23], our control law contains a proportional-integral-derivative (PID) feedback term, and the constant learning law is differential-difference-type, which is helpful to improve the control performance of closed-loop system.

4). Form the viewpoint of stability analysis, due to the uncertain time-delay terms, we construct a Lyapunov-Krasovskii-like CEF to analyze the closed-loop stability, which is also different from [23].

The rest of this paper is organized as follows. The system description and problem formulation are given in Section 2. In Section 3, the observer and controller are designed, and the stability of closed-loop system is analyzed. In Section 4, a simulation example is provided to verify the feasibility of control approach. Finally, we conclude the work of this paper in Section 5.

Throughout this paper, $\mathcal{N}$ denotes the set of nonnegative integers, $\|\cdot\|$ denotes Euclidean norm of a vector or its induced matrix norm, and $\|\cdot\|_s =: \max_{t\in[a,b]}\|\cdot\|$ represents the uniform norm over the interval $[a,b]$. For a finite positive constant T, $\mathcal{C}^p\{\mathcal{R}^{m\times n},[0,T]\}$ denotes a set of matrix-valued continuous functions ($p=0$) and continuously differentiable functions ($p=1$). For a 2-D signal $r_k(t), t\in[0,T], k\in\mathcal{N}$, we say that $r_k(t)$ converges to zero in $\mathcal{L}_T^2$ norm if $\lim_{k\to\infty}\int_0^T\|r_k(\sigma)\|^2d\sigma=0$, and that $r_k(t)$ is uniformly bounded if $\max_{(k,t)\in\mathcal{N}\times[0,T]}\|r_k(t)\|<\infty$.

2 Systems description and problem formulation

Consider a class of uncertain nonlinear systems with the delayed output:

$$\left\{\begin{array}{rcl}\dot{x} &=& Ax+f(x,t)+B\left[u+\Theta(t)\xi(x,t)\right.\\ && \left.+\eta\left(y(t-\tau(t)),\upsilon(t)\right)\right]\\ y &=& Cx\\ y(t) &=& \varpi(t), t\in[-\tau_{\max},0]\end{array}\right. \tag{1}$$

where $x\in R^n$ is the system state vector, $y\in R^m$ is the system output vector, and $u\in R^m$ is the system input vector; $f: R^{n+1}\to R^n$ and $\xi: R^{n+1}\to R^{n_1}$ are known vector-valued functions; $\Theta(t)\in\mathcal{C}^0(R^{m\times n_1},[0,T])$ and $\upsilon(t)\in\mathcal{C}^0(R^{1\times n_2},[0,T])$ represent the time-varying parametric uncertainties; $\eta: R^{m+1}\to R^m$ represents the time-varying and time-delay nonlinearly parameterized uncertainty; $\tau(t)\in\mathcal{C}^1(R^1,[-\tau_{\max},0])$ is the unknown time-varying delay of the output $y(t)$ with $\tau_{\max}\geq 0$ being an unknown constant; $\varpi(t)\in\mathcal{C}^0(R^m,[-\tau_{\max},0])$ denotes the initial function of system (1); A, B, and C are known constant matrices of appropriate dimensions with rank $(CB)=m$; Only the output $y(t)$ is physically accessible.

Remark 1. The adaptive learning control problem of system (1) become more difficult than that studied in [23] due to the known mismatched nonlinear function $f(x,t)$ and the uncertain matched time-varying, time-delay function $\eta(y(t-\tau(t)),\upsilon(t))$. The main feature of this paper is to deal with these nonlinear terms in designing observer and controller. It is obvious that system (1) can be considered as an extension of system in [23]. For example, if $f(x,t)=0$ and $\eta(y(t-\tau(t)),\upsilon(t))=0$, system (1) become the system considered in [23].

Throughout this paper, we make the following assumptions on system (1).

Assumption 1. The time-delay term $\eta(y(t-\tau),\upsilon(t))$ satisfies the following inequality:

$$\begin{array}{rl} & ||\eta(\varrho_1,\upsilon(t))-\eta(\varrho_2,\upsilon(t))||\\ \leq & ||\varrho_1-\varrho_2||\phi(\varrho_1,\varrho_2)\vartheta, \forall\varrho_1,\varrho_2\in R^m\end{array} \tag{2}$$

where $\phi(\varrho_1,\varrho_2)$ is a known continuous nonlinear function, and ϑ is an unknown constant.

Assumption 2. The time-varying delay $\tau(t)$ satisfies $\dot{\tau}(t)\leq\mu<1$, i.e., $-\frac{1-\dot{\tau}(t)}{1-\mu}<-1$.

Assumption 3. $f(x,t)$ and $\xi(x,t)$ are global Lipschitz continuous, i.e., $\forall\chi_1,\chi_2\in R^n$,

$$||f(\chi_1,t)-f(\chi_2,t)|| \leq ||\chi_1-\chi_2||\rho \tag{3}$$

$$||\xi(\chi_1,t)-\xi(\chi_2,t)|| \leq ||\chi_1-\chi_2||l \tag{4}$$

where ρ is a known Lipschitz constant, and l is an unknown Lipschitz constant.

Remark 2. The inequality (4) in Assumption 3 is from [23]. Assumption 1 and Assumption 2 will be used to construct a Lyapunov-Krasovskii-like CEF. The inequality (3) in Assumption 3 will be used to design the nonlinear observer.

Our AILC problem for system (1) is formulated as follows. For a given desired trajectory $y_r(t)\in C^1(R^m,[0,T])$, find an appropriate control input series $u_k(t)$, such that the system tracking error $e_k=y_k-y_r$ converges to zero in $\mathcal{L}_T^2$ norm.

To facilitate the subsequent derivations, two trace properties are given later [23]. For $A_1,A_2,A_4,W\in R^{m\times n_1}$, $A_3\in R^{m\times m}$, $\omega_1\in R^{n_1}$, and $\omega_2\in R^m$.

$$\begin{array}{rcl}P_1^0 &:& \mathrm{trace}[(A_1-A_2)^{\mathrm{T}}A_3(A_1-A_2)]\\ && -\mathrm{trace}[(A_1-A_4)^{\mathrm{T}}A_3(A_1-A_4)]\\ &=& 2\mathrm{trace}[(A_4-A_2)^{\mathrm{T}}A_3(A_1-A_2)]\\ && -\mathrm{trace}[(A_4-A_2)^{\mathrm{T}}A_3(A_4-A_2)]\\ P_2^0 &:& \mathrm{trace}(\omega_1\omega_2^{\mathrm{T}}W)=\omega_2^{\mathrm{T}}W\omega_1.\end{array}$$

The following lemma is used in this paper.

Lemma 1[43] (Schur compliment lemma). The LMI

$$S=\left[\begin{array}{cc}S_{11} & S_{12}\\ S_{21} & S_{22}\end{array}\right]<0$$

where $S_{11}=S_{11}^{\mathrm{T}}$ and $S_{22}=S_{22}^{\mathrm{T}}$ is equivalent to

$$S_{22}<0,\quad S_{11}-S_{12}S_{22}^{-1}S_{12}^{\mathrm{T}}<0.$$

3 Observer and controller design, stability analysis

3.1 Observer design

Since the system states cannot be obtained, the following nonlinear observer is used to estimate the system states

$$\left\{\begin{array}{rcl}\hat{x} &=& \nu-Dy\\ \dot{\nu} &=& (FA-LC)\nu+Ff(\hat{x},t)\\ && +\left[L(I_m+CD)-FAD\right]y\end{array}\right. \tag{5}$$

where $\hat{x}$ denotes the estimate of x; $\nu \in R^n$, $D = -B(CB)^{-1} \in R^{n\times m}$, $F = I_n + DC \in R^{n\times n}$; I_m and I_n are unit matrices of appropriate dimension; $L \in R^{n\times m}$ can be chosen to render the following inequality holds

$$(FA-LC)^{\mathrm{T}}P + P(FA-LC) + \iota^{-1}P^2 + \iota\rho^2||F||^2 I_n < -Q \quad (6)$$

where P, Q are positive definite matrices, ι is a positive constant.

By defining the state estimation error $\varepsilon = x - \hat{x}$, it can be easily derived from (1) and (5) that

$$\dot{\varepsilon} = (FA-LC)\varepsilon + F(f(x,t) - f(\hat{x},t)). \quad (7)$$

Remark 3. Different from [23], the observer (5) is nonlinear, which leads to the requirement that the matrix inequality (6) must be satisfied. In fact, based on Lemma 1, inequality (6) is equivalent to the following LMI

$$\begin{bmatrix} M & P \\ P & -\iota I_n \end{bmatrix} < 0$$

where $M = (FA)^{\mathrm{T}}P - C^{\mathrm{T}}W^{\mathrm{T}} + PFA - WC + \iota\rho^2\|F\|^2 I_n + Q$, and Q is a given positive definite matrix. Further, we can use LMI tool box in Matlab to obtain P, W and ι, and then we compute the observer gain matrix $L = P^{-1}W$.

3.2 Controller design

In order to design the control law, we rewrite the dynamic of the output tracking error $e_k = y_k - y_r$ at the k-th iteration as follows

$$\begin{aligned} \dot{e}_k &= C\left[Ax_k + f(x_k,t) + B(u_k + \Theta(t)\xi(x_k,t) + \eta(y_k(t-\tau(t)),\upsilon(t))\right] - \dot{y}_r \\ &= h(x_k,t) + CB\left[u_k + \beta(t)\Xi(x_k,t) + \Lambda_k\right] \\ &= CB\left[u_k + \beta(t)\Xi(\hat{x}_k,t)\right] + h(\hat{x}_k,t) + g_k \end{aligned} \quad (8)$$

where

$$\begin{aligned} h(x_k,t) &= CAx_k + Cf(x_k,t), \\ \beta(t) &= [\Theta(t) \mid \eta(y_r(t-\tau(t)),\upsilon(t)) - (CB)^{-1}\dot{y}_r(t)], \\ \Xi(x_k,t) &= \begin{bmatrix} \xi(x_k,t) \\ 1 \end{bmatrix}, \\ \Lambda_k &= \eta(y_k(t-\tau(t),\upsilon(t)) - \eta(y_r(t-\tau(t),\upsilon(t)), \\ g_k &= h(x_k,t) - h(\hat{x}_k,t) + CB\beta(t)(\Xi(x_k,t) - \Xi(\hat{x}_k,t)) + CB\Lambda_k. \end{aligned} \quad (9)$$

Based on (8), the control law is designed as

$$\begin{aligned} u_k &= -\hat{\beta}_k(t)\Xi(\hat{x}_k,t) + (CB)^{-1}\Big(-h(\hat{x}_k,t) - \hat{\psi}_k e_k - K_P e_k - K_I\int_0^t e_k(\sigma)d\sigma - K_D\dot{e}_k - \frac{\lambda}{4(1-\mu)}e_k\phi^2(y_k,y_r)\Big) \end{aligned} \quad (10)$$

$$\hat{\beta}_k(t) = \hat{\beta}_{k-1}(t) + \Gamma(CB)^{\mathrm{T}}e_k\Xi^{\mathrm{T}}(\hat{x}_k,t) \quad (11)$$

$$\begin{aligned} (1-\varsigma)\dot{\hat{\psi}}_k &= -\varsigma\hat{\psi}_k + \varsigma\hat{\psi}_{k-1} + \Upsilon||e_k||^2, \\ &\hat{\psi}_{-1}(t) = 0,\ \hat{\psi}_k(0) = \hat{\psi}_{k-1}(T) \end{aligned} \quad (12)$$

where $K_P, K_I, K_D \in R^{m\times m}$ are positive definite matrices with the minimum eigenvalue γ; λ is the minimum eigenvalue of Q; $\Gamma \in R^{m\times m}$ is a diagonal, positive gain matrix; $\hat{\beta}_k(t)$ is to approximate $\beta(t)$ and $\hat{\psi}_k$ is used to estimate an unknown constant $\psi = \frac{(||CA||+||C||\rho+||CB||\beta_m l)^2+||CB||^2\vartheta^2)}{\lambda}$ with $\beta_m =: \max_{t\in[0,T]}\beta(t)$; $0 < \varsigma < 1$ and $\Upsilon > 0$ are design parameters.

In the control law (10), the term $-\hat{\beta}_k(t)\Xi(\hat{x}_k,t)$ is used to encounter the term $\beta(t)\Xi(\hat{x}_k,t)$ in (8); the terms $-h(\hat{x}_k,t)$ and $-\hat{\psi}_k e_k$ are used to compensate for the term $h(\hat{x}_k,t)$ and g_k, respectively; the term $-\frac{\lambda}{4(1-\mu)}e_k\phi^2(y_k,y_r)\Big)$ is employed to compensate for the time-delay term Λ_k in g_k (see eq.(17) later).

Remark 4. Note that different from [23], our control law (10) contains a PID feedback term in time domain. It has been proved that the PID control is indeed superior to proportional(P)-type control law if the designer can suitably choose their coefficients. The reason can be explained as follows: The traditional P-type controller only provides an overall control action proportional to the error signal through the all-pass gain factor. The proportional term responds immediately to the current error, yet typically cannot achieve the desired accuracy without an unacceptably large gain. For plants with significant dead time, the effects of previous control actions are poorly represented in the current error. This situation may lead to large transient errors when the pure proportional controller is used. However, in PID controller, the integral term reduces steady-state errors through low-frequency compensation by an integrator. In general, the integral term yields zero steady-state error in tracking a reference signal, and also enables the complete rejection of constant disturbances. The derivative term improves the transient response through high-frequency compensation by a differentiator. So, in general, PID controller is superior to the P-type controller if the designer can suitably choose coefficients, which has been proved by the development of PID control. Moreover, how to tune PID coefficients is an open issue especially for nonlinear systems. Some existing methods for linear systems may be further considered for nonlinear systems, for example, adaptive method [5]. Based on the existing literature, a guideline is given as follows. If the external noises appear, the D coefficient is usually chosen small owing to the sensitiveness of derivatives on the noises; if the control task requires the fast convergence speed, then the designer should increase the I coefficient since I term can remove the static error and accelerate the convergence, which can be verified in simulation (see Fig. 2). Overall, the choice of PID coefficients is different for different control tasks and systems. How to determine PID coefficients is an issue studied in the further work.

Remark 5. The adaptive learning law (12) is a differential-difference learning law which is originally proposed in [24]. In general, the adaptive law (12) will become the pure differentiation-type learning law if $\varsigma = 0$, or pure difference-type learning law if $\varsigma = 1$. In general, the differential-difference-type learning law is better than the difference-type learning law. The reason is given as follows. The traditional difference-type learning law is used to update the estimate of the unknown parameter only in the iteration domain. That is, the adaptive law in the current trial is adjusted by the information from the past trials. However, the proposed differential-difference-type learning law updates the estimation value of unknown parameter using the information not only from the past trials but also from the current trial. So, the information used to update the estimation values is more exact and effective, which in general, makes the differential-difference-type learning law superior to the difference-type one. The other advantage of the differential-difference-type learning law is that the differential-difference-type learning law, in fact, contains a leakage term $\varsigma/(\varsigma-1)\hat{\psi}$ that helps eliminate the parameter drift in practical applications.

Then, substituting (10) into (8) leads to

$$\begin{aligned}\dot{e}_k &= -K_P e_k - K_I\int_0^t e_k(\sigma)d\sigma - K_D\dot{e}_k \\ &\quad +CB\tilde{\beta}_k(t)\Xi(\hat{x}_k,t) - \frac{\lambda\phi_k^2(y_k,y_r)}{4(1-\mu)}e_k \\ &\quad +g_k - \hat{\psi}_k e_k \qquad (13)\end{aligned}$$

where $\tilde{\beta}_k(t)$ denotes the estimation error of $\beta(t)$, i.e., $\tilde{\beta}_k(t) = \beta(t) - \hat{\beta}_k(t)$.

3.3 Convergence analysis

Theorem 1. Under Assumptions 1-3, if the identical initial condition is satisfied, i.e., $x_k(0) = \hat{x}_k(0)$ and $y_k(t) = y_r(t), t \in [-\tau_{\max}, 0]$, the control law (10), the learning law (11) and (12) ensure the state estimation error and the output tracking error converge to zeros in $\mathcal{L}_T^2$ norm, while keeping all closed-loop signals $y_k(t), x_k(t), \nu_k(t), \hat{x}_k(t), \hat{\psi}_k(t)$, $\int_0^t \text{trace}[\hat{\beta}^{\mathrm{T}}(\sigma)\hat{\beta}(\sigma)]d\sigma$ and $\int_0^t \|u_k(\sigma)\|^2 d\sigma$ are bounded.

Proof. First, consider a Lyapunov-Krasovskii functional candidate as follows

$$\begin{aligned}V_k(t) &= \varepsilon_k^{\mathrm{T}}P\varepsilon_k + \frac{1}{2}e_k^{\mathrm{T}}e_k + \frac{1-\varsigma}{2\Upsilon}\tilde{\psi}_k^2 + \frac{1}{2}e_k^{\mathrm{T}}K_D e_k \\ &\quad + \frac{\lambda}{4(1-\mu)}\int_{t-\tau(t)}^t ||e_k(\sigma)||^2\phi^2(y_k(\sigma),y_r(\sigma))d\sigma \\ &\quad + \frac{1}{2}\Big(\int_0^t e_k(\sigma)d\sigma\Big)^{\mathrm{T}}K_I\Big(\int_0^t e_k(\sigma)d\sigma\Big) \qquad (14)\end{aligned}$$

where $\tilde{\psi}_k = \psi_k - \hat{\psi}_k$. The differentiation of (14) is given by

$$\begin{aligned}\dot{V}_k(t) &= \varepsilon_k^{\mathrm{T}}P\dot{\varepsilon}_k + \dot{\varepsilon}_k^{\mathrm{T}}P\varepsilon_k + e_k^{\mathrm{T}}\dot{e}_k + \frac{1-\varsigma}{\Upsilon}\tilde{\psi}_k\dot{\tilde{\psi}}_k \\ &\quad + e_k^{\mathrm{T}}K_D\dot{e}_k + \frac{\lambda}{4(1-\mu)}||e_k||^2\phi^2(y_k,y_r) - \\ &\quad \frac{\lambda(1-\dot{\tau}(t))}{4(1-\mu)}||e_k(t-\tau)||^2\phi^2(y_k(t-\tau),y_r(t-\tau)) \\ &\quad + \frac{1}{2}\Big(\int_0^t e_k(\sigma)d\sigma\Big)^{\mathrm{T}}K_I e_k(t). \qquad (15)\end{aligned}$$

Substituting (7), (12) and (13) into (15) yields

$$\begin{aligned}\dot{V}_k(t) &= \varepsilon_k^{\mathrm{T}}[(FA-LC)^{\mathrm{T}}P + P(FA-LC)]\varepsilon_k \\ &\quad +2\varepsilon_k^{\mathrm{T}}PF(f(x_k,t)-f(\hat{x}_k,t)) \\ &\quad +e_k^{\mathrm{T}}[CB\tilde{\beta}_k(t)\Xi(\hat{x}_k,t) - K_P e_k - \hat{\psi}_k e_k + g_k] - \\ &\quad \frac{\lambda(1-\dot{\tau}(t))}{4(1-\mu)}||e_k(t-\tau)||^2\phi^2(y_k(t-\tau),y_r(t-\tau)) \\ &\quad -\frac{1}{\Upsilon}\tilde{\psi}_k[-\varsigma\hat{\psi}_k + \varsigma\hat{\psi}_{k-1} + \Upsilon||e_k||^2]. \qquad (16)\end{aligned}$$

Using inequality $2a^{\mathrm{T}}b \le \iota^{-1}a^{\mathrm{T}}a + \iota b^{\mathrm{T}}b$, $a, b \in R^q$ for arbitrary $\iota > 0$, and recalling inequality (3), we have

$$\begin{aligned}&2\varepsilon_k^{\mathrm{T}}PF(f(x_k,t)-f(\hat{x}_k,t)) \\ &\le \iota^{-1}\varepsilon_k^{\mathrm{T}}P^2\varepsilon_k + \iota\rho^2||F||^2\varepsilon_k^{\mathrm{T}}\varepsilon_k. \qquad (17)\end{aligned}$$

It is easily derived from (9) that

$$\begin{aligned}e_k^{\mathrm{T}}g_k &= e_k^{\mathrm{T}}\Big[h(x_k,t) - h(\hat{x}_k,t) + CB\beta(t)(\Xi(x_k,t) \\ &\quad -\Xi(\hat{x}_k,t)) + CB\Lambda_k\Big] \\ &\le \|e_k\|\Big[\|h(x_k,t)-h(\hat{x}_k,t)\| \\ &\quad +\|CB\|\|\beta(t)\|\|(\Xi(x_k,t)-\Xi(\hat{x}_k,t))\| \\ &\quad +\|CB\|\|\Lambda_k\|\Big]. \qquad (18)\end{aligned}$$

Using Young's inequality $\bar{a}\bar{b} \le c\bar{a}^2 + \frac{1}{4c}\bar{b}^2$ with $c > 0$. Set $c = \frac{1}{\lambda}$ and note (2)-(4). We further have

$$\begin{aligned}e_k^{\mathrm{T}}g_k &\le (||CA|| + ||C||\rho + ||CB||\beta_m l)\|e_k\|\|\varepsilon_k\| \\ &\quad +||CB||\vartheta\|e_k\|\|e_k(t-\tau)\|\phi(y_k(t-\tau),y_r(t-\tau)) \\ &\le \frac{(||CA|| + ||C||\rho + ||CB||\beta_m l)^2}{\lambda}||e_k||^2 \\ &\quad +\frac{\lambda}{4}||\varepsilon_k||^2 + \frac{\|CB\|^2\vartheta^2}{\lambda}\|e_k\|^2 \\ &\quad +\frac{\lambda}{4}\|e_k(t-\tau)\|^2\phi^2(y_k(t-\tau),y_r(t-\tau)) \\ &= \psi||e_k||^2 + \frac{\lambda}{4}||\varepsilon_k||^2 \\ &\quad +\frac{\lambda}{4}||e_k(t-\tau)||^2\phi^2(y_k(t-\tau),y_r(t-\tau)). \qquad (19)\end{aligned}$$

where $\beta_m =: \max_{t\in[0,T]}\beta(t)$. Substituting (17) and (19) back into (16), noting inequality (6) and Assumption 2, we have

$$\begin{aligned}\dot{V}_k(t) &\le -\varepsilon_k^{\mathrm{T}}Q\varepsilon_k + \frac{\lambda}{4}\varepsilon_k^{\mathrm{T}}\varepsilon_k + e_k^{\mathrm{T}}CB\tilde{\beta}_k(t)\Xi(\hat{x}_k,t) \\ &\quad -\gamma||e_k||^2 + \tilde{\psi}_k||e_k||^2 \\ &\quad -\frac{1}{\Upsilon}\tilde{\psi}_k[-\varsigma\hat{\psi}_k + \varsigma\hat{\psi}_{k-1} + \Upsilon||e_k||^2] \\ &= -\frac{3\lambda}{4}\varepsilon_k^{\mathrm{T}}\varepsilon_k + e_k^{\mathrm{T}}CB\tilde{\beta}_k(t)\Xi(\hat{x}_k,t) - \gamma||e_k||^2 \\ &\quad -\frac{\varsigma}{\Upsilon}\tilde{\psi}_k[\tilde{\psi}_k - \tilde{\psi}_{k-1}] \\ &\le -\frac{3\lambda}{4}\varepsilon_k^{\mathrm{T}}\varepsilon_k + e_k^{\mathrm{T}}CB\tilde{\beta}_k(t)\Xi(\hat{x}_k,t) - \gamma||e_k||^2 \\ &\quad -\frac{\varsigma}{2\Upsilon}[\tilde{\psi}_k^2 - \tilde{\psi}_{k-1}^2] \qquad (20)\end{aligned}$$

where γ is the minimum eigenvalue of K_P. Note that in the derivation of (20) we employ inequality $\tilde{\psi}_k\tilde{\psi}_{k-1} \le \frac{1}{2}\tilde{\psi}_k^2 + \frac{1}{2}\tilde{\psi}_{k-1}^2$. Then, we define the following Lyapunov-Krasovskii-like CEF

$$\begin{aligned}E_k(t) &= V_k(t) + \frac{1}{2}\int_0^t \text{trace}[\tilde{\beta}_k^{\mathrm{T}}(\sigma)\Gamma^{-1}\tilde{\beta}_k(\sigma)]d\sigma \\ &\quad +\frac{\varsigma}{2\Upsilon}\int_0^t \tilde{\psi}_k^2(\sigma)d\sigma \qquad (21)\end{aligned}$$

The proof consists of three parts: Part A derives the difference of the CEF; Part B proves the convergence of the tracking error; and Part C examines the boundedness property of the system.

Part A: Difference of CEF: For any $t \in [0,T]$, noting $V_k(t) = \int_0^t \dot{V}_k(\sigma)d\sigma + V_k(0)$, the difference of the CEF is

$$\begin{aligned}\Delta E_k(t) &=: E_k(t) - E_{k-1}(t)\\ &= \int_0^t \dot{V}_k(\sigma)d\sigma + V_k(0) - V_{k-1}(t)\\ &\quad+\frac{1}{2}\int_0^t \{\mathrm{trace}[\tilde{\beta}_k^{\mathrm{T}}(\sigma)\Gamma^{-1}\tilde{\beta}_k(\sigma)]\\ &\quad-\mathrm{trace}[\tilde{\beta}_{k-1}^{\mathrm{T}}(\sigma)\Gamma^{-1}\tilde{\beta}_{k-1}(\sigma)]\}d\sigma\\ &\quad+\frac{\varsigma}{2\Upsilon}\int_0^t [\tilde{\psi}_k^2(\sigma) - \tilde{\psi}_{k-1}^2(\sigma)]d\sigma. \end{aligned}\tag{22}$$

Using the trace properties P_1^0, P_2^0 and considering the learning law (11), we derive

$$\begin{aligned}&\frac{1}{2}\int_0^t \{\mathrm{trace}[\tilde{\beta}_k^{\mathrm{T}}(\sigma)\Gamma^{-1}\tilde{\beta}_k(\sigma)]\\ &-\mathrm{trace}[\tilde{\beta}_{k-1}^{\mathrm{T}}(\sigma)\Gamma^{-1}\tilde{\beta}_{k-1}(\sigma)]\}d\sigma\\ =& \int_0^t \{\mathrm{trace}[(\hat{\beta}_{k-1}(\sigma) - \hat{\beta}_k(\sigma))^{\mathrm{T}}\Gamma^{-1}\tilde{\beta}_k(\sigma)]\\ &-\mathrm{trace}[(\hat{\beta}_{k-1}(\sigma) - \hat{\beta}_k(\sigma))^{\mathrm{T}}\Gamma^{-1}(\hat{\beta}_{k-1}(\sigma) - \hat{\beta}_k(\sigma))]\}d\sigma\\ \le& -\int_0^t \mathrm{trace}\{\Xi(\hat{x}_k(\sigma),\sigma)[(CB)^{\mathrm{T}}e_k(\sigma)]^{\mathrm{T}}\tilde{\beta}_k(\sigma)\}d\sigma\\ =& -\int_0^t e_k^{\mathrm{T}}(\sigma)CB\tilde{\beta}_k(\sigma)\Xi(\hat{x}_k(\sigma),\sigma)d\sigma. \end{aligned}\tag{23}$$

Substituting (20) and (23) into (22) results in

$$\begin{aligned}\Delta E_k(t) &\le -\frac{3\lambda}{4}\int_0^t ||\varepsilon_k(\sigma)||^2 d\sigma - \gamma\int_0^t ||e_k(\sigma)||^2 d\sigma\\ &\quad+V_k(0) - V_{k-1}(t). \end{aligned}\tag{24}$$

Noting the identical initial condition $\varepsilon_k(0) = 0$ and $e_k(0) = 0, t \in [-\tau_{\max}, 0]$, we have $V_k(0) = \frac{1-\varsigma}{2\Upsilon}\tilde{\psi}_k^2(0) = \frac{1-\varsigma}{2\Upsilon}\tilde{\psi}_{k-1}^2(T) \le V_{k-1}(T)$. Let $t = T$ in (24), we further obtain

$$\begin{aligned}\Delta E_k(T) &\le -\frac{3\lambda}{4}\int_0^T ||\varepsilon_k(\sigma)||^2 d\sigma\\ &\quad-\gamma\int_0^T ||e_k(\sigma)||^2 d\sigma \le 0. \end{aligned}\tag{25}$$

Part B: Convergence of Analysis: Applying (25) repeatedly, we have

$$\begin{aligned}E_k(T) &= E_0(T) + \sum_{j=1}^k \Delta E_j(T)\\ &\le E_0(T) - \gamma\sum_{j=1}^k \int_0^T ||e_j(\sigma)||^2 d\sigma\\ &\quad-\frac{3\lambda}{4}\sum_{j=1}^k \int_0^T ||\varepsilon_j(\sigma)||^2 d\sigma. \end{aligned}\tag{26}$$

The previous relationship holds for any k, thus

$$\begin{aligned}\lim_{k\to\infty} E_k(T) &\le E_0(T) - \lim_{k\to\infty}\gamma\sum_{j=1}^k \int_0^T ||e_j(\sigma)||^2 d\sigma\\ &\quad-\lim_{k\to\infty}\frac{3\lambda}{4}\sum_{j=1}^k \int_0^T ||\varepsilon_j(\sigma)||^2 d\sigma. \end{aligned}\tag{27}$$

or equivalently

$$\begin{aligned}&\lim_{k\to\infty}\gamma\sum_{j=1}^k \int_0^T ||e_j(\sigma)||^2 d\sigma\\ &+\lim_{k\to\infty}\frac{3\lambda}{4}\sum_{j=1}^k \int_0^T ||\varepsilon_j(\sigma)||^2 d\sigma\\ \le\ & E_0(T) - \lim_{k\to\infty} E_k(T). \end{aligned}\tag{28}$$

As $E_k(T)$ is positive, if we can further prove that $E_0(T)$ is finite, then we conclude that the series $\sum_{j=1}^{\infty}\int_0^T ||e_j(\sigma)||^2 d\sigma$ and $\sum_{j=1}^{\infty}\int_0^T ||\varepsilon_j(\sigma)||^2 d\sigma$ will converge. According to the necessary condition of the convergence of the series, we have $\lim_{k\to\infty}\int_0^T ||e_k(\sigma)||^2 d\sigma = 0$ and $\lim_{k\to\infty}\int_0^T ||\varepsilon_k(\sigma)||^2 d\sigma = 0$. Therefore, as k approaches infinity, $\hat{x}_k$ converges to x and y_k converges to $y_r(t)$ asymptotically in L_T^2 norm.

Now, Let us check the finiteness property of $E_0(T)$. For any $t \in [0,T]$, from (21) (let $k = 0$), the derivative of $E_0(t)$ is

$$\dot{E}_0(t) = \dot{V}_0(t) + \frac{1}{2}\mathrm{trace}(\tilde{\beta}_0^{\mathrm{T}}\Gamma^{-1}\tilde{\beta}_0) + \frac{\varsigma}{2\Upsilon}\tilde{\psi}_0^2. \tag{29}$$

Note that in (29), the first term on the right-hand side has been derived from (20) as follows

$$\begin{aligned}\dot{V}_0(t) &\le -\frac{3\lambda}{4}\varepsilon_0^{\mathrm{T}}\varepsilon_0 + e_0^{\mathrm{T}}CB\tilde{\beta}_0(t)\Xi(\hat{x}_0,t) - \gamma||e_0||^2\\ &\quad-\frac{\varsigma}{2\Upsilon}\tilde{\psi}_0^2 + \frac{\varsigma}{2\Upsilon}\psi^2. \end{aligned}\tag{30}$$

For the second term on the right-hand side of (29), by substituting the learning law (11) and using the trace property P_2^0, we have

$$\begin{aligned}&\frac{1}{2}\mathrm{trace}(\tilde{\beta}_0^{\mathrm{T}}\Gamma^{-1}\tilde{\beta}_0)\\ =& \frac{1}{2}\mathrm{trace}(\beta^{\mathrm{T}}\Gamma^{-1}\beta) - \mathrm{trace}(\hat{\beta}_0^{\mathrm{T}}\Gamma^{-1}\tilde{\beta}_0)\\ &-\frac{1}{2}\mathrm{trace}(\hat{\beta}_0^{\mathrm{T}}\Gamma^{-1}\hat{\beta}_0)\\ \le& \frac{1}{2}\mathrm{trace}(\beta^{\mathrm{T}}\Gamma^{-1}\beta) - e_0^{\mathrm{T}}CB\tilde{\beta}_0(t)\Xi(\hat{x}_0,t). \end{aligned}\tag{31}$$

Substituting (30) and (31) into (29) yields

$$\begin{aligned}\dot{E}_0(t) &\le -\frac{3\lambda}{4}||\varepsilon_0||^2 - \gamma||e_0||^2 + \frac{1}{2}\mathrm{trace}(\beta^{\mathrm{T}}\Gamma^{-1}\beta)\\ &\quad+\frac{\varsigma}{2\Upsilon}\psi^2\\ &\le \frac{1}{2}\mathrm{trace}(\beta^{\mathrm{T}}\Gamma^{-1}\beta) + \frac{\varsigma}{2\Upsilon}\psi^2. \end{aligned}\tag{32}$$

The boundedness of β and ψ leads to the boundedness of $\dot{E}_0(t)$, which implies that $\forall t \in [0,T]$, $|E_0(T)| \le |E_0(0)| + \int_0^T |\dot{E}_0(\sigma)|d\sigma$ is bounded.

Part C: Boundedness Property: Up to now, we only prove the boundedness of $E_k(T)$ for $k \in \mathcal{N}$. Now, let us check the boundedness of $E_k(t)$ for any $t \in [0, T]$ and $k \in \mathcal{N}$. According to the definition of $E_k(t)$ and the finiteness of $E_k(T)$, the boundedness of $\int_0^T \text{trace}(\tilde{\beta}_k^{\rm T}(\sigma)\Gamma^{-1}\tilde{\beta}_k(\sigma))d\sigma$, $\int_0^T \tilde{\psi}_k^2(\sigma)d\sigma$ and $\tilde{\psi}_k^2(T)$ are guaranteed for all iterations. Therefore, $\forall k \in \mathcal{N}$ and $t \in [0, T]$, there exist finite constants M_1 and M_2 satisfying

$$\begin{aligned} & \frac{1}{2}\int_0^t \text{trace}[\tilde{\beta}_k^{\rm T}(\sigma)\Gamma^{-1}\tilde{\beta}_k(\sigma)]d\sigma + \frac{\varsigma}{2\Upsilon}\int_0^t \tilde{\psi}_k^2(\sigma)d\sigma \\ \leq\ & \frac{1}{2}\int_0^T \text{trace}[\tilde{\beta}_k^{\rm T}(\sigma)\Gamma^{-1}\tilde{\beta}_k(\sigma)]d\sigma + \frac{\varsigma}{2\Upsilon}\int_0^T \tilde{\psi}_k^2(\sigma)d\sigma \\ \leq\ & M_1 \end{aligned} \tag{33}$$

and

$$V_{k+1}(0) = \frac{1-\varsigma}{2\Upsilon}\tilde{\psi}_{k+1}^2(0) = \frac{1-\varsigma}{2\Upsilon}\tilde{\psi}_k^2(T) \leq M_2. \tag{34}$$

Then, from (21)and (33), we have

$$E_k(t) = V_k(t) + M_1. \tag{35}$$

On the other hand, from (25) and (34), we have

$$\begin{aligned} & \Delta E_{k+1}(t) \\ \leq\ & -\frac{3\lambda}{4}\int_0^t ||\varepsilon_{k+1}(\sigma)||^2 d\sigma - \gamma\int_0^t ||e_{k+1}(\sigma)||^2 d\sigma \\ & +V_{k+1}(0) - V_k(t) \\ \leq\ & M_2 - V_k(t). \end{aligned} \tag{36}$$

Combining (35) and (36) yields

$$E_{k+1}(t) \leq M_1 + M_2. \tag{37}$$

As we have shown $E_0(t)$ is bounded, for $\forall k \in \mathcal{N}, t \in [0, T]$, $E_k(t)$ is uniformly bounded, hence $\varepsilon_k(t), e_k(t), \tilde{\psi}_k(t)$ and $\int_0^t \text{trace}(\tilde{\beta}_k^{\rm T}(\sigma)\tilde{\beta}_k(\sigma))d\sigma$ are uniformly bounded, which further implies the uniform boundedness of $y_k(t), \hat{\psi}_k(t)$ and $\int_0^t \text{trace}(\hat{\beta}_k^{\rm T}(\sigma)\hat{\beta}_k(\sigma))d\sigma$. In the sequel, consider the Lyapunov function $W_k = \nu_k^{\rm T} P \nu_k$, which derivative is given by

$$\begin{aligned} \dot{W}_k &= \nu_k^{\rm T}[P(FA - LC) + (FA - LC)^{\rm T} P]\nu_k \\ &\quad +2\nu_k^{\rm T} PF[f(\nu_k - Dy_k, t) - f(-Dy_k, t)] \\ &\quad +2\nu_k^{\rm T} P\Delta(y_k) \\ &\leq -\nu_k^{\rm T} Q\nu_k + \frac{\lambda}{2}\nu_k^{\rm T}\nu_k + \frac{2}{\lambda}||P||^2||\Delta(y_k)||^2 \\ &= -\frac{\lambda}{2}\nu_k^{\rm T}\nu_k + \frac{2}{\lambda}||P||^2||\Delta(y_k)||^2 \end{aligned} \tag{38}$$

where $\Delta(y_k) = Ff(-Dy_k) + [L(I_m + CD) - FAD]\, y_k$ is a continuous vector-valued function. According to (38), the uniformly bounded $y_k(t)$ ensures the uniform boundedness of $\nu_k(t)$, in the sequel the uniform boundedness of $\hat{x}_k(t)$, then $x_k(t) = \hat{x}_k(t) + \varepsilon_k(t)$ is also uniformly bounded. Finally, from (13), we have

$$\begin{aligned} \dot{e}_k &= (I + K_D)^{-1}\Big[- K_P e_k - K_I\int_0^t e_k(\sigma)d\sigma \\ &\quad +CB\tilde{\beta}_k(t)\Xi(\hat{x}_k, t) - \frac{\lambda\phi^2(y_k, y_r)}{4(1-\eta)}e_k \\ &\quad +g_k - \hat{\psi}_k e_k\Big]. \end{aligned} \tag{39}$$

The uniform boundedness of $e_k(t)$, $x_k(t), \hat{x}_k(t)$, $\hat{\psi}_k(t), y_k(t), \int_0^t \text{trace}[\hat{\beta}_k^{\rm T}(\sigma)\hat{\beta}_k(\sigma)]d\sigma$ and the boundednsess of $y_r(t), \dot{y}_r(t)$ further imply that $\dot{e}_k(t)$ is uniformly bounded. Therefore, according to the control law (10), $\int_0^t \|u_k(\sigma)\|^2 d\sigma$ is uniformly bounded.

Remark 6. Compared with [23], due to the nonlinear term $f(\hat{x}, t)$ in (5), it is more difficult to obtain the uniform boundedness of ν. Therefore, we must make additional efforts to establish the inequality (38) to ensure the uniform boundedness of ν. Similarly, due to appearance of PID-type feedback term in the control law (10), we also have to establish the Eq.(39) to ensure the uniform boundedness of $\dot{e}_k(t)$.

4 Simulation study

In this section, we provide a simulation example to illustrate the control approach proposed in this paper. Consider the following system at the k-th iteration

$$\begin{aligned} \dot{x}_k &= Ax_k + f(x_k) \\ &\quad +B\{u_k + \Theta(t)\xi(x_k) + \eta(y_k(t - \tau(t)), t)\} \\ y_k &= Cx_k \end{aligned} \tag{40}$$

with

$$\begin{aligned} & x_k = \begin{bmatrix} x_{k,1} \\ x_{k,2} \end{bmatrix}, A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}, \\ & f(x_k) = \begin{bmatrix} \frac{\sqrt{2}}{2}\sin x_{k,1} \\ \frac{\sqrt{2}}{2}\cos x_{k,2} \end{bmatrix}, \xi(x_k) = \begin{bmatrix} e^{-x_{k,1}^2} \\ x_{k,2} \end{bmatrix}, \\ & \Theta(t) = [|\sin t| \quad \cos^3(t)], \\ & \eta(y_k(t - \tau(t)), t) = \sin^2(t) y_k^2(t - \tau(t)), \\ & \tau(t) = 2 + 0.5\sin t, \\ & B = [1 \quad 1]^{\rm T}, C = [1 \quad 1]. \end{aligned}$$

The control objective is to make the system output $y_k(t)$ track the desired trajectory $y_r(t) = \sin(t) + \sin(2t)$ for $t \in [0, 2\pi]$. It is easy to verify that Assumptions 1-3 are satisfied, and $\rho = 1$, $l = \sqrt{2}$, $\vartheta = 1$ and $\phi(y_k, y_r) = |y_k + y_r|$. Based on the LMI in Remark 3 with $Q = I$, by using the LMI tool box in Matlab we can obtain $L = [2.0289; 1.0289]$. Then, according to (5), the observer is given as

$$\left\{ \begin{aligned} \hat{x}_k &= \nu_k - \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix} y_k \\ \dot{\nu}_k &= \begin{bmatrix} -4.0289 & 0.9711 \\ 0.9711 & -4.0289 \end{bmatrix}\nu_k + \\ &\quad \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}\begin{bmatrix} \frac{\sqrt{2}}{2}\sin\hat{x}_{k,1} \\ \frac{\sqrt{2}}{2}\cos\hat{x}_{k,2} \end{bmatrix} + \\ &\quad \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} y_k. \end{aligned} \right. \tag{41}$$

Based on (10), the control law is designed as

$$\begin{aligned} u_k &= \frac{1}{2}\Big(- h(\hat{x}_k) - \hat{\psi}_k e_k - K_P e_k - K_I\int_0^t e_k(\sigma)d\sigma \\ &\quad -K_D\dot{e}_k - \frac{1}{2}e_k(y_k + y_r)^2\Big) - \hat{\beta}_k(t)\Xi(\hat{x}_k) \end{aligned} \tag{42}$$

with $e_k = y_k - y_r$, and

$$\Xi(\hat{x}_k) = \begin{bmatrix} e^{-\hat{x}^2_{k,1}} \\ \hat{x}_{k,2} \\ 1 \end{bmatrix},$$

$$h(\hat{x}_k) = [1\ 1]\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} \hat{x}_{k,1} \\ \hat{x}_{k,2} \end{bmatrix} + [1\ 1]\begin{bmatrix} \frac{\sqrt{2}}{2}\sin\hat{x}_{k,1} \\ \frac{\sqrt{2}}{2}\cos\hat{x}_{k,2} \end{bmatrix}.$$

The parameter learning laws are designed as follows

$$\hat{\beta}_k(t) = \hat{\beta}_{k-1}(t) + \Gamma 2 e_k \Xi^{\mathrm{T}}(\hat{x}_k), \hat{\beta}_{-1}(t) = 0 \quad (43)$$

$$(1-\varsigma)\dot{\hat{\psi}}_k = -\varsigma\hat{\psi}_k + \varsigma\hat{\psi}_{k-1} + \Upsilon||e_k||^2,$$
$$\hat{\psi}_{-1}(t) = 0, \hat{\psi}_k(0) = \hat{\psi}_{k-1}(T). \quad (44)$$

In simulation, the control parameters are set to be $K_P = 0.5, K_I = 0.5$ and $K_D = 0.5$, the design parameters are specified as $\varsigma = 0.5$, $\Upsilon = 0.5$, $\Gamma = 0.25$. All initial conditions are set to be zeros. The simulation results are shown in Fig. 1.

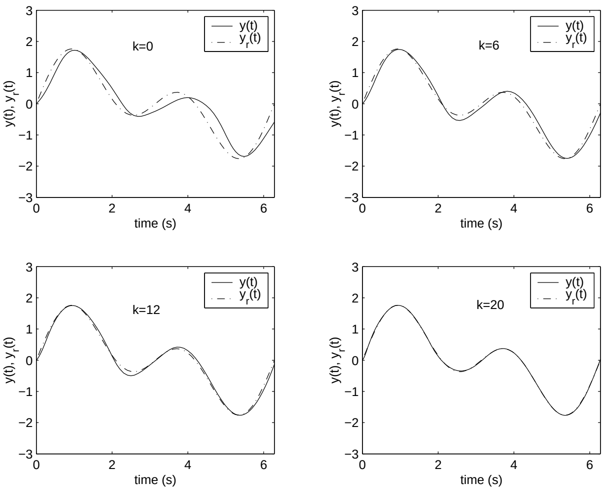

Fig. 1 Simulation results for system (37)

From Fig. 1, it can be seen that as the iteration number k increases, the tracking error $\|e(t)\|_{L^2_T}$ can converge to zero. When $k = 20$, the system output $y(t)$ can match the reference output $y_r(t)$ very well, which further verifies the effectiveness of the control scheme proposed in this paper.

To show the effect of PID coefficients on the control performance, we give the simulation curves of $|e(t)|_{L^2_T}$ for four cases which are shown in Fig. 2. It can be seen from Fig. 2 that compared with P-type controller, PI-type controller can obtain the fast convergence, but increase the initial tracking error; PD-type controller can decrease the initial tracking error, but converge slowly; PID-type controller can obtain a good balance between the initial tracking error and the convergence speed. So, in the applications, the designer should choose the suitable PID coefficients in order to obtain the satisfactory control performance.

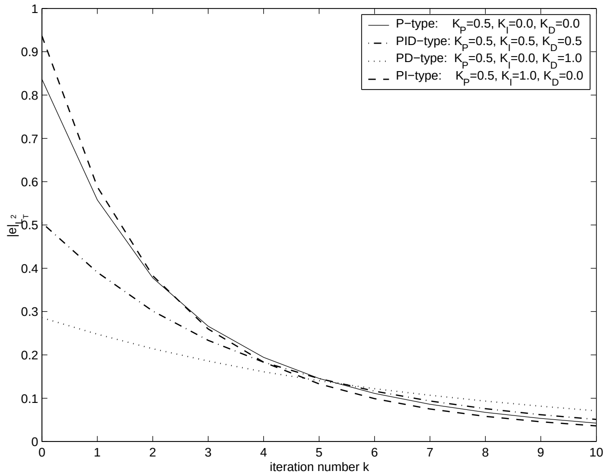

Fig. 2 Simulation comparison of $|e(t)|_{L^2_T}$

5 Conclusions

In this paper, we extend the CEF-based AILC approach to the nonlinear time-varying time-delay systems. We develop a novel observer-based AILC scheme to deal with nonlinearly parameterized uncertainties depending on unknown time-varying parameters and unknown time-varying delays. For simplicity, we only focus on AILC in this paper, but the proposed idea is also applied to solve ARC problem without any difficulty.

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