Chapter 4 Frequency Estimation

4.1 Historical overview of frequency measurement

Power system frequency measurement has been in use since the advent of alternating current generators and systems. The speed of rotation of generator rotors is directly related to the frequency of the voltages they generate. The Watt-type fly-ball governor of steam turbines (Figure 4.1) is essentially a frequency measuring device which is used in a feedback control system to keep the machine speed within a limited range around the nominal value. However, this measurement is available only at the generating stations, and there is need for measuring frequency of power system at network buses away from the generating stations.

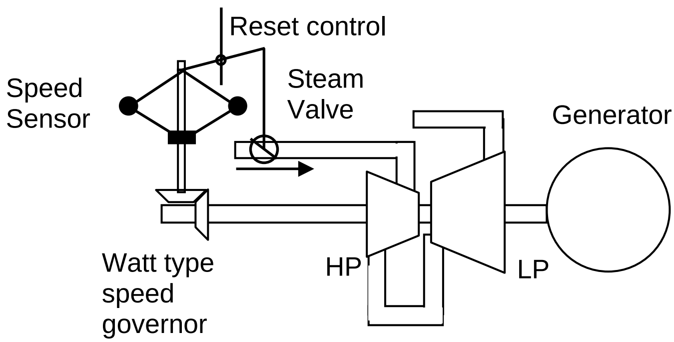

Fig. 4.1 Mechanical speed sensing used in a Watt-type speed governor of a steam turbine.

The earliest frequency measurement for power frequency voltages was performed by mechanical devices which employed mechanical resonators (similar to tuning forks) tuned to a range of frequencies around the nominal power frequency [1]. Such a frequency meter of mid-1950s vintage is shown in Figure 4.2 a. Another frequency measuring instrument of about the same period is a resonance-type device, whereby tuned resonant cir-

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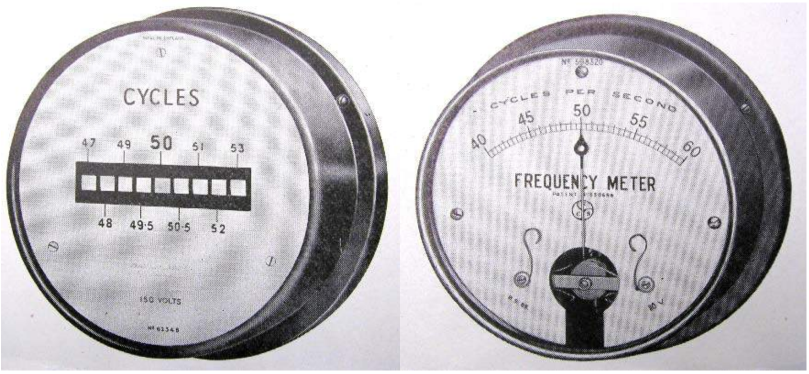
cuits at different frequencies are energized by the secondary voltage obtained from a voltage transformer, and the circuit which is in resonance provides the frequency measurement (Figure 4.2b) [1]. Typical resolution of these meters was of the order of 0.25 Hz.



(a) (b)

Fig. 4.2 (**a**) A mechanical resonance-type frequency meter. (**b**) An electrical resonance-type frequency meter. These instruments are for a 50-Hz power system.

The next advance in frequency measurement came with the introduction of precise time measurement techniques. By measuring the time interval between consecutive zero crossings of the voltage waveform the frequency of the voltage could be determined. Clearly the accuracy of such a measurement depends upon the precision of time measurement, as well as on the accuracy with which the zero crossing of the waveform could be determined. This latter measurement is affected by the presence of noise in the measurement, varying harmonic frequencies and levels, and the performance of the zero-crossing detector circuits.

Synchronized phasor measurements offer an opportunity for measuring power system frequency which eliminates many of these error sources. It should be noted that the frequency measurement on a power system is primarily dedicated to estimating rotor speed(s) of connected generators. As such, the positive-sequence voltage measurement is an ideal vehicle for frequency measurement. In addition, phasors reflect the fundamental frequency components of the voltages, and harmonics do not affect frequency measurement based upon phasors. Techniques for measuring frequency from phasors are described in the following sections.

4.2 Frequency estimates from balanced three-phase inputs

Frequency and rate of change of frequency can be estimated from the phase angles of phasor estimates [2]. It was pointed out in Chapter 3 that positive-sequence phasor estimated from balanced inputs at off-nominal frequencies has a minor attenuation in phasor magnitude, and both the magnitude and phase angle estimates are free from a ripple of approximate second harmonic. Setting the negative-sequence component of the input $X_2 = 0$ in Eq. (3.22) the estimate of the positive-sequence voltage is given by

$$X'_{r1} = PX_1\varepsilon^{jr(\omega-\omega_0)\Delta t}\,. \tag{4.1}$$

The magnitude of P is the attenuation factor, and phase angle of P is a constant offset in the measured phase angles. The angle of the phasor X'_{r1} advances at each sample time by $(\omega - \omega_0)\Delta t$, where ω is the signal frequency, ω_0 is the nominal system frequency, and Δt is the sampling interval.

It should be clear from Eq. (4.1) that the first and second derivatives of phase angle of the phasor estimate would provide an estimate of $\Delta\omega = (\omega - \omega_0)$, and the rate of change of frequency. Since there are errors of estimation in phasor calculation, it is desirable to use a weighted least-squares approach over a reasonable data window for calculating the derivatives of the phase angle.

Assume that the positive-sequence phasors are estimated over one period of the nominal frequency, and that the phasors calculated with several consecutive data windows over a span of 3–6 cycles are used for frequency and rate of change of frequency estimation.

Let $[\phi_k]$ $\{k = 0,1,\ldots,N-1\}$ be the vector of "N" samples of the phase angles of the positive-sequence measurement. The vector $[\phi_k]$ is assumed to be monotonically changing over the window of "N" samples. As the phase angles of the phasor estimate may be restricted to a range of 0–2π, it may be necessary to adjust the angles to make them monotonic over the entire spanning period by correcting any offsets of 2π radians which may exist. This is illustrated in Figure 4.3.

If the frequency deviation from the nominal value, and the rate of change of frequency at $t = 0$ are $\Delta\omega$ and ω', respectively, the frequency at any time "t" is given by

$$\omega(t) = (\omega_0 + \Delta\omega + t\omega'). \tag{4.2}$$

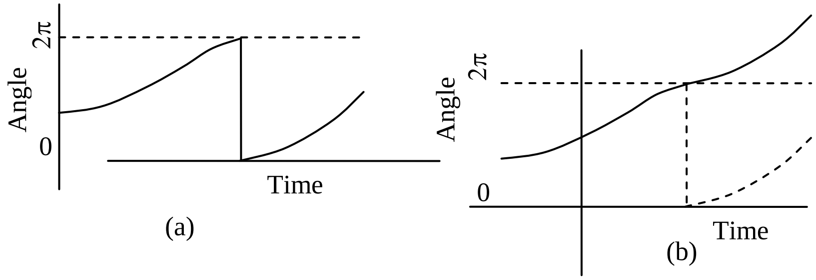


Fig. 4.3 (**a**) Phasor estimates produce angles which are restricted to a range of 0–2π. (**b**) For estimating frequency and rate of change of frequency, the offset of 2π in the phase angle estimates are removed.

The phase angle is the integral of the frequency:

$$\phi(t) = \int \omega dt = \int (\omega_0 + t\Delta\omega + t\omega')dt = \phi_0 + t\omega_0 + t\Delta\omega + \frac{1}{2}t^2\omega', \quad (4.3)$$

ϕ_0 being the initial value of the angle. Assuming that the recursive algorithm is used for estimating the phasors, the term $t\omega_0$ is suppressed from the estimated phase angles (see Section 2.2.2). Thus, the phase angle as a function of time becomes

$$\phi(t) = \phi_0 + t\Delta\omega + \frac{1}{2}t^2\omega'. \quad (4.4)$$

If $\phi(t)$ is assumed to be a second degree polynomial of time:

$$\phi(t) = a_0 + a_1 t + a_2 t^2 \quad (4.5)$$

it follows that at $t = 0$,

$$\Delta\omega = a_1, \quad \omega' = 2a_2, \quad (4.6)$$

or, in terms of Hz and Hz s^{-1},

$$\Delta f_0 = a_1/(2\pi) \text{ and } f' = a_2/(\pi). \quad (4.7)$$

The vector of "*N*" angle measurements is given by

$$\begin{bmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \Delta t & \Delta t^2 \\ 1 & 2\Delta t & 2^2 \Delta t^2 \\ \vdots & \vdots & \vdots \\ 1 & (N-1)\Delta t & (N-1)^2 \Delta t^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}. \tag{4.8}$$

In matrix notation

$$[\boldsymbol{\phi}]=[\mathbf{B}][\mathbf{A}], \tag{4.9}$$

where **[B]** is the coefficient matrix in Eq. (4.8). The unknown vector **[A]** is calculated by the weighted least-squares (WLS) technique:

$$[\mathbf{A}] = [\mathbf{B}^{\mathbf{T}}\mathbf{B}]^{-1}\mathbf{B}^{\mathbf{T}}\ [\boldsymbol{\phi}] = [\mathbf{G}]\ [\boldsymbol{\phi}], \tag{4.10}$$

where

$$[\mathbf{G}] = [\mathbf{B}^{\mathbf{T}}\mathbf{B}]^{-1}\mathbf{B}^{\mathbf{T}} \tag{4.11}$$

The matrix **[G]** is pre-calculated and stored for use in real time. It has "*N*" rows and three columns. In real time, **[G]** is multiplied by **[ϕ]** to obtain the vector **[A],** and from that the frequency and rate of change of frequency at any time t (which is a multiple of Δt) can be calculated. This time is usually associated with the time tag for which the measurement is posted.

Example 4.1
Numerical example of frequency and rate of change of frequency estimation.

Consider an input with a frequency of 60.5 Hz and a rate of change of frequency of 1 Hz s^{-1}. The polynomial for phase angles is given by

$$\phi(t) = \phi_0 + 2\pi\times0.5\times t + (1/2)t^2\times1\times2\pi.$$

The initial angle ϕ_0 is assumed to be 0.1 radian. Assuming that the phasors are calculated at a sampling rate of 24 samples per cycle of the nominal power system frequency, the time step is Δt = (1/1440) second. Phase angles over a span of four cycles are tabulated below with and without a Gaussian random noise with zero mean and a standard deviation of 0.01 radian (Table 4.1).

Table 4.1 Partial list of 96 phase angle samples with and without noise

Sample no.	Phase angles without noise	Phase angles with noise
1	0.1000	0.1007
2	0.1022	0.1017
3	0.1044	0.1062
4	0.1066	0.1077
5	0.1088	0.1075
6	0.1109	0.1103
…	…	…
88	0.3013	0.3015
89	0.3037	0.3033
90	0.3062	0.3056
91	0.3086	0.3087
92	0.3111	0.3123
93	0.3135	0.3110
94	0.3160	0.3166
95	0.3185	0.3175
96	0.3209	0.3219

The estimated frequency and rate of change of frequency using the weighted least-squares formulation is found to be Δf = 0.5000 Hz and $f'(0)$ = 1.0000 Hz s^{-1} with no noise in the phase angle measurement, and Δf = 0.4968 Hz and $f'(0)$ = 1.0550 Hz s^{-1} with the noise.

The estimates of Δf and f' for different amounts of random noise in phase angle measurements are shown in Table 4.2.

Table 4.2 Effect of random noise on frequency and rate of change of frequency estimation

σ of random noise (radians)	Mean of frequency estimate (Hz)	σ of frequency estimate(Hz)	Mean of rate of change of frequency estimate(Hz s^{-1})	σ of rate of change of frequency estimate (Hz)
0.0001	0.5000	0.0003	1.0000	0.0096
0.0005	0.5001	0.0017	0.9979	0.0498
0.0010	0.5000	0.0033	0.9986	0.0982
0.0050	0.5000	0.0167	1.0038	0.4888
0.0100	0.4998	0.0331	1.0033	0.9660

The results in Table 4.2 are for 1000 Monte Carlo trials with the specified standard deviation of the noise. It is clear that the rate of change of frequency is more sensitive to the amount of noise in the input. Also, the estimates are essentially zero-mean processes.

4.3 Frequency estimates from unbalanced inputs

The effect of unbalance in input signals has been analyzed in Section 3.5. Equation (3.22) provides the formula for the estimate of the positive-sequence component when there is a negative-sequence component present in the input signal:

$$X'_{r1} = PX_1\varepsilon^{jr(\omega-\omega_0)\Delta t} + QX_2^*\varepsilon^{-jr(\omega+\omega_0)\Delta t}, \tag{4.12}$$

where Q is given by Eq. (3.11), and X_2 is the negative-sequence component in the input signals. The effect of the second term of Eq. (4.12) is to produce a ripple in the angle estimate of the positive-sequence component. This ripple can be eliminated by one of the filtering techniques described in Section 3.3. When the ripple in angle is eliminated, the frequency and rate of change of frequency can be estimated as in Section 4.2. The error performance of the estimates is then identical to that corresponding to balanced input signals.

4.4 Nonlinear frequency estimators

It is possible to formulate the frequency and rate of change of frequency estimation problem as a nonlinear estimation problem from the input signal waveform [3, 4]. Consider a single-phase input having a frequency deviation of $\Delta\omega$ and a rate of change of frequency ω' (as in Eq. 4.4):

$$x(t) = X\cos\left\{\phi_0 + t\Delta\omega + \frac{1}{2}t^2\omega'\right\}. \tag{4.13}$$

"N" samples of this signal at a sampling interval of Δt are $\{x_k, k = 0,1,\ldots,N - 1\}$. It is assumed that there are four unknowns in the data samples:

$$z = \begin{bmatrix} X \\ \phi_0 \\ \Delta\omega \\ \omega' \end{bmatrix}. \tag{4.14}$$

The function $x(t)$ is a nonlinear function of the four unknowns, and if "N" is greater than four, a nonlinear weighted least-squares iterative technique can be used to solve for the four unknowns.

Assuming reasonable initial values of the four unknowns as $[z_0]$, the initial estimates of the function $x(t)$ are $[x_0]$. Using first-order terms of Taylor's series to represent the nonlinear function around $[z_0]$

$$[x - x_0] = \begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial x}{\partial \phi_0} & \frac{\partial x}{\partial \Delta\omega} & \frac{\partial x}{\partial \omega'} \end{bmatrix}_{z=z_0} [\Delta z].,\tag{4.15}$$

where the partial derivatives are columns of "N" rows evaluated at the assumed value of the unknown vector $[z_0]$. Representing the matrix of partial derivatives by the Jacobian matrix $[J]$, the weighted least-squares solution for $[\Delta z]$ is

$$[\Delta z] = [J^T J]^{-1} J^T \,[x - x_0].\tag{4.16}$$

The four partial derivatives in Eq. (4.15) are obtained by differentiating the expression for $x(t)$:

$$\begin{aligned}
J_1 &= \frac{\partial x}{\partial X} = \cos(\phi_0 + t\Delta\omega + \frac{1}{2}t^2\omega'),\\
J_2 &= \frac{\partial x}{\partial \phi_0} = -X\sin(\phi_0 + t\Delta\omega + \frac{1}{2}t^2\omega'),\\
J_3 &= \frac{\partial x}{\partial \Delta\omega} = -Xt\sin(\phi_0 + t\Delta\omega + \frac{1}{2}t^2\omega'),\\
J_4 &= \frac{\partial x}{\partial \omega'} = -X\,\frac{t^2}{2}\sin(\phi_0 + t\Delta\omega + \frac{1}{2}t^2\omega').
\end{aligned}\tag{4.17}$$

Having calculated the corrections $[\Delta z]$ in Eq. (4.16), they are added to $[z_0]$ to produce the answer at the end of first iteration. The process is repeated until the residual $[x - x_0]$ becomes smaller than a suitable tolerance.

Example 4.2 Numerical example of nonlinear frequency and rate of change of frequency estimation.

Consider a single-phase input with an amplitude of 1.1, a frequency 60.5 Hz at $t = 0$, a rate of change of frequency of 1 Hz s^{-1}, and a phase angle ϕ_0 of $\pi/8$. The initial values for starting the iteration are assumed to be

$$\begin{aligned}
X &= 1.0,\\
\phi_0 &= 0,\\
\Delta\omega &= 0,\\
\omega' &= 0.
\end{aligned}$$

The sampling rate is assumed to be 1440 Hz, and 96 samples of the input signal are used to estimate the signal parameters.

Table 4.3 lists first 10 values of the input signal, the estimated signal with the vector [z_0], and the first 10 entries of the Jacobian matrix.

Table 4.3 First 10 values of [x], [x_0], and [J] at the beginning of the iteration

	[x]	[x_0]	[J_1]	[J_2]	[J_3]	[J_4] × 10^4
1	1.0163	1.0000	1.0000	0	0	0
2	0.8712	0.9659	0.9659	–0.2588	–0.0002	–0.0006
3	0.6658	0.8660	0.8660	–0.5000	–0.0007	–0.0048
4	0.4143	0.7071	0.7071	–0.7071	–0.0015	–0.0153
5	0.1340	0.5000	0.5000	–0.8660	–0.0024	–0.0334
6	–0.1555	0.2588	0.2588	–0.9659	–0.0034	–0.0582
7	–0.4343	0.0000	0.0000	–1.0000	–0.0042	–0.0868
8	–0.6830	–0.2588	–0.2588	–0.9659	–0.0047	–0.1141
9	–0.8843	–0.5000	–0.5000	–0.8660	–0.0048	–0.1336
10	–1.0244	–0.7071	–0.7071	–0.7071	–0.0044	–0.1381

The corrections vector at the end of first iteration are

$$\begin{aligned} \Delta(X) &= -0.0365, \\ \Delta(\phi_0) &= 0.4072, \\ \Delta(\Delta\omega) &= 4.2767, \\ \Delta(\omega') &= -32.2073. \end{aligned}$$

At the end of four iterations correct values for the unknowns are obtained.

It is necessary to consider the effect of noise in the sampled data on the performance of the nonlinear frequency estimator. Zero-mean normally distributed random noise was added to the sampled data of the above example, and the effect on the results obtained after five iterations evaluated. Thousand Monte Carlo trials produce the result shown in Table 4.4. It can be seen that the mean of the parameter estimation is very close to the true value, although the standard deviation of the estimation increases very rapidly as the size of the noise added increases. The rate of change of frequency is practically unusable when the noise exceeds 1% of the signal peak value. The amplitude and phase angle estimates are quite good even for very large sample errors.

Table 4.4 Effect of sample noise on estimation of signal parameters

σ sample noise	Mean X	Mean ϕ_0	Mean $\Delta\omega$	Mean ω'	σX	$\sigma\phi_0$	$\sigma\Delta\omega$	$\sigma\omega'$
0.0	1.1000	0.3927	0.5000	1.0000	0	0	0	0
0.01	1.1000	0.3929	0.4982	1.0511	0.0014	0.0037	0.0430	1.3061
0.02	1.1000	0.3924	0.5011	0.9927	0.0028	0.0076	0.0878	2.6582
0.03	1.1001	0.3923	0.5028	0.8986	0.0043	0.0112	0.1323	4.0170
0.04	1.1001	0.3926	0.4992	1.0135	0.0056	0.0144	0.1620	4.9061
0.05	1.1001	0.3928	0.5045	0.8704	0.0072	0.0187	0.2153	6.4638
0.06	1.1007	0.3930	0.5068	0.7559	0.0086	0.0223	0.2543	7.7155
0.07	1.1004	0.3930	0.5028	0.8911	0.0099	0.0269	0.3029	9.0593
0.08	1.1010	0.3910	0.5185	0.4604	0.0119	0.0293	0.3424	10.4239
0.09	1.1003	0.3942	0.4814	1.6546	0.0129	0.0337	0.3877	11.7737
0.10	1.1006	0.3943	0.4957	1.0495	0.0143	0.0376	0.4451	13.5048

The signals are the same as for Example 4.1.

4.5 Other Techniques for frequency measurements

A number of other techniques for measuring power system frequency have been published in the technical literature [5, 6, 7, 8]. These references are provided for the interested reader as a sample of what is available, and is by no means a complete listing of papers dealing with frequency measurement. In general, the faster approaches (measurements made within one or two periods of the power frequency signal) tend to have greater errors than those using longer data windows. It is better to keep in mind that a traditional use of frequency measurement is in under-frequency load-shedding. Relays used for that purpose tend to have operating times of the order of 5–6 cycles of the nominal power frequency. This is probably a good size for a data window to be used in frequency estimation.

One should not have excessively long data windows for frequency estimation in order to improve the accuracy of the estimate. During transient stability swings, the frequency of the power system may change rapidly. Thus, a long window may include significantly different frequencies over the window span, and once again the frequency estimation may be in error. We will consider the effect of changing frequency due to transients in Chapter 6.

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