ADAPTIVE MULTIUSER DETECTION FOR FREQUENCY SELECTIVE CHANNEL USING IMM

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Abstract - In this paper, an adaptive multiuser detector based on interacting multiple model (IMM) is introduced to estimate the transmitted sequence corrupted by MAI (multiple access interference), multipath fading and noise. The proposed algorithm presents a novel multipath combining scheme based on a multiple model concepts and used in frequency selective Raleigh fading channels. The performance of the IMM multiuser detector is studied and compared with the adaptive Per-survivor detector of [1]

Keywords - CDMA, Interacting multiple models, Multipath diversity.

I. Introduction

In a CDMA (code division multiple access) system, several users share a common channel by modulating antipodally a set of signature waveforms. The received signal consists of a superposition of signals from all users plus additive noise. The optimum multiuser detector was obtained in [2] where the near-far problem was eliminated by a more complex receiver. This receiver is impossible to implement except in simple cases. The complexity of this receiver, which is exponential in the number of users, has motivated the development of number of detectors. The decorrelating detector [3] is a simple receiver which easily outperforms the conventional receiver and does not require the estimation of user powers. But the inversion performed by the detector enhances the system noise, deteriorating the system performance. Another linear receiver is the minimum mean square error equalizer [4] which retains some MAI (multiple access interference) in exchange for more noise reduction.

There are many nonlinear techniques for multiuser detection. In decision-feedback (DF) detectors [5], all symbols except the symbol of interest would have been estimated and the effect of the future symbols are eliminated by triangular matrix factorization. [6], [7], [8] use neural networks for multiuser detection in CDMA systems.

In many CDMA communications systems, such as cellular mobile radio systems, indoor wireless communication systems and aeronautical radio channels, channels exhibit severe multipath fading, which limits the system performance. Many recent papers on multiuser detection have addressed the multiple-access fading channels. The conventional strategy to fading channel problems is to design an adaptive equalizer by using the recursive least-squares (RLS) algorithm [9]. Another approach is that the adaptation of the algorithm is performed indirectly via a channel estimator [1], [10]. A joint multiuser detection/channel is presented in papers [1], [11]. In [1], the MAI is eliminated by a decorrelating detector and a Kalman filter is used to estimate the channel. The fading channels are directly estimated in [11] and thus eliminating the full rank condition in the decorrelating based receivers. Our proposed method is also based on a decorrelating detector followed by a whitening filter to obtain a white noise model as in [5]. The proposed method is related to the adaptive Per-survivor of [1] but does not need Viterbi algorithm to estimate the transmitted symbol sequence. We are using a multiple model approach based on IMM (Interacting Multiple Model) which unlike the Viterbi algorithm does not incur a decision delay and computationally quite simpler.

The paper is organized as follows: In Section 2 we define our signal model and formulate the problem. In Section 3, our IMM based adaptive multiuser detector is presented. Simulation studies are presented in Section 4 and some conclusions are drawn in Section 5.

II. Problem Formulation

Consider a DS/CDMA mobile radio network with K users, employing normalized spreading waveforms $s_1,\ s_2,\cdots s_K$ and transmitting sequences of binary symbols through their respective multipath channels. Each signature waveform is restricted to a symbol duration T with normalized energy $\int_0^T s_k^2(t)dt = 1$ and input symbols of each user takes on independent antipodal binary values ($b_k(n) \in \mathcal{B} \equiv \{+1,-1\}$) with equal probability. The received signal in a frequency selective Raleigh fading channel can be modeled as [1],

$$r(t) = \sum_{k=0}^{K} e^{j\phi_k} A_k \sum_{n=0}^{M-1} b_k(n) \sum_{l=1}^{L} c_{kl}(t) \, s_k(t - nT - \frac{l-1}{W}) + v(t) \quad (1)$$

where $v(t)$ is a zero mean white Gaussian noise power spectral density $\sigma^2/2$; M is the number of data bits transmitted and A_k and $e^{j\phi_k}$ are respectively the amplitude and carrier phase of the k-th user's signal. The number of resolvable paths are $L = T_m W$, where T_m is the channel multipath spread and $c_{kl}(t)$ is the channel gain for the lth path for the user k; W is the spread-spectrum bandwidth.

It is assumed that the symbol interval T is smaller than the channel coherence time, such that the fading attenuation can be assumed to be constant for the duration of at least one of symbol interval. It is also assumed that KL channel fading processes $\{c_{kl}(n)\}$ are statistically independent of each other. The time-varying frequency selective channel for each user is modeled as a tapped delay line with tap spacing $1/W$. Assuming that the symbol duration is much longer than the multipath delay spread, i.e. $T >> T_m \approx L/W$, any intersymbol interference (ISI) due to channel dispersion can be neglected [12]. Therefore, (1) can be reduced to a model which can be viewed as equivalent to a synchronous mulitiuser system with KL users:

$$r(t) = \sum_{k=0}^{K} e^{j\phi_k} A_k \sum_{n=0}^{M-1} b_k(n) \sum_{l=1}^{L} c_{kl}(t) s_{kl}(t - nT) + v(t) \quad (2)$$

where $s_{kl}(t) \triangleq s_k(t - \frac{l-1}{W})$.

The received signal is passed through a matched filter bank whose $[(k-1)L + l]$th output component is given by,

$$[\mathbf{z}(n)]_{(k-1)L+l} \triangleq z_{kl}(n) = \int_{nT}^{(n+1)T} r(t) s_{kl}(t - nT) dt$$
$$k = 1, ..., K,\ l = 1, ..., L,\ n = 0, ..., M-1 \quad (3)$$

Note that for the synchronous AWGN channel, $\mathbf{z}(n)$ is a sufficient statistic for detecting the nth bits of the K users.

From (2) and (3), we can write the (KL) vector output, $\mathbf{z}(n)$ as

$$\mathbf{z}(n) = \mathbf{R}\Phi\mathbf{A}\mathbf{B}(n)\mathbf{c}(n) + \mathbf{v}(n) \quad (4)$$

where $\mathbf{v}(n)$ is a $(KL \times 1)$ Gaussian noise vector with $(KL \times KL)$ autocorrelation matrix $\sigma^2\mathbf{R}$ and $\mathbf{R}$ is the $(KL \times KL)$ normalized cross-correlation matrix,
$[\mathbf{R}]_{(k'-1)L+l',(k-1)L+l} = \rho_{(k',l')(k,l)}$;

$$\rho_{(k',l')(k,l)} \triangleq \int_0^T s_{k'l'}(t) s_{kl}(t) dt \quad (5)$$

$$\mathbf{A} \triangleq \mathrm{diag}(A_1\mathbf{I}, \cdots, A_K\mathbf{I}) \quad (6)$$

$$\mathbf{B}(n) \triangleq \mathrm{diag}(b_1(n)\mathbf{I}, \cdots, b_K(n)\mathbf{I}) \quad (7)$$

$$\mathbf{\Phi} \triangleq \mathrm{diag}(e^{j\phi_1}\mathbf{I}, \cdots, e^{j\phi_K}\mathbf{I}) \quad (8)$$

$\mathbf{I}$ is the $L \times L$ identity matrix; $\mathbf{c}$(i) is the (KL) vector of channel gain. We can obtain a white noise model by applying a filter with a response $(\mathbf{S}^T)^{-1}$ to the output of the matched filter bank (4), where $\mathbf{S}$ is a lower triangular matrix obtained by factorising $\mathbf{R}$ as $\mathbf{R} = \mathbf{S}^T\mathbf{S}$ [5]. This operation completely the eliminates the MAI from the user 1 and can remove MAI of other users if we can detect correctly the symbols of preceding users. (For example, symbols of all users $k' < k$ need to be known to remove the MAI for the user k).

$$\mathbf{y}(n) = \mathbf{S}\Phi\mathbf{A}\mathbf{B}(n)c(n) + \mathbf{n}(n) \quad (9)$$

where $\mathbf{y}(n) = \mathbf{S}^{-T}\mathbf{z}(n)$ and $\mathbf{n}(n)$ is a white Gaussian noise vector with the autocorrelation matrix, $\sigma^2\mathbf{I}$ ($\mathbf{I}$ is a $KL \times KL$ identity matrix).

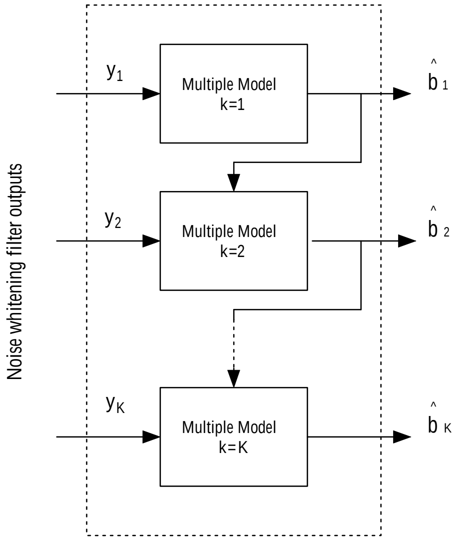

Fig. 1. Structure of the IMM based multiuser detector

III. IMM BASED ADAPTIVE MULTIUSER DETECTOR

In this section, we will show how the model posterior probablity $P\{m_j(n)|Y^n\}$ can be used detect the received symbol sequence in a fading environment. We denote the L-state vector of channel fading gain for user k as

$$\theta^k(n) = [c_{k1}(n), \cdots, c_{kL}(n)]^T \quad (10)$$

As shown in the figure 1, the proposed algorithm operates successively. It consists of K number of IMM models corresponding to each user. The first IMM gives the symbol and fading gain of the user 1. The estimated bit symbol $\hat{b}_1$ is fed into the 2nd IMM. Thus the IMM model corresponding to user k gets the symbol estimates of previous users ($< k$) from preceding IMM models. The following description applies to the kth IMM model corresponding to user k and the index k is suppressed for clarity. (e.g. $\theta(n) = \theta^k(n)$)

Assumption 1 The number of distinct symbols in the input alphabet, $\mathcal{B}$ of a transmission does not vary and is fixed; For binary transmission, this equals to two.

Assumption 2 The received signal model effective at time nT is caused by one of the symbols under Assumption 1.

Corresponding to number of probable models at a time, we can get a set of models within the hybrid model:

$$\zeta_j = \{F(j,n),\ G(j,n),\ H(j,n),\ w(j,n),\ v(j,n)\}$$
$$j = 1, \cdots, r \quad (11)$$

We can use a random walk model to model the unknown state variation (channel fading gain) as we are considering a slowly fading environment. Then we can get the following hybrid model

$$\begin{aligned} \theta^k(n+1) &= \theta^k(n) + w(n) \\ y_k(n) &= H(j,n)\theta^k(n) + v(n) \ for \ j = 1, \cdots, r \end{aligned}$$

where $w(n)$ is a disturbance noise, $y_k(n)$ is the L-vector component of $\mathbf{y}(n)$ corresponding to user k ($y_k(n) = \mathbf{y}(n)_{(k-1)L+1:kL}$) and $H(j,n)$ is the observation matrix corresponding to model j given by

$$H(j,n) = \overline{\mathbf{S\Phi A}}\mathbf{B}(n) = \tilde{\mathbf{S}}\mathbf{B}(\mathbf{n}) \tag{12}$$

where $\overline{\mathbf{S\Phi A}} = \tilde{\mathbf{S}}$ is the $L \times KL$ submatrix of $\mathbf{S\Phi A}$ corresponding to user k.

Remark The received symbol at time nT is given by the symbol corresponding to the model $P\{m(n)|Y^n\}$ (effective model given by the maximum of $P\{m(n)_j|Y^n\}$ $j = 1, \cdots, r$) and the channel fading is given by the state vector $(\hat{\theta}_k(n))$ of the IMM algorithm.

This Remark follows directly by taking the number of models within an IMM are equal to the number of symbols and then $P\{m(n)|Y^n\}$ is the best model effecitve at time nT. The symbol corresponding to the best model is the received symbol.

Above Remark can now be used to detect the symbol sequence by having r number of Kalman filters in parallel at a time and the effective model obtained using the IMM algorithm gives the received symbol at the time. We successfully used this idea of modeling the input bit sequence by a set of multiple models for joint symbol and channel equalization for single user systems [13].

Due to the lower triangular matrix $\tilde{\mathbf{S}}$ at the output of the whitening filter, the user signals are partially decorrelated. We can observe that the first user is free from MAI but corrupted by multipath fading from different paths. We first detect the symbol for the first user and then successively estimate for others. This results in the following $KL \times KL$ estimated symbol matrix $\tilde{\mathbf{B}}_j(n)$

$$\tilde{\mathbf{B}}_j(n) = \text{diag}(\hat{b}_1(n)\mathbf{I}, \cdots, \overline{b}_j\mathbf{I}, \mathbf{0}, \cdots, \mathbf{0}) \tag{13}$$

where $\mathbf{0}$ and $\mathbf{I}$ are $L \times L$ zero and identity matrices respectively. The $\overline{b}_j$ denotes the symbol corresponding to model j for the user k and chosen such that $\cup_{j=1}^r\{\overline{b}_j\} \equiv \mathcal{B}$ and $\cap_{j=1}^r\{\overline{b}_j\} = \phi$. But for user k' ($< k$), the already estimated bit symbol, $\hat{b}_{k'}$ is used. For other users, due to the lower triangular nature of $\tilde{\mathbf{S}}$, we do not need to know their bit symbols when detecting the kth user's bit symbol.

The IMM algorithm as used in this problem is outlined below. For a detailed discussion of IMM algorithm, readers can refer to [11]. The IMM based receiver is based on following simplifying assumptions. At any time $n-1$, the distribution of the state vector conditioned on all past observations (Y^{n-1}) and the most recent model state is Gaussian with known mean and variance denoted, respectively, by $\hat{\theta}^j(n-1|n-1)$ and $P^j(n-1|n-1)$. The conditional probabilities of the most recent model states are also known and denoted by $\mu_j(n-1)$.

We begin by using the transition probability matrices to calculate the predicted state probabilities

$$\begin{aligned} \mu_{i|j}(n-1|n-1) &= P(m_i(n-1)|m_j(n), Y^{n-1}) \\ &= \frac{1}{c}P(m_j(n)|m_i(n-1), Y^{y-1}) \\ &\quad .P(m_i(n-1)|Y^{n-1}) \\ &= \frac{1}{c}\pi_{ij}\mu_i(n-1) \quad i,j = 1,..,r \end{aligned} \tag{14}$$

where c is the normalizing constant and $\mu_i(n-1)$ is the posterior symbol probability at time $n-1$.

We then use these predicted model state probabilities to combine the state estimates and covariances

$$\begin{aligned} \hat{\theta}^{0j}(n-1|n-1) &= \sum_{i=1}^{r} \hat{\theta}^i(n-1|n-1) \\ &\quad .\mu_{i|j}(n-1|n-1) \ \forall \ j = 1,..,r \end{aligned} \tag{15}$$

$$\begin{aligned} &P^{0j}(n-1|n-1) \\ &= \sum_{i=1}^{r} \mu_{i|j}(n-1|n-1)\{P^i(n-1|n-1) \\ &\quad +(\hat{\theta}^i(n-1|n-1) - \hat{\theta}^{0j}(n-1|n-1)) \\ &\quad (\hat{\theta}^i(n-1|n-1) - \hat{\theta}^{0j}(n-1|n-1))'\} \\ &\qquad \forall \ j = 1,..,r \end{aligned} \tag{16}$$

where $\hat{\theta}^j(n|n)$ and $P^i(n|n)$ are the state estimate and covariance of the state estimate at time n. Based only on the transition probabilities, (14)-(16) merge the state estimates and model probabilities and generate a set of r mean/variance pairs. The IMM algorithm makes the simplifying assumption that each pair corresponds to a Gaussian density. This assumption, along with the merging strategy, forms the foundation of the IMM algorithm. Each pair is then used as a prior statistics for $r = 2$ Kalman filters to obtain the $\hat{\theta}^j(n|n)$ and $P^j(n|n)$. These are the outputs of the Kalman filter and the Kalman filter equations can be found in [14]. The likelihood corresponding to two filters,

$$\begin{aligned} \Lambda_j(n) &= p(y_k(n)|m_j(n), \\ &\quad \hat{\theta}^{0j}(n-1|n-1), P^{0j}(n-1|n-1)) \end{aligned} \tag{17}$$

are also computed. For Gaussian noise this reduces to

$$\Lambda_j(n) \quad \propto \quad \exp(-0.5\, e(n)^T R_e(n)^{-1} e(n)) \tag{18}$$

where $e(n)$ and $R_e(n)$ are respectively the L-innovation vector and innovation covariance given by the Kalman filter.

The final step is to complete the propagation of model probabilities using

$$\begin{aligned} \mu_j(n) &= P(m_j(n)|Y^n) \quad (19) \\ &= \frac{1}{\bar{c}}\Lambda_j(n)\sum_{i=1}^{r}\pi_{ij}\mu_i(n-1) \\ &\quad \forall\ j = 1,..,r \end{aligned}$$

where $\overline{c}$ is the normalization constant. The mode at time n can now be estimated as

$$m(n) = \arg\max_j \mu_j(n) \quad (20)$$

Thus the symbol at time n is the symbol corresponding to mode $m(n)$ and the following mixture equation gives the estimated channel fading gain, $\hat{\theta}(k)$:

$$\hat{\theta}(k) = \sum_{j=1}^{r} \hat{\theta}^j(k|k)\mu_j(k) \quad (21)$$

Above algorithm gives symbols and channel fading gains of all users after repeating it for $k = 1, \cdots, K$. Note that at each stage of the algorithm we obtain fading gain estimates of all users. But only the fading gain estimate corresponding to the current IMM filter stage is correct. For example, at the IMM filter stage k, we can only accurately estimate the fading gain of user k. The reason for this is clear from the figure 1, where the kth filter is conditoned only on y_i $(i \leq k)$ and past observations.

IV. Simulation Results

A synchronous three-user ($K = 3$) system with processing gain $N = 31$ is employed in the simulation. All users are assumed to have the same signal power. The number of resolvable paths is $L = 3$. The signature waveforms corresponding to the second and third paths of the original signal of a user is the signature waveform of that user shifted by one and two chips respectively.

The Raleigh fading channel process $\{c_{kl}(n)\}$ is simulated by filtering two independent real iid Gaussian processes with two identical 3rd-order Butterworth filters. The 3-dB fading bandwidth f_D normalized to the symbol rate $1/T$ is used as a measure of fading rate. The symbol rate $1/T$ is set at 10 Kb/s. The fading rate $f_D T = 0.05$ is considered in our simulations corresponding to a Doppler shift of 500 Hz.

The figure 2 shows the tracking capability of the IMM based filter. It seen that the IMM filter is capable of closely tracking the channel variations. For comparision, we also simulated the adaptive per-survivor detector (PSP) proposed in [1]. Simulations for both methods are performed alternatively on training and decision-directed modes. The length of training subsequence was 10 and the length of the symbol subsequence was also 10. Spacing of the training subsequence was needed to be denser in order to obtain an acceptable performance. The transmitted sequence was an iid binary sequence with a transition probability matrix, π_{ij} is a 2×2 matrix with all elements being $.5$. The simulation parameters of the adaptive per-survivor detector are as given in [1] (Number of states in the trellis was 8). To obtain average values of BER, 10000 symbols were repeated 10 times for Monte Carlo simulations. The bit-error rate (BER) performance of the two methods are plotted in figure 3. As seen from the figure, the proposed algorithm compares well with the adaptive per-survivor receiver. But the proposed algorithm requires only r filters compared to r^Q (Q is the memory length of the channel) filters required for the adaptive PSP. For our case, adaptive PSP required eight filters whereas the proposed method requires only two irrespective of the channel length. And the proposed method does not incur decision delays that are inherent in all Viterbi based methods and thus storage requirements are quite minimal.

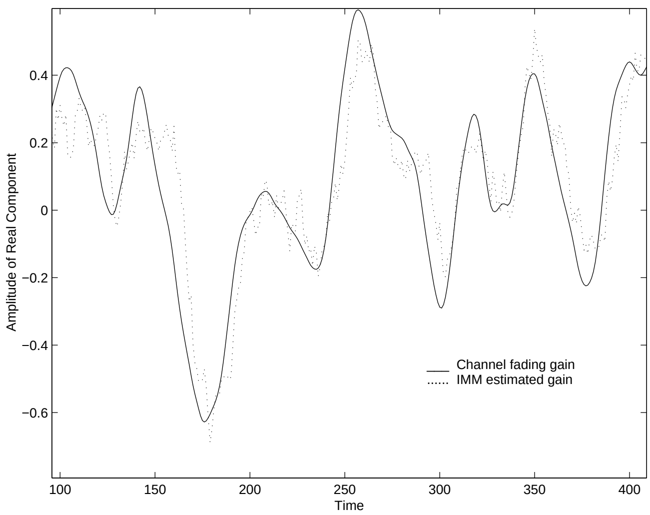

Fig. 2. A snap-shot of one output sequence of the IMM estimated channel gain, corresponding to one fading channel and with the true channel fading process (SNR=15dB)

V. Conclusions

In this paper, we considered the problem of joint multiuser detection and channel estimation in a Raleigh fading channel. We obtained our receiver by modeling the received bit sequence as a sequence of models due to each bit symbol. While the effective model at a time instant corresponds to the received bit symbol and states of the hybrid model tracks the fading channel gain. Computer simulations were carried out to demonstrate the merits of the proposed IMM based multiuser detector. They showed similar BER performance compared to the adaptive per-survivor receiver presented in [1] but with a lower computationl cost.

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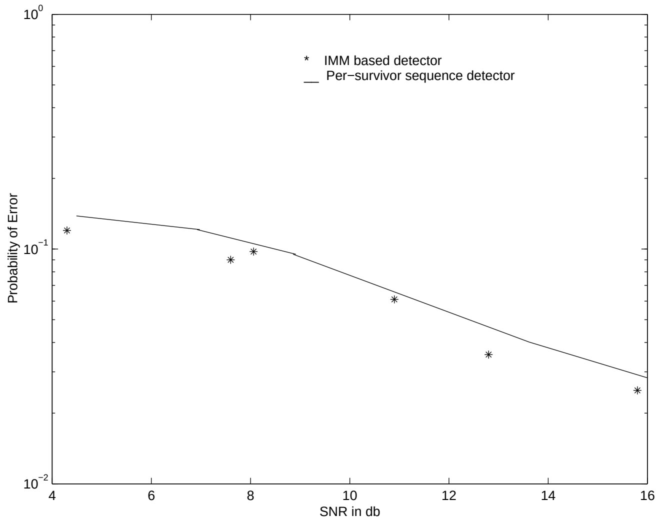

Fig. 3. Simulated BER performance of the IMM based multiuser detector and adaptive PSP for $f_D T = 0.05$

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