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Delay-dependent stability analysis for uncertain neutral systems with time-varying delays

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Abstract

This paper investigates the stability of neutral delay-differential systems with mixed multiple time-varying delay arguments. Based on the Lyapunov functional method, and the relationship between the system states and the derivatives of these states, we present a new asymptotical stability criterion and a new robust stability criterion in terms of only one simple linear matrix inequality (LMI), which guarantees stability for such systems with time-varying delays. This LMI can be easily solved by various convex optimization algorithms. Two examples are given to illustrate the advantages of the proposed methods over the existing ones.

Keywords: Neutral systems; Multiple time-varying delays; Linear matrix inequality; Lyapunov method

1. Introduction

Dynamical systems with time delays have been of considerable interest for decades, and in particular stability analysis of various neutral delay-differential systems has received much attention. A number of delay-independent sufficient conditions for the asymptotic stability of neutral delay differential systems have been presented by various researchers [1,6,10,11,14,19]. Also, rather fewer delay-dependent sufficient conditions have been shown in [8,9,20]. However, in each case above, the time delays considered are constant. Han's stability criterion in [7] is neutral delay-independent, and is applicable only to systems with a time-varying state delay argument and invariant neutral delay. Park [18] gave a new delay-dependent stability criterion for neutral differential systems with mixed multiple time-varying delay arguments, but it is very conservative.

In this paper, we investigate the stability of neutral delay differential systems with multiple time-varying delay arguments. In order to establish a new delay-dependent criterion for asymptotic stability and robust stability of the systems, using the Lyapunov method, various slack matrices are introduced to express the relationship between system states and the derivatives of the system states. Since this criterion is both neutral delay-dependent and discrete delay-dependent, it is less conservative than existing criteria for neutral differential systems. Stability criteria derived in this

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paper are expressed using linear matrix inequalities (LMIs). We also note that these criteria can be applied under more relaxed assumptions than those made in previous work. A solution to these LMIs can easily be found using various optimization algorithms. Numerical examples are given to illustrate the proposed results.

2. Main results

In this paper, we are interested in the following linear system of neutral type with mixed multiple time-varying delay arguments:

$$\dot{x}(t) = Ax(t) + B_1 x(t - d_1(t)) + B_2 x(t - d_2(t)) + C_1 \dot{x}(t - h_1(t)) + C_2 \dot{x}(t - h_2(t)) \tag{1}$$

with the initial condition function

$$x(\theta) = \phi(\theta), \quad \forall \theta \in [-\rho, 0] \tag{2}$$

where $x(t) \in R^n$ is the state vector, A, B_i, C_i are constant real system matrices, $d_i(t)$ and $h_i(t)$ are positive time-varying bounded delays satisfying

$$\begin{cases} 0 < d_i(t) \le d_i < \infty, & \dot{d}_i(t) \le \mu_i < 1, \\ 0 < h_i(t) \le h_i < \infty, & \dot{h}_i(t) \le \mu_{i+2} < 1, \end{cases} \quad \text{for } i = 1, 2. \tag{3}$$

$\phi(\theta)$ is a given continuously differentiable function on $[-\rho, 0]$. We write $d = \max(d_1, d_2)$, $h = \max(h_1, h_2)$, $\rho = \max(d, h)$. We assume the system matrix A is a Hurwitz matrix. Using $\|\cdot\|$ to denote the matrix norm, we assume that matrices C_1 and C_2 satisfy

Assumption 1. $\|C_1\| + \|C_2\| < 1$.

This assumption guarantees that we can apply the Lyapunov–Krasovskii method to the stability of neutral type systems with time-varying delays [13]—see, e.g., [3].

The goals of this paper are to find criteria for asymptotic stability and robust stability of Eq. (1) using the Lyapunov method in conjunction with LMI techniques.

For simplicity, in the rest of the paper, in symmetric block matrices or long matrix expressions, we use '$*$' to represent some term that is induced by symmetry.

Theorem 1. *For given scalars* d_1, d_2, h_1, h_2 *and* μ_i $(i = 1 \ldots 4)$, *under the assumptions given, the neutral system in* Eq. (1) *is asymptotically stable if there exist positive definite matrices* $P > 0$, $Q_i > 0$, $Z_i > 0$, $W_i > 0$, $M_i > 0$, *and* $R_i > 0$ for $i = 1, 2$ *and any appropriately dimensioned matrices* $Y_j, L_j, T_j, N_j, (j = 1, \ldots, 7)$ *such that the following linear matrix inequality (LMI) holds*:

$$\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & -T_1 + G_4^{\mathrm{T}} & PC_1 + G_5^{\mathrm{T}} & -N_1 + G_6^{\mathrm{T}} & PC_2 + G_7^{\mathrm{T}} & -d_1 Y_1 & -d_2 L_1 & -h_1 T_1 & -h_2 N_1 & A^{\mathrm{T}} S \\
* & \Omega_{22} & \Omega_{23} & -T_2 - Y_4^{\mathrm{T}} & -Y_5^{\mathrm{T}} & -N_2 - Y_6^{\mathrm{T}} & -Y_7^{\mathrm{T}} & -d_1 Y_2 & -d_2 L_2 & -h_1 T_2 & -h_2 N_2 & B_1^{\mathrm{T}} S \\
* & * & \Omega_{33} & -T_3 - L_4^{\mathrm{T}} & -L_5^{\mathrm{T}} & -N_3 - L_6^{\mathrm{T}} & -L_7^{\mathrm{T}} & -d_1 Y_3 & -d_2 L_3 & -h_1 T_3 & -h_2 N_3 & B_2^{\mathrm{T}} S \\
* & * & * & \Omega_{44} & -T_5^{\mathrm{T}} & -N_4 - T_6^{\mathrm{T}} & -T_7^{\mathrm{T}} & -d_1 Y_4 & -d_2 L_4 & -h_1 T_4 & -h_2 N_4 & 0 \\
* & * & * & * & \Omega_{55} & -N_5 & 0 & -d_1 Y_5 & -d_2 L_5 & -h_1 T_5 & -h_2 N_5 & C_1^{\mathrm{T}} S \\
* & * & * & * & * & \Omega_{66} & -N_7^{\mathrm{T}} & -d_1 Y_6 & -d_2 L_6 & -h_1 T_6 & -h_2 N_6 & 0 \\
* & * & * & * & * & * & \Omega_{77} & -d_1 Y_7 & -d_2 L_7 & -h_1 T_7 & -h_2 N_7 & C_2^{\mathrm{T}} S \\
* & * & * & * & * & * & * & -d_1 Z_1 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -d_2 Z_2 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \star & * & -h_1 W_1 & 0 & 0 \\
* & * & * & * & * & * & * & * & \star & * & -h_2 W_2 & 0 \\
* & * & * & * & * & * & * & * & \star & * & * & -S
\end{bmatrix} < 0 \tag{4}$$

where $\Omega_{11} = A^{\mathrm{T}}P + PA + Q_1 + Q_2 + M_1 + M_2 + G_1 + G_1^{\mathrm{T}}; \Omega_{12} = PB_1 - Y_1 + G_2^{\mathrm{T}}; \Omega_{13} = PB_2 - L_1 + G_3^{\mathrm{T}}; \Omega_{22} = -(1-\mu_1)Q_1 - Y_2 - Y_2^{\mathrm{T}}; \Omega_{23} = -L_2 - Y_3^{\mathrm{T}}; \Omega_{33} = -(1-\mu_2)Q_2 - L_3 - L_3^{\mathrm{T}}; \Omega_{44} = -(1-\mu_3)M_1 - T_4 - T_4^{\mathrm{T}}; \Omega_{55} = -(1-\mu_3)R_1; \Omega_{66} = -(1-\mu_4)M_2 - N_6 - N_6^{\mathrm{T}}; \Omega_{77} = -(1-\mu_4)R_2; G_i = Y_i + L_i + T_i + N_i; S = \sum_{i=1}^{2}[d_iZ_i + h_iW_i + R_i]$.

Proof. Let the Lyapunov functional candidate be

$$V(x_t) = V_1 + V_2 + V_3 + V_4 \tag{5}$$

where

$$V_1 = x^{\mathrm{T}}(t)Px(t), \tag{6}$$

$$V_2 = \sum_{i=1}^{2} \int_{-d_i}^{0}\int_{t+s}^{t} \dot{x}^{\mathrm{T}}(\alpha)Z_i\dot{x}(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}s + \int_{t-d_i(t)}^{t} x^{\mathrm{T}}(s)Q_ix(s)\,\mathrm{d}s, \tag{7}$$

$$V_3 = \sum_{i=1}^{2} \int_{-h_i}^{0}\int_{t+s}^{t} \dot{x}^{\mathrm{T}}(\alpha)W_i\dot{x}(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}s + \int_{t-h_i(t)}^{t} x^{\mathrm{T}}(s)M_ix(s)\,\mathrm{d}s, \tag{8}$$

$$V_4 = \sum_{i=1}^{2} \int_{t-h_i(t)}^{t} \dot{x}^{\mathrm{T}}(s)R_i\dot{x}(s)\,\mathrm{d}s. \tag{9}$$

The time derivative of V along the trajectories of System (1) is given by

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4. \tag{10}$$

From (6) to (9), we have

$$\begin{aligned}\dot{V}_1 &= 2x^{\mathrm{T}}(t)P\dot{x}(t) = 2x^{\mathrm{T}}(t)PAx(t) + 2x^{\mathrm{T}}(t)PB_1x(t-d_1(t)) + 2x^{\mathrm{T}}(t)PB_2x(t-d_2(t)) + 2x^{\mathrm{T}}(t)PC_1\dot{x}(t-h_1(t))\\ &\quad +2x^{\mathrm{T}}(t)PC_2\dot{x}(t-h_2(t))\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_2 &= \sum_{i=1}^{2} d_i\dot{x}^{\mathrm{T}}(t)Z_i\dot{x}(t) - \int_{t-d_i}^{t}\dot{x}^{\mathrm{T}}(s)Z_i\dot{x}(s)\,\mathrm{d}s + x^{\mathrm{T}}(t)Q_ix(t) - (1-\dot{d}_i(t))x^{\mathrm{T}}(t-d_i(t))Q_ix(t-d_i(t))\\ &\le \sum_{i=1}^{2} d_i\dot{x}^{\mathrm{T}}(t)Z_i\dot{x}(t) - \frac{1}{d_i(t)}\int_{t-d_i(t)}^{t} d_i(t)\dot{x}^{\mathrm{T}}(s)\frac{1}{d_i}Z_i\dot{x}(s)d_i(t)\,\mathrm{d}s + x^{\mathrm{T}}(t)Q_ix(t)\\ &\quad -(1-\dot{d}_i(t))x^{\mathrm{T}}(t-d_i(t))Q_ix(t-d_i(t))\\ &\le \sum_{i=1}^{2} d_i\dot{x}^{\mathrm{T}}(t)Z_i\dot{x}(t) - \frac{1}{d_i(t)}\int_{t-d_i(t)}^{t} d_i(t)\dot{x}^{\mathrm{T}}(s)\frac{1}{d_i}Z_i\dot{x}(s)d_i(t)\,\mathrm{d}s\\ &\quad +x^{\mathrm{T}}(t)Q_ix(t) - (1-\mu_i)x^{\mathrm{T}}(t-d_i(t))Q_ix(t-d_i(t))\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{V}_3 &\le \sum_{i=1}^{2} h_i\dot{x}^{\mathrm{T}}(t)W_i\dot{x}(t) - \frac{1}{h_i(t)}\int_{t-h_i(t)}^{t} h_i(t)\dot{x}^{\mathrm{T}}(s)\frac{1}{h_i}W_i\dot{x}(s)h_i(t)\,\mathrm{d}s + x^{\mathrm{T}}(t)M_ix(t)\\ &\quad -(1-\mu_{i+2})x^{\mathrm{T}}(t-h_i(t))M_ix(t-h_i(t))\end{aligned} \tag{13}$$

$$\begin{aligned}\dot{V}_4 &= \sum_{i=1}^{2} \dot{x}^{\mathrm{T}}(t)R_i\dot{x}(t) - [1-\dot{h}_i(t)]\dot{x}^{\mathrm{T}}(t-h_i(t))R_i\dot{x}(t-h_i(t))\\ &\le \sum_{i=1}^{2} \dot{x}^{\mathrm{T}}(t)R_i\dot{x}(t) - [1-\mu_{i+2}]\dot{x}^{\mathrm{T}}(t-h_i(t))R_i\dot{x}(t-h_i(t)).\end{aligned} \tag{14}$$

Adding (11)–(14) gives

$$\begin{aligned}\dot V_1+\dot V_2+\dot V_3+\dot V_4 &\le 2x^{\mathrm T}(t)PAx(t)+2x^{\mathrm T}(t)PB_1x(t-d_1(t))+2x^{\mathrm T}(t)PB_2x(t-d_2(t))\\
&+2x^{\mathrm T}(t)PC_1\dot x(t-h_1(t))+2x^{\mathrm T}(t)PC_2\dot x(t-h_2(t))+\sum_{i=1}^{2}\dot x^{\mathrm T}(t)[d_iZ_i+h_iW_i+R_i]\dot x(t)\\
&+x^{\mathrm T}(t)[Q_i+M_i]x(t)-(1-\mu_i)x^{\mathrm T}(t-d_i(t))Q_ix(t-d_i(t))-(1-\mu_{i+2})x^{\mathrm T}(t-h_i(t))M_ix(t-h_i(t))\\
&-(1-\mu_{i+2})\dot x^{\mathrm T}(t-h_i(t))R_i\dot x(t-h_i(t))-\frac{1}{d_i(t)}\int_{t-d_i(t)}^{t}d_i(t)\dot x^{\mathrm T}(s)\frac{1}{d_i}Z_i\dot x(s)d_i(t)\,\mathrm ds\\
&-\frac{1}{h_i(t)}\int_{t-h_i(t)}^{t}h_i(t)\dot x^{\mathrm T}(s)\frac{1}{h_i}W_i\dot x(s)h_i(t)\,\mathrm ds\\
&=x^{\mathrm T}(t)[PA+A^{\mathrm T}P+Q_1+Q_2+M_1+M_2]x(t)\\
&+2x^{\mathrm T}(t)PB_1x(t-d_1(t))+2x^{\mathrm T}(t)PB_2x(t-d_2(t))+2x^{\mathrm T}(t)PC_1\dot x(t-h_1(t))+2x^{\mathrm T}(t)PC_2\dot x(t-h_2(t))\\
&+\dot x^{\mathrm T}(t)S\dot x(t)-(1-\mu_1)x^{\mathrm T}(t-d_1(t))Q_1x(t-d_1(t))-(1-\mu_2)x^{\mathrm T}(t-d_2(t))Q_2x(t-d_2(t))\\
&-(1-\mu_3)x^{\mathrm T}(t-h_1(t))M_1x(t-h_1(t))-(1-\mu_4)x^{\mathrm T}(t-h_2(t))M_2x(t-h_2(t))\\
&-(1-\mu_3)\dot x^{\mathrm T}(t-h_1(t))R_1\dot x(t-h_1(t))-(1-\mu_4)\dot x^{\mathrm T}(t-h_2(t))R_2\dot x(t-h_2(t))\\
&-\frac{1}{d_1(t)}\int_{t-d_1(t)}^{t}d_1(t)\dot x^{\mathrm T}(s)\frac{1}{d_1}Z_1\dot x(s)d_1(t)\,\mathrm ds-\frac{1}{d_2(t)}\int_{t-d_2(t)}^{t}d_2(t)\dot x^{\mathrm T}(\alpha)\frac{1}{d_2}Z_2\dot x(\alpha)d_2(t)\,\mathrm d\alpha\\
&-\frac{1}{h_1(t)}\int_{t-h_1(t)}^{t}h_1(t)\dot x^{\mathrm T}(\beta)\frac{1}{h_1}W_1\dot x(\beta)h_1(t)\,\mathrm d\beta-\frac{1}{h_2(t)}\int_{t-h_2(t)}^{t}h_2(t)\dot x^{\mathrm T}(r)\frac{1}{h_2}W_2\dot x(r)h_2(t)\,\mathrm dr. \qquad (15)\end{aligned}$$

Let

$$\xi(t)=[x^{\mathrm T}(t),x^{\mathrm T}(t-d_1(t)),x^{\mathrm T}(t-d_2(t)),x^{\mathrm T}(t-h_1(t)),\dot x^{\mathrm T}(t-h_1(t)),x^{\mathrm T}(t-h_2(t)),\dot x^{\mathrm T}(t-h_2(t))]^{\mathrm T}.$$

Then

$$\begin{aligned}\dot x(t)&=Ax(t)+B_1x(t-d_1(t))+B_2x(t-d_2(t))+C_1\dot x(t-h_1(t))+C_2\dot x(t-h_2(t))\\
&=[A,B_1,B_2,0,C_1,0,C_2]\xi(t). \qquad (16)\end{aligned}$$

Therefore,

$$\dot x^{\mathrm T}(t)S\dot x(t)=\xi^{\mathrm T}(t)[A,B_1,B_2,0,C_1,0,C_2]^{\mathrm T}S[A,B_1,B_2,0,C_1,0,C_2]\xi(t) \qquad (17)$$

$$=\xi^{\mathrm T}(t)\begin{bmatrix}A^{\mathrm T}SA & A^{\mathrm T}SB_1 & A^{\mathrm T}SB_2 & 0 & A^{\mathrm T}SC_1 & 0 & A^{\mathrm T}SC_2\\ * & B_1^{\mathrm T}SB_1 & B_1^{\mathrm T}SB_2 & 0 & B_1^{\mathrm T}SC_1 & 0 & B_1^{\mathrm T}SC_2\\ * & * & B_2^{\mathrm T}SB_2 & 0 & B_2^{\mathrm T}SC_1 & 0 & B_2^{\mathrm T}SC_2\\ * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & C_1^{\mathrm T}SC_1 & 0 & C_1^{\mathrm T}SC_2\\ * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & C_2^{\mathrm T}SC_2\end{bmatrix}\xi(t). \qquad (18)$$

Since

$$x(t)-x(t-d_1(t))=\int_{t-d_1(t)}^{t}\dot x(s)\,\mathrm ds,$$

$$2\xi^{\mathrm{T}}(t)\begin{bmatrix}Y_1\\Y_2\\Y_3\\Y_4\\Y_5\\Y_6\\Y_7\end{bmatrix}[x(t)-x(t-d_1(t))]-2\xi^{\mathrm{T}}(t)\begin{bmatrix}Y_1\\Y_2\\Y_3\\Y_4\\Y_5\\Y_6\\Y_7\end{bmatrix}\int_{t-d_1(t)}^{t}\dot{x}(s)\,\mathrm{d}s=0,$$

hence,

$$2\xi^{\mathrm{T}}(t)\begin{bmatrix}Y_1 & -Y_1 & 0 & 0 & 0 & 0 & 0\\Y_2 & -Y_2 & 0 & 0 & 0 & 0 & 0\\Y_3 & -Y_3 & 0 & 0 & 0 & 0 & 0\\Y_4 & -Y_4 & 0 & 0 & 0 & 0 & 0\\Y_5 & -Y_5 & 0 & 0 & 0 & 0 & 0\\Y_6 & -Y_6 & 0 & 0 & 0 & 0 & 0\\Y_7 & -Y_7 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\xi(t)-2\int_{t-d_1(t)}^{t}\xi^{\mathrm{T}}(t)\begin{bmatrix}Y_1\\Y_2\\Y_3\\Y_4\\Y_5\\Y_6\\Y_7\end{bmatrix}\dot{x}(s)\,\mathrm{d}s=0. \tag{19}$$

Similarly, we have

$$2\xi^{\mathrm{T}}(t)\begin{bmatrix}L_1 & 0 & -L_1 & 0 & 0 & 0 & 0\\L_2 & 0 & -L_2 & 0 & 0 & 0 & 0\\L_3 & 0 & -L_3 & 0 & 0 & 0 & 0\\L_4 & 0 & -L_4 & 0 & 0 & 0 & 0\\L_5 & 0 & -L_5 & 0 & 0 & 0 & 0\\L_6 & 0 & -L_6 & 0 & 0 & 0 & 0\\L_7 & 0 & -L_7 & 0 & 0 & 0 & 0\end{bmatrix}\xi(t)-2\int_{t-d_2(t)}^{t}\xi^{\mathrm{T}}(t)\begin{bmatrix}L_1\\L_2\\L_3\\L_4\\L_5\\L_6\\L_7\end{bmatrix}\dot{x}(\alpha)\,\mathrm{d}\alpha=0, \tag{20}$$

$$2\xi^{\mathrm{T}}(t)\begin{bmatrix}T_1 & 0 & 0 & -T_1 & 0 & 0 & 0\\T_2 & 0 & 0 & -T_2 & 0 & 0 & 0\\T_3 & 0 & 0 & -T_3 & 0 & 0 & 0\\T_4 & 0 & 0 & -T_4 & 0 & 0 & 0\\T_5 & 0 & 0 & -T_5 & 0 & 0 & 0\\T_6 & 0 & 0 & -T_6 & 0 & 0 & 0\\T_7 & 0 & 0 & -T_7 & 0 & 0 & 0\end{bmatrix}\xi(t)-2\int_{t-h_1(t)}^{t}\xi^{\mathrm{T}}(t)\begin{bmatrix}T_1\\T_2\\T_3\\T_4\\T_5\\T_6\\T_7\end{bmatrix}\dot{x}(\beta)\,\mathrm{d}\beta=0, \tag{21}$$

$$2\xi^{\mathrm{T}}(t)\begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & -N_1 & 0 \\ N_2 & 0 & 0 & 0 & 0 & -N_2 & 0 \\ N_3 & 0 & 0 & 0 & 0 & -N_3 & 0 \\ N_4 & 0 & 0 & 0 & 0 & -N_4 & 0 \\ N_5 & 0 & 0 & 0 & 0 & -N_5 & 0 \\ N_6 & 0 & 0 & 0 & 0 & -N_6 & 0 \\ N_7 & 0 & 0 & 0 & 0 & -N_7 & 0 \end{bmatrix}\xi(t) - 2\int_{t-h_2(t)}^{t}\xi^{\mathrm{T}}(t)\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{bmatrix}\dot{x}(r)\,\mathrm{d}r = 0. \tag{22}$$

Denote $G_i = Y_i + L_i + T_i + N_i$ for $i = 1\dots 7$. From Eqs. (19)–(22), we have

$$\xi^{\mathrm{T}}(t)\begin{bmatrix} G_1+G_1^{\mathrm{T}} & -Y_1+G_2^{\mathrm{T}} & -L_1+G_3^{\mathrm{T}} & -T_1+G_4^{\mathrm{T}} & G_5^{\mathrm{T}} & -N_1+G_6^{\mathrm{T}} & G_7^{\mathrm{T}} \\ * & -Y_2-Y_2^{\mathrm{T}} & -L_2-Y_3^{\mathrm{T}} & -T_2-Y_4^{\mathrm{T}} & -Y_5^{\mathrm{T}} & -N_2-Y_6^{\mathrm{T}} & -Y_7^{\mathrm{T}} \\ * & * & -L_3-L_3^{\mathrm{T}} & -T_3-L_4^{\mathrm{T}} & -L_5^{\mathrm{T}} & -N_3-L_6^{\mathrm{T}} & -L_7^{\mathrm{T}} \\ * & * & * & -T_4-T_4^{\mathrm{T}} & -T_5^{\mathrm{T}} & -N_4-T_6^{\mathrm{T}} & -T_7^{\mathrm{T}} \\ * & * & * & * & 0 & -N_5 & 0 \\ * & * & * & * & * & -N_6-N_6^{\mathrm{T}} & -N_7^{\mathrm{T}} \\ * & * & * & * & * & * & 0 \end{bmatrix}\xi(t)$$

$$-2\int_{t-d_1(t)}^{t}\xi^{\mathrm{T}}(t)\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \end{bmatrix}\dot{x}(s)\,\mathrm{d}s - 2\int_{t-d_2(t)}^{t}\xi^{\mathrm{T}}(t)\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \end{bmatrix}\dot{x}(\alpha)\,\mathrm{d}\alpha - 2\int_{t-h_1(t)}^{t}\xi^{\mathrm{T}}(t)\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{bmatrix}\dot{x}(\beta)\,\mathrm{d}\beta$$

$$-2\int_{t-h_2(t)}^{t}\xi^{\mathrm{T}}(t)\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{bmatrix}\dot{x}(r)\,\mathrm{d}r = 0. \tag{23}$$

Let $\eta(t,s,\alpha,\beta,r) = [\xi^{\mathrm{T}}(t), d_1(t)\dot{x}^{\mathrm{T}}(s), d_2(t)\dot{x}^{\mathrm{T}}(\alpha), h_1(t)\dot{x}^{\mathrm{T}}(\beta), h_2(t)\dot{x}^{\mathrm{T}}(r)]^{\mathrm{T}}$. Thus,

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 + 0 \\ &\leq \frac{1}{d_1(t)d_2(t)h_1(t)h_2(t)}\int_{t-d_1(t)}^{t}\mathrm{d}s\int_{t-d_2(t)}^{t}\mathrm{d}\alpha\int_{t-h_1(t)}^{t}\mathrm{d}\beta\int_{t-h_2(t)}^{t}\eta^{\mathrm{T}}(t,s,\alpha,\beta,r)\bar{\Omega}\eta(t,s,\alpha,\beta,r)\,\mathrm{d}r\end{aligned} \tag{24}$$

where

$$\bar{\Omega} = \begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & -T_1 + G_4^{\mathrm{T}} & A^{\mathrm{T}}SC_1 + PC_1 + G_5^{\mathrm{T}} & -N_1 + G_6^{\mathrm{T}} & A^{\mathrm{T}}SC_2 + PC_2 + G_7^{\mathrm{T}} & -Y_1 & -L_1 & -T_1 & -N_1 \\ * & \bar{\Omega}_{22} & \bar{\Omega}_{23} & -T_2 - Y_4^{\mathrm{T}} & B_1^{\mathrm{T}}SC_1 - Y_5^{\mathrm{T}} & -N_2 - Y_6^{\mathrm{T}} & B_1^{\mathrm{T}}SC_2 - Y_7^{\mathrm{T}} & -Y_2 & -L_2 & -T_2 & -N_2 \\ * & * & \bar{\Omega}_{33} & -T_3 - L_4^{\mathrm{T}} & B_2^{\mathrm{T}}SC_1 - L_5^{\mathrm{T}} & -N_3 - L_6^{\mathrm{T}} & B_2^{\mathrm{T}}SC_2 - L_7^{\mathrm{T}} & -Y_3 & -L_3 & -T_3 & -N_3 \\ * & * & * & \bar{\Omega}_{44} & -T_5^{\mathrm{T}} & -N_4 - T_6^{\mathrm{T}} & -T_7^{\mathrm{T}} & -Y_4 & -L_4 & -T_4 & -N_4 \\ * & * & * & * & \bar{\Omega}_{55} & -N_5 & C_1^{\mathrm{T}}SC_2 & -Y_5 & -L_5 & -T_5 & -N_5 \\ * & * & * & * & * & \bar{\Omega}_{66} & -N_7^{\mathrm{T}} & -Y_6 & -L_6 & -T_6 & -N_6 \\ * & * & * & * & * & * & \bar{\Omega}_{77} & -Y_7 & -L_7 & -T_7 & -N_7 \\ * & * & * & * & * & * & * & -Z_1/d_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -Z_2/d_2 & 0 & 0 \\ * & * & * & * & * & * & * & \star & * & -W_1/h_1 & 0 \\ * & * & * & * & * & * & * & * & \star & * & -W_2/h_2 \end{bmatrix} \tag{25}$$

and

$$\begin{aligned}
\bar{\Omega}_{11} &= A^{\mathrm{T}}SA + A^{\mathrm{T}}P + PA + Q_1 + Q_2 + M_1 + M_2 + G_1 + G_1^{\mathrm{T}};\\
\bar{\Omega}_{12} &= A^{\mathrm{T}}SB_1 + PB_1 - Y_1 + G_2^{\mathrm{T}};\\
\bar{\Omega}_{13} &= A^{\mathrm{T}}SB_2 + PB_2 - L_1 + G_3^{\mathrm{T}};\\
\bar{\Omega}_{22} &= B_1^{\mathrm{T}}SB_1 - (1-\mu_1)Q_1 - Y_2 - Y_2^{\mathrm{T}};\\
\bar{\Omega}_{23} &= B_1^{\mathrm{T}}SB_2 - L_2 - Y_3^{\mathrm{T}};\\
\bar{\Omega}_{33} &= B_2^{\mathrm{T}}SB_2 - (1-\mu_2)Q_2 - L_3 - L_3^{\mathrm{T}};\\
\bar{\Omega}_{44} &= -(1-\mu_3)M_1 - T_4 - T_4^{\mathrm{T}};\\
\bar{\Omega}_{55} &= C_1^{\mathrm{T}}SC_1 - (1-\mu_3)R_1;\\
\bar{\Omega}_{66} &= -(1-\mu_4)M_2 - N_6 - N_6^{\mathrm{T}};\\
\bar{\Omega}_{77} &= C_2^{\mathrm{T}}SC_2 - (1-\mu_4)R_2;\\
G_i &= Y_i + L_i + T_i + N_i.
\end{aligned}$$

It can be seen that $\dot{V}$ is negative if LMI $\bar{\Omega} < 0$ holds. Multiplying both sides of this LMI by the matrix diag$(I, I, I, I, I, I, I, d_1I, d_2I, h_1I, h_2I)$, and using Schur complement, we find that $\bar{\Omega} < 0$ is equivalent to LMI (4).

We will prove that all conditions of Theorem 1.6 in [13] are satisfied. In fact, denote

$$\|x_t\|_w = \left(|x(t)| + \int_{-\rho}^{0} |\dot{x}(t+s)|^2\,\mathrm{d}s\right)^{1/2}, \quad \|x_t\|_s = \sup_{-\rho \le \theta \le 0} |x(t+\theta)|.$$

For simplicity, the subscript of $\|\cdot\|_s$ is usually omitted.

Since $x(t+\theta) = x(t) - \int_{\theta}^{0} \dot{x}(t+s)\,\mathrm{d}s$ for $-\rho \le \theta \le 0$,

$$|x(t+\theta)| \le |x(t)| + \int_{-\rho}^{0} |\dot{x}(t+s)|\,\mathrm{d}s.$$

So

$$\|x_t\|^2 \le \left(|x(t)| + \int_{-\rho}^{0} |\dot{x}(t+s)|\,\mathrm{d}s\right)^2 \le 2|x(t)|^2 + 2\left(\int_{-\rho}^{0} |\dot{x}(t+s)|\,\mathrm{d}s\right)^2.$$

Using Hölder's inequality gives

$$\|x_t\|^2 \leq 2|x(t)|^2 + 2\rho\int_{-\rho}^{0}|\dot{x}(t+s)|^2\,\mathrm{d}s.$$

Hence,

$$\|x_t\|^2 \leq 2(1+\rho)\|x_t\|_w^2.$$

By simple computation, we have $\lambda_{\min}(P)|x(t)|^2 \leq V(x_t) \leq \{\lambda_{\max}(P) + \sum_{i=1}^{2}[\rho(\lambda_{\max}(Z_i) + \lambda_{\max}(W_i)) + \lambda_{\max}(R_i) + 2(1+\rho)(Q_i + M_i)]\}\|x_t\|_w^2$. The neutral system in Eq. (1) can be given in the following form:

$$\dot{x}(t) = f(t, x_t, \dot{x}_t),$$

where $f(t, x_t, \dot{x}_t) = Ax(t) + B_1x(t-d_1(t)) + B_2x(t-d_2(t)) + C_1\dot{x}(t-h_1(t)) + C_2\dot{x}(t-h_2(t))$. It is obvious that $f(t, 0, 0) = 0$ and $f(t, x_t, \dot{x}_t)$ satisfies a Lipschitz condition on the second argument x_t. Now, we prove that $f(t, x_t, \dot{x}_t)$ satisfies a Lipschitz condition on the third argument $\dot{x}_t$ with Lipschitz constant less than 1. For simplicity, denote $f(t, \varphi, \psi) = A\varphi(0) + B_1\varphi(-d_1(t)) + B_2\varphi(-d_2(t)) + C_1\psi(-h_1(t)) + C_2\psi(-h_2(t))$. Since Assumption 1 holds, i.e. $\|C_1\| + \|C_2\| < 1$, then, for any $\varphi, \psi_1, \psi_2 \in C([-\rho, 0], R^n)$, we have that

$$\begin{aligned}
&|f(t,\varphi,\psi_2) - f(t,\varphi,\psi_1)|\\
&\quad = |C_1\psi_2(-h_1(t)) + C_2\psi_2(-h_2(t)) - C_1\psi_1(-h_1(t)) - C_2\psi_1(-h_2(t))|\\
&\quad\leq |C_1\psi_2(-h_1(t)) - C_1\psi_1(-h_1(t))| + |C_2\psi_2(-h_2(t)) - C_2\psi_1(-h_2(t))|\\
&\quad\leq \|C_1\||\psi_2(-h_1(t)) - \psi_1(-h_1(t))| + \|C_2\||\psi_2(-h_2(t)) - \psi_1(-h_2(t))|\\
&\quad\leq \|C_1\|\|\psi_2 - \psi_1\| + \|C_2\|\|\psi_2 - \psi_1\| = (\|C_1\| + \|C_2\|)\|\psi_2 - \psi_1\| < \|\psi_2 - \psi_1\|.
\end{aligned}$$

So, $f(t, x_t, \dot{x}_t)$ satisfies a Lipschitz condition on the third argument $\dot{x}_t$ with Lipschitz constant less than 1. $\dot{V}$ is negative if LMI (4) hold. Hence, by Theorem 1.6 in [13], the existence of $V > 0$ such that $\dot{V} < 0$ guarantees asymptotic stability of neutral system given in Eq. (1). This completes the proof. □

In the rest of this section, we develop a new robust stability criterion for a neutral system with uncertainties. We first state a useful lemma.

Lemma 1. *From [21]. Given matrices $Q = Q^{\mathrm{T}}$, H, E and $R = R^{\mathrm{T}} > 0$ of appropriate dimensions,*

$$Q + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$$

for all F satisfying $F^{\mathrm{T}}F \leq R$, if and only if there exists some $\lambda > 0$ such that

$$Q + \lambda HH^{\mathrm{T}} + \lambda^{-1}E^{\mathrm{T}}RE < 0.$$

Consider the following uncertain neutral system

$$\dot{x}(t) = (A + \Delta A(t))x(t) + \sum_{i=1}^{2}(B_i + \Delta B_i(t))x(t-d_i(t)) + \sum_{i=1}^{2}C_i\dot{x}(t-h_i(t)) \tag{26}$$

where $x(t)$, $\phi(\theta)$, ρ, A, B_i, C_i, $d_i(t)$, $h_i(t)$, d_i, h_i, μ_j, for $i = 1, 2$ and $j = 1, \ldots, 4$ are defined as in System (1). The time-varying structured uncertainties are of the form

$$[\Delta A(t), \Delta B_1(t), \Delta B_2(t)] = DF(t)[E, E_1, E_2], \tag{27}$$

where D, E, E_1, E_2 are constant matrices with appropriate dimensions. $F(t)$ is an unknown and possibly time-varying real matrix with Lebesgue measurable elements and whose Euclidean norm satisfies

$$\|F(t)\| \leq 1 \quad \forall t. \tag{28}$$

Theorem 2. *For given scalars d_1, d_2, h_1, h_2 and μ_i $(i = 1, 2, 3, 4)$, and* Assumption 1, *the neutral system in* Eq. (26) *is robustly stable if there exist positive definite matrices $P > 0$, $Q_i > 0$, $Z_i > 0$, $W_i > 0$, $M_i > 0$, $R_i > 0$, for*

$i = 1, 2$ and any appropriately dimensioned matrices Y_j, L_j, T_j, N_j $(j = 1 \ldots 7)$ such that the following linear matrix inequality (LMI) holds:

$$\begin{bmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} & \hat{\Omega}_{14} & \hat{\Omega}_{15} & -N_1+G_6^{\mathrm{T}} & PC_2+G_7^{\mathrm{T}} & -d_1Y_1 & -d_2L_1 & -h_1T_1 & -h_2N_1 & A^{\mathrm{T}}S & PD \\ * & \hat{\Omega}_{22} & \hat{\Omega}_{23} & \hat{\Omega}_{24} & \hat{\Omega}_{25} & -N_2-Y_6^{\mathrm{T}} & -Y_7^{\mathrm{T}} & -d_1Y_2 & -d_2L_2 & -h_1T_2 & -h_2N_2 & B_1^{\mathrm{T}}S & 0 \\ * & * & \hat{\Omega}_{33} & \hat{\Omega}_{34} & \hat{\Omega}_{35} & -N_3-L_6^{\mathrm{T}} & -L_7^{\mathrm{T}} & -d_1Y_3 & -d_2L_3 & -h_1T_3 & -h_2N_3 & B_2^{\mathrm{T}}S & 0 \\ * & * & * & \hat{\Omega}_{44} & \hat{\Omega}_{45} & -N_4-T_6^{\mathrm{T}} & -T_7^{\mathrm{T}} & -d_1Y_4 & -d_2L_4 & -h_1T_4 & -h_2N_4 & 0 & 0 \\ * & * & * & * & \hat{\Omega}_{55} & -N_5 & 0 & -d_1Y_5 & -d_2L_5 & -h_1T_5 & -h_2N_5 & C_1^{\mathrm{T}}S & 0 \\ * & * & * & * & * & \hat{\Omega}_{66} & -N_7^{\mathrm{T}} & -d_1Y_6 & -d_2L_6 & -h_1T_6 & -h_2N_6 & 0 & 0 \\ * & * & * & * & * & * & \hat{\Omega}_{77} & -d_1Y_7 & -d_2L_7 & -h_1T_7 & -h_2N_7 & C_2^{\mathrm{T}}S & 0 \\ * & * & * & * & * & * & * & -d_1Z_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -d_2Z_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \star & * & -h_1W_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \star & * & -h_2W_2 & 0 & 0 \\ * & * & * & * & * & * & * & * & \star & * & * & -S & S^{\mathrm{T}}D \\ * & * & * & * & * & * & * & * & \star & * & * & * & -I \end{bmatrix} < 0. \tag{29}$$

where

$$\hat{\Omega}_{11} = A^{\mathrm{T}}P + PA + Q_1 + Q_2 + M_1 + M_2 + G_1 + G_1^{\mathrm{T}} + E^{\mathrm{T}}E;$$
$$\hat{\Omega}_{12} = PB_1 - Y_1 + G_2^{\mathrm{T}} + E^{\mathrm{T}}E_1;$$
$$\hat{\Omega}_{13} = PB_2 - L_1 + G_3^{\mathrm{T}} + E^{\mathrm{T}}E_2;$$
$$\hat{\Omega}_{14} = -T_1 + G_4^{\mathrm{T}};$$
$$\hat{\Omega}_{15} = PC_1 + G_5^{\mathrm{T}};$$
$$\hat{\Omega}_{22} = -(1-\mu_1)Q_1 - Y_2 - Y_2^{\mathrm{T}} + E_1^{\mathrm{T}}E_1;$$
$$\hat{\Omega}_{23} = -L_2 - Y_3^{\mathrm{T}} + E_1^{\mathrm{T}}E_2;$$
$$\hat{\Omega}_{24} = -T_2 - Y_4^{\mathrm{T}};$$
$$\hat{\Omega}_{25} = -Y_5^{\mathrm{T}};$$
$$\hat{\Omega}_{33} = -(1-\mu_2)Q_2 - L_3 - L_3^{\mathrm{T}} + E_2^{\mathrm{T}}E_2;$$
$$\hat{\Omega}_{34} = -T_3 - L_4^{\mathrm{T}};$$
$$\hat{\Omega}_{35} = -L_5^{\mathrm{T}};$$
$$\hat{\Omega}_{44} = -(1-\mu_3)M_1 - T_4 - T_4^{\mathrm{T}};$$
$$\hat{\Omega}_{45} = -T_5^{\mathrm{T}};$$
$$\hat{\Omega}_{55} = -(1-\mu_3)R_1;$$
$$\hat{\Omega}_{66} = -(1-\mu_4)M_2 - N_6 - N_6^{\mathrm{T}};$$
$$\hat{\Omega}_{77} = -(1-\mu_4)R_2;$$
$$G_i = Y_i + L_i + T_i + N_i;$$
$$S = \sum_{i=1}^{2}[d_iZ_i + h_iW_i + R_i].$$

Proof. Replace A, B_1, B_2 in Theorem 1 by $A + DF(t)E$, $B_1 + DF(t)E_1$, $B_2 + DF(t)E_2$, respectively. If $U = [E \quad E_1 \quad E_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$, $X = [D^{\mathrm{T}}P \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad D^{\mathrm{T}}S]$ then LMI (4) for the uncertain system in Eq. (26) is equivalent to the following condition

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & -T_1 + G_4^{\mathrm{T}} & PC_1 + G_5^{\mathrm{T}} & -N_1 + G_6^{\mathrm{T}} & PC_2 + G_7^{\mathrm{T}} & -d_1Y_1 & -d_2L_1 & -h_1T_1 & -h_2N_1 & A^{\mathrm{T}}S \\ * & \Omega_{22} & \Omega_{23} & -T_2 - Y_4^{\mathrm{T}} & -Y_5^{\mathrm{T}} & -N_2 - Y_6^{\mathrm{T}} & -Y_7^{\mathrm{T}} & -d_1Y_2 & -d_2L_2 & -h_1T_2 & -h_2N_2 & B_1^{\mathrm{T}}S \\ * & * & \Omega_{33} & -T_3 - L_4^{\mathrm{T}} & -L_5^{\mathrm{T}} & -N_3 - L_6^{\mathrm{T}} & -L_7^{\mathrm{T}} & -d_1Y_3 & -d_2L_3 & -h_1T_3 & -h_2N_3 & B_2^{\mathrm{T}}S \\ * & * & * & \Omega_{44} & -T_5^{\mathrm{T}} & -N_4 - T_6^{\mathrm{T}} & -T_7^{\mathrm{T}} & -d_1Y_4 & -d_2L_4 & -h_1T_4 & -h_2N_4 & 0 \\ * & * & * & * & \Omega_{55} & -N_5 & 0 & -d_1Y_5 & -d_2L_5 & -h_1T_5 & -h_2N_5 & C_1^{\mathrm{T}}S \\ * & * & * & * & * & \Omega_{66} & -N_7^{\mathrm{T}} & -d_1Y_6 & -d_2L_6 & -h_1T_6 & -h_2N_6 & 0 \\ * & * & * & * & * & * & \Omega_{77} & -d_1Y_7 & -d_2L_7 & -h_1T_7 & -h_2N_7 & C_2^{\mathrm{T}}S \\ * & * & * & * & * & * & * & -d_1Z_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -d_2Z_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \star & * & -h_1W_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & \star & * & -h_2W_2 & 0 \\ * & * & * & * & * & * & * & * & \star & * & * & -S \end{bmatrix}$$

$$+ X^{\mathrm{T}}F(t)U + U^{\mathrm{T}}F(t)X < 0. \qquad (30)$$

By Lemma 1, a necessary and sufficient condition for LMI (30) to hold for the uncertain system in Eq. (26) is that there exists a $\lambda > 0$ such that

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & -T_1 + G_4^{\mathrm{T}} & PC_1 + G_5^{\mathrm{T}} & -N_1 + G_6^{\mathrm{T}} & PC_2 + G_7^{\mathrm{T}} & -d_1Y_1 & -d_2L_1 & -h_1T_1 & -h_2N_1 & A^{\mathrm{T}}S \\ * & \Omega_{22} & \Omega_{23} & -T_2 - Y_4^{\mathrm{T}} & -Y_5^{\mathrm{T}} & -N_2 - Y_6^{\mathrm{T}} & -Y_7^{\mathrm{T}} & -d_1Y_2 & -d_2L_2 & -h_1T_2 & -h_2N_2 & B_1^{\mathrm{T}}S \\ * & * & \Omega_{33} & -T_3 - L_4^{\mathrm{T}} & -L_5^{\mathrm{T}} & -N_3 - L_6^{\mathrm{T}} & -L_7^{\mathrm{T}} & -d_1Y_3 & -d_2L_3 & -h_1T_3 & -h_2N_3 & B_2^{\mathrm{T}}S \\ * & * & * & \Omega_{44} & -T_5^{\mathrm{T}} & -N_4 - T_6^{\mathrm{T}} & -T_7^{\mathrm{T}} & -d_1Y_4 & -d_2L_4 & -h_1T_4 & -h_2N_4 & 0 \\ * & * & * & * & \Omega_{55} & -N_5 & 0 & -d_1Y_5 & -d_2L_5 & -h_1T_5 & -h_2N_5 & C_1^{\mathrm{T}}S \\ * & * & * & * & * & \Omega_{66} & -N_7^{\mathrm{T}} & -d_1Y_6 & -d_2L_6 & -h_1T_6 & -h_2N_6 & 0 \\ * & * & * & * & * & * & \Omega_{77} & -d_1Y_7 & -d_2L_7 & -h_1T_7 & -h_2N_7 & C_2^{\mathrm{T}}S \\ * & * & * & * & * & * & * & -d_1Z_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -d_2Z_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \star & * & -h_1W_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & \star & * & -h_2W_2 & 0 \\ * & * & * & * & * & * & * & * & \star & * & * & -S \end{bmatrix}$$

$$+ \lambda^{-1}X^{\mathrm{T}}X + \lambda U^{\mathrm{T}}U < 0. \qquad (31)$$

Multiplying both sides of LMI (31) by λ and replacing matrices $\lambda P, \lambda Q_i, \lambda Z_i, \lambda W_i, \lambda M_i, \lambda R_i$ for $i = 1, 2$ and matrices $\lambda Y_j, \lambda L_j, \lambda T_j, \lambda N_j$ $(j = 1, \ldots, 7)$ by matrices $P, Q_i, Z_i, W_i, M_i, R_i$ for $i = 1, 2$ and matrices Y_j, L_j, T_j, N_j $(j = 1 \ldots 7)$, respectively, and applying Schur complements, we obtain LMI (29), completing our proof of Theorem 2. □

Remark 1. A descriptor model transformation was introduced for analysis of delay-dependent stability of neutral systems in [2]. Fridman and Shaked [5] extended the results in [2] to the case of systems with time-varying discrete delays by finding tighter bounds on the cross terms introduced by Park in [17]. This method produces less conservative criteria than those in [12]. However, since the basic approach in [5] is based on the substitution of $x(t) - \int_{t-\tau}^{t} \dot{x}(s)\,ds$ for $x(t-\tau)$, and Park's inequality for bounding of the cross terms, it does not entirely overcome the conservatism of the methods given by Park [17]. The stability criteria obtained in [5] are neutral-delay-independent. Furthermore, these stability criteria can not be applied to neutral systems with time-varying neutral delays. Our paper presents a new approach to establishing both neutral-delay-dependent and discrete-delay-dependent stability criteria for time-varying-delay systems basing on the free weighting matrix method without requiring use of Park's inequality [17] or Moon's inequality [16].

Remark 2. Michiels and Vyhlidal [15] studied the stabilization of linear time delay systems of neutral type using an eigenvalue-based approach. Computing the radius of the essential spectrum of the solution operator of the neutral equation, they reduced the stabilization problem of the neutral equation to a problem involving only a finite number of characteristic roots. Stabilization is achieved by shifting the rightmost or unstable characteristic roots to the left half-plane in a quasi-continuous way. However, this eigenvalue-based approach is limited to systems with constant neutral and discrete delays. The stability criterion obtained in our paper is suitable for systems with time-varying neutral and discrete delays.

3. Numerical examples

To illustrate the usefulness of the proposed method, we present the following examples.

Example 1. Consider the time-delay system

$$\dot{x}(t) = Ax(t) + B_1x(t-d_1(t)) + B_2x(t-d_2(t)) + C_1\dot{x}(t-h_1(t)) + C_2\dot{x}(t-h_2(t))$$

where

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

and it is assumed that the time derivatives of delays $d_1(t)$, $d_2(t)$, $h_1(t)$ and $h_2(t)$ are bounded by $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\mu_3 = 0.3$ and $\mu_4 = 0.2$, respectively. Solving LMI (4) in Theorem 1 in our paper we obtain a solution with $d_1 = 0.5429$, $d_2 = 0.5429$, $h_1 = 0.5429$ and $h_2 = 0.5429$. The maximum allowable bound ρ guaranteeing asymptotic stability of this system is 0.5429.

Note that the criteria of Chen [1] and Hui and Hu [11] are not applicable to this system even were it to have time-invariant delay. Also, since the criterion of Park and Won [20] is not satisfied, their method cannot decide whether this system is stable or not. However, in [18], Park established delay-independent and delay-dependent criteria for asymptotic stability of the system in Eq. (1), and used the former criterion to prove the system in this example is asymptotically stable, i.e. it has a maximum allowable delay bound ρ of $+\infty$. He did not investigate the asymptotic stability of this system using his delay-dependent asymptotic stability criterion.

Our delay-dependent stability criterion can also be used to investigate the asymptotic stability of this system. Generally speaking, delay-independent criteria are more conservative than delay-dependent criteria when the delay is small. However, it should be noted that the maximum allowable delay bound ρ we give above for this system is more conservative than that obtained in [18] since the system is asymptotically stable for infinite delays $d_i(t) = h_i(t) = +\infty$, $i = 1, 2$. In fact, the sufficient asymptotic stability condition in our paper is more conservative than the delay-independent criterion in Theorem 1 in [18] when we analyze the stability of the above system with large delays. In such cases, some conservatism of our sufficient condition results from replacements of $d_i(t)$ by d_i (see, for example, the second part of Inequality (12)) and from replacements of $h_i(t)$ by h_i (see, for example, Inequality (13)) when we compute the time derivative of the Lyapunov functional V.

Remark 3. If we set $d_1(t) = 0.5\sin(0.6t)$, $d_2(t) = 0.4\sin(0.5t)$ which ensure that the time derivatives of delays $d_1(t)$ and $d_2(t)$ are bounded by $\mu_1 = 0.3$, $\mu_2 = 0.2$, respectively in Example 1, and if the time derivatives of delays $h_1(t)$ and $h_2(t)$ are bounded by $\mu_3 = 0.3$ and $\mu_4 = 0.2$,, respectively, iteratively solving the LMI (4) in Theorem 1 gives a maximum allowable upper bound of neutral delay for $h_1(t)$ of $h_1 = 0.3156$, and a maximum allowable upper bound h_2 for neutral delay $h_2(t)$ of more than 10,000. In fact, the actual maximum allowable upper bound for $h_2(t)$ is $+\infty$.

Example 2. Consider the following system

$$\dot{x}(t) = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t - d_1(t)) + \begin{bmatrix} 0.1 & -0.05 \\ 0.05 & 0.1 \end{bmatrix} x(t - d_2(t)) + \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix} \dot{x}(t - h_1(t)) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \dot{x}(t - h_2(t))$$

in which it is assumed that the time derivatives of delay $d_1(t)$, $d_2(t)$, $h_1(t)$ and $h_2(t)$ are bounded by $\mu_1 = 0.4$, $\mu_2 = 0.4$, $\mu_3 = 0.3$ and $\mu_4 = 0.3$, respectively. We can compute the maximum allowable bound ρ for guaranteeing asymptotic stability of the above system by iteratively solving LMI (4), giving $\rho = 4.98$. This implies the system is stable for time delays $d_1(t) \leq 4.98$, $d_2(t) \leq 4.98$, $h_1(t) \leq 4.98$ and $h_2(t) \leq 4.98$ with the given μ_i. However, the delay-dependent stability criterion in [18] provides a maximum allowable bound $\rho = 1.07$ for this system to be asymptotically stable. This example shows that the stability criterion in this paper gives much less conservative results than the one in Park [18].

Remark 4. If we set $d_1(t) = 5\ \sin(0.08t)$, $d_2(t) = 8\ \sin(0.05t)$ which ensure that the time derivatives of delays $d_1(t)$ and $d_2(t)$ are bounded by $\mu_1 = 0.4$, $\mu_2 = 0.4$, respectively, and if the upper bounds of the derivatives of delays $h_1(t)$ and $h_2(t)$ are $\mu_3 = 0.3$, $\mu_4 = 0.3$, respectively, in Example 2, iteratively solving the LMI (4) in Theorem 2 gives a maximum allowable upper bound of neutral delay for $h_1(t)$ of $h_1 = 4.98$, and a maximum allowable upper bound h_2 for neutral delay $h_2(t)$ is more than 10,000. Again, the actual maximum allowable upper bound for $h_2(t)$ is $+\infty$.

Remark 5. In Example 2, when the time derivatives of delay $d_1(t)$, $d_2(t)$, $h_1(t)$ and $h_2(t)$ are bounded by $\mu_1 = 0.4$, $\mu_2 = 0.4$, $\mu_3 = 0.3$ and $\mu_4 = 0.3$, respectively, Park [18] obtained the maximum allowable bound $\rho = 1.07$ such that this system is asymptotically stable. However, the maximum allowable bound ρ for guaranteeing asymptotic stability of the above system using our Theorem 1 is $\rho = 4.98$. Let us consider the influence of the free weighting matrices in Theorem 1 for this particular case, i.e. $\mu_1 = 0.4$, $\mu_2 = 0.4$, $\mu_3 = 0.3$ and $\mu_4 = 0.3$. If we set $Y_1 = Y_2 = Y_3 = Y_4 = Y_5 = Y_6 = Y_7 = L_1 = L_2 = L_3 = L_4 = L_5 = L_6 = L_7 = N_1 = N_2 = N_3 = N_4 = N_5 = N_6 = N_7 = T_6 = 0$ we obtain a solution to Inequality (4) giving a maximum allowable bound $\rho = 4.98$. The free weighting matrices T_1, T_3, T_4, T_5, T_7 lead to the improvement over the result in Park [18]. In fact,

- If $T_7 = 0$ in Inequality (4), then the maximum allowable bound ρ guaranteeing the asymptotic stability of the system given in Example 2 decreases from $\rho = 4.98$ to 4.74.
- If $T_7 = T_3 = 0$ in Inequality (4), then the maximum allowable bound $\rho = 4.37$.
- If $T_7 = T_3 = T_5 = 0$ in Inequality (4), then the maximum allowable bound $\rho = 3.51$.
- If $T_7 = T_3 = T_5 = T_4 = 0$ in Inequality (4), then the maximum allowable bound $\rho = 2.36$.
- If $T_1 = 0$ in Inequality (4), then the maximum allowable bound decreases from $\rho = 4.98$ to 3.57.

Thus, the free weighting matrices T_1, T_3, T_4, T_5 and T_7 in Theorem 1 clearly contribute to the improvement in the maximum allowable bound ρ for guaranteeing asymptotic stability of the system given in Example 2.

4. Conclusion

The stability of a class of linear neutral systems with mixed multiple time-varying delay arguments has been investigated. New stability criteria have been obtained which are applicable to linear neutral systems. Numerical examples have shown that the results derived using this new criterion significantly improve the stability bounds compared to existing results in the literature.

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