

II. SLIDING MODE OBSERVER

A. Flux/Current Observer of PMSM

In sensorless position/speed control, the rotor position cannot be detected, and therefore d-q axis mathematical model cannot be applied directly. Most approaches are based on the estimation of the back electromotive force (EMF) in the stationary reference frame α - β . The proposed PMSM mathematical model of sliding mode observer is reflected in an estimated reference frame γ - δ rotating at an estimated angular velocity $\hat{\omega}$ and lagging behind the d-q reference frame by electrical angle error $\bar{\theta}$. Fig. 2 shows the relations between the synchronous reference model (dq-axis) and the estimated reference model ($\gamma\delta$ -axis) used in this study. The mathematical model of PMSM in dq-axis synchronous rotating reference frame is presented by the following flux/current state equations.

$$\dot{\lambda}_d = -r_s \cdot i_d + u_d + \omega \cdot \lambda_q \quad (1)$$

$$\dot{\lambda}_q = -r_s \cdot i_q + u_q - \omega \cdot \lambda_d \quad (2)$$

$$\lambda_d = L_d \cdot i_d + \lambda_m \quad (3)$$

$$\lambda_q = L_q \cdot i_q \quad (4)$$

The conventional d-q axis model can be transformed to γ - δ axis as follows (see Appendix A).

$$\dot{\lambda}_\gamma = -r_s i_\gamma + u_\gamma + (\omega - \dot{\bar{\theta}}) \lambda_\delta \quad (5)$$

$$\dot{\lambda}_\delta = -r_s i_\delta + u_\delta - (\omega - \dot{\bar{\theta}}) \lambda_\gamma \quad (6)$$

$$\lambda_\gamma = L_d \cdot i_\gamma + \lambda_{m\gamma} \quad (7)$$

$$\lambda_\delta = L_q \cdot i_\delta + \lambda_{m\delta} \quad (8)$$

The partial fluxes $\lambda_{m\gamma}$ and $\lambda_{m\delta}$ are functions of λ_m , $\bar{\theta}$ and L_2 , where $L_2 = (L_q - L_d)/2$. In the proposed sensorless scheme, this property of the partial fluxes is utilized to extract information of rotor speed and position error.

The flux/current state observer is expressed by the following equations using as sliding surfaces $s_\gamma = L_d(i_\gamma - \hat{i}_\gamma) = L_d \bar{i}_\gamma$ and $s_\delta = L_q(i_\delta - \hat{i}_\delta) = L_q \bar{i}_\delta$.

$$\dot{\hat{\lambda}}_\gamma = -\hat{r}_s i_\gamma + u_\gamma + (\omega - \dot{\bar{\theta}}) \hat{\lambda}_\delta + K_\gamma \text{sgn}(L_d \bar{i}_\gamma) \quad (9)$$

$$\dot{\hat{\lambda}}_\delta = -\hat{r}_s i_\delta + u_\delta - (\omega - \dot{\bar{\theta}}) \hat{\lambda}_\gamma + K_\delta \text{sgn}(L_q \bar{i}_\delta) \quad (10)$$

$$\hat{\lambda}_\gamma = L_d \cdot \hat{i}_\gamma + \hat{\lambda}_m \quad (11)$$

$$\hat{\lambda}_\delta = L_q \cdot \hat{i}_\delta \quad (12)$$

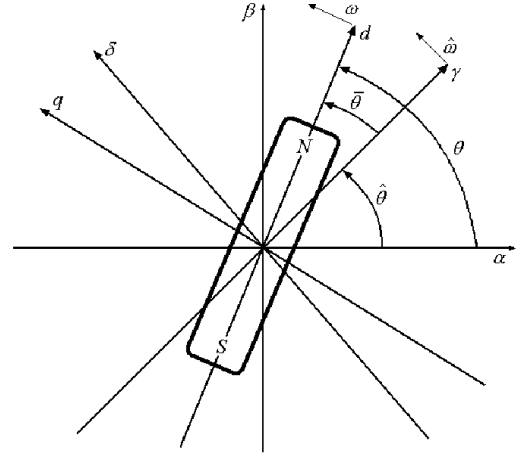


Figure 2. The stationary α - β , the synchronous rotating d-q and the estimated γ - δ reference frames.

Here K_γ , K_δ are adaptable positive gains. Using an appropriate Lyapunov function candidate V_{irs} determined by

$$V_{irs} = \frac{1}{2} \left[(L_d \bar{i}_\gamma)^2 + (L_q \bar{i}_\delta)^2 + \bar{r}_s^2 \right] \quad (13)$$

and analyzing the flux/current error dynamics, the V_{irs} derivative is expressed by

$$\begin{aligned} \dot{V}_{irs} = & \frac{1}{\gamma_r} \bar{r}_s \left[\dot{\bar{r}}_s - \gamma_r (i_\gamma L_d \bar{i}_\gamma + i_\delta L_q \bar{i}_\delta) \right] \\ & - \dot{\bar{\lambda}}_{m\gamma} L_d \bar{i}_\gamma + (\omega - \dot{\bar{\theta}}) \bar{\lambda}_{m\delta} L_d \bar{i}_\gamma - K_\gamma |L_d \bar{i}_\gamma| \\ & - \dot{\bar{\lambda}}_{m\delta} L_q \bar{i}_\delta - (\omega - \dot{\bar{\theta}}) \bar{\lambda}_{m\gamma} L_q \bar{i}_\delta - K_\delta |L_q \bar{i}_\delta| \end{aligned} \quad (14)$$

Here $\gamma_r > 0$ represents the stator resistance observer gain.

The system is asymptotically stable, if the following conditions are valid.

$$\dot{\bar{r}}_s = \gamma_r (i_\gamma L_d \bar{i}_\gamma + i_\delta L_q \bar{i}_\delta) \quad (15)$$

$$K_\gamma > \left| -\dot{\bar{\lambda}}_{m\gamma} + (\omega - \dot{\bar{\theta}}) \cdot \bar{\lambda}_{m\delta} \right| \quad (16)$$

$$K_\delta > \left| -\dot{\bar{\lambda}}_{m\delta} - (\omega - \dot{\bar{\theta}}) \cdot \bar{\lambda}_{m\gamma} \right| \quad (17)$$

Therefore the global current error convergence to the origin of the current observer is succeeded, that is $(\bar{i}_\gamma, \dot{\bar{i}}_\gamma) = 0$ and $(\bar{i}_\delta, \dot{\bar{i}}_\delta) = 0$ for $t \geq t_n$ ($t_n =$ time required to reach the sliding surface).

B. Stator Resistance Observer

The stator resistance observer is based on the condition expressed by Eq. (15). This is the adaptation law of stator resistance error leading to $\bar{r}_s = 0$, $\dot{\bar{r}}_s = 0$ and therefore the estimated stator resistance dynamics could be expressed by

$$\dot{\hat{r}}_s = -\gamma_r (i_\gamma L_d \bar{i}_\gamma + i_\delta L_q \bar{i}_\delta) \quad (18)$$

The stator resistance observer is embedded in the flux/current observer and efficiently improves the flux/current estimation.

C. Modified Back EMF, Rotor Angle and Speed Observer

When sliding mode occurs in the flux/current and stator resistance observers, the error dynamics of the resulting system is expressed by means of partial fluxes.

$$-\dot{\bar{\lambda}}_{m\gamma} + (\omega - \dot{\bar{\theta}}) \bar{\lambda}_{m\delta} - (K_\gamma \text{sgn } \bar{i}_\gamma)_{eq} = 0 \quad (19)$$

$$-\dot{\bar{\lambda}}_{m\delta} - (\omega - \dot{\bar{\theta}}) \bar{\lambda}_{m\gamma} - (K_\delta \text{sgn } \bar{i}_\delta)_{eq} = 0 \quad (20)$$

The last terms of the left-hand side of both equations represent the modified back EMF observer errors in γ - δ reference frame. These expressions are rearranged in such a way that the first right-hand terms are errors.

$$\bar{E}_\gamma = E_\gamma - \hat{E}_\gamma = -(K_\gamma \text{sgn } \bar{i}_\gamma)_{eq} \quad (21)$$

$$\bar{E}_\delta = E_\delta - \hat{E}_\delta = -(K_\delta \text{sgn } \bar{i}_\delta)_{eq} \quad (22)$$

Here $E_\gamma = -\omega \lambda_m \sin \bar{\theta}$ and $E_\delta = \omega \lambda_m \cos \bar{\theta}$.

By defining

$$\dot{\hat{E}}_\gamma = (\bar{\omega} - \dot{\bar{\theta}}) \bar{E}_\delta + c_1 \bar{E}_\gamma \quad (23)$$

$$\dot{\hat{E}}_\delta = -(\omega - \dot{\bar{\theta}}) \bar{E}_\gamma + c_2 \bar{E}_\delta \quad (24)$$

and using the Lyapunov function candidate $V_{E\omega}$ determined by

$$V_{E\omega} = \frac{1}{2} \left[\bar{E}_\gamma^2 + \bar{E}_\delta^2 + \frac{1}{\gamma_\omega} \bar{\omega}^2 \right] \quad (25)$$

then

$$\dot{V}_{E\omega} = -c_1 \bar{E}_\gamma^2 + \frac{1}{\gamma_\omega} \bar{\omega} \left[\dot{\bar{\omega}} - \gamma_\omega (\hat{E}_\delta \bar{E}_\gamma - \hat{E}_\gamma \bar{E}_\delta) \right] - c_2 \bar{E}_\delta^2 \quad (26)$$

Here, c_1, c_2 and $\gamma_\omega > 0$ are the modified back EMF and speed observer gains.

In order $\dot{V}_{E\omega}$ to be negative, the following speed adaptation law is assumed to be valid.

$$\dot{\bar{\omega}} = \gamma_\omega (\hat{E}_\delta \bar{E}_\gamma - \hat{E}_\gamma \bar{E}_\delta) \quad (27)$$

The above leads to the estimated speed dynamics expressed by

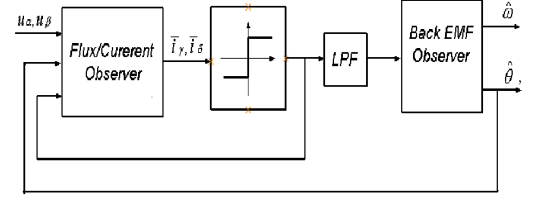


Figure 3. Block diagram of the entire sliding mode observer, in detailed form.

$$\dot{\hat{\omega}} = -\gamma_\omega (\hat{E}_\delta \bar{E}_\gamma - \hat{E}_\gamma \bar{E}_\delta) \quad (28)$$

From the previous analysis results that the rotor angle error could be calculated by

$$\bar{\theta} = \tan^{-1} \left(-\frac{E_\gamma}{E_\delta} \right) = \tan^{-1} \left(-\frac{\hat{E}_\gamma}{\hat{E}_\delta} \right) \quad (29)$$

If rotor angle error is known and by considering $\dot{\bar{\theta}} = \dot{\bar{\omega}} + K_\theta \text{sgn } \bar{\theta}$, the error dynamics for $\bar{\theta}$ is expressed as follows.

$$\dot{\bar{\theta}} = \bar{\omega} - K_\theta \text{sgn } \bar{\theta} \quad (30)$$

The angle error converges to zero for $K_\theta > |\bar{\omega}|$.

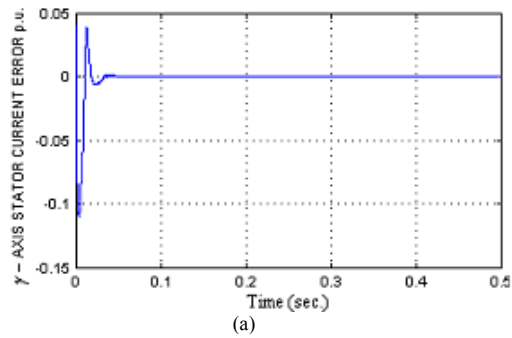
From (30) it is obvious that the term $(\omega - \dot{\bar{\theta}})$ used in the flux/error sliding mode observer is given by

$$(\omega - \dot{\bar{\theta}}) = \hat{\omega} + K_\theta \text{sgn } \bar{\theta} \quad (31)$$

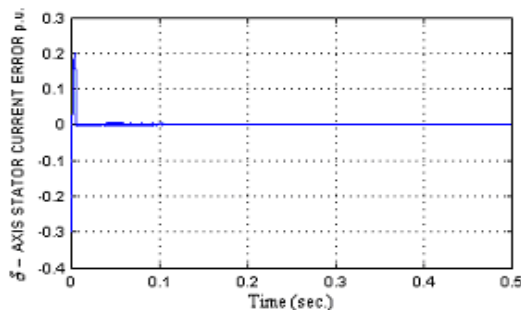
The above sliding mode observer analysis ensures the observer stability and the considerably fast tracking convergence. Fig. 3 shows that the modified back EMF observer is in cascade with the flux/current observer using as inputs the stator current errors.

III. SIMULATION RESULTS

The presented method was tested and verified using Matlab/Simulink facility on a PMSM voltage model. Simulation results are presented first without external torque disturbance. Afterwards it is considered that an external torque disturbance (1.0 p.u.) is applied at time $t_1=1.5s$ and it is removed at time $t_2=2.5s$. Fig. 5 (b) shows the estimated ($\hat{\omega}$) and real (ω) speed responses of PMSM with external disturbance applied. In the simulation, it is assumed that the stator resistance changes between 0.8 and 1.5 of the nominal value. It is found that the estimated stator resistance \hat{r}_s follows any variation of the real stator resistance r_s . This permits the overall observer to work at high and low speed as well, even close to zero.

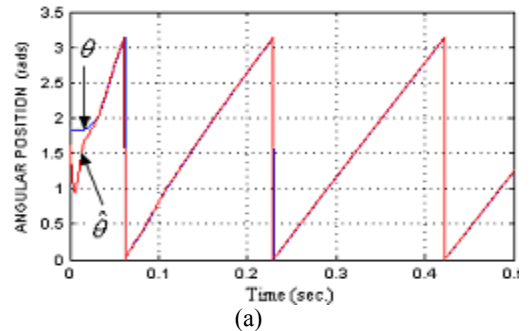


(a)

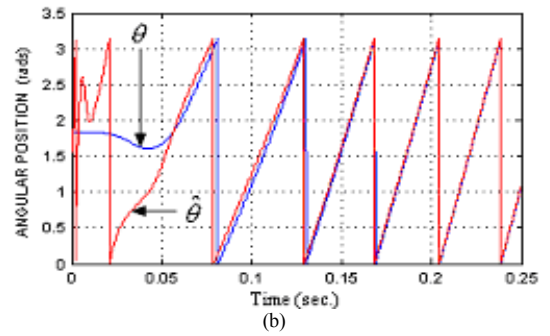


(b)

Figure 4. γ -axis stator current error response for step change of speed reference 0-0.04pu with initial position error $\pi/12$ and (b) δ -axis stator current error response for step change of speed reference 0-0.04pu with initial position error $\pi/12$

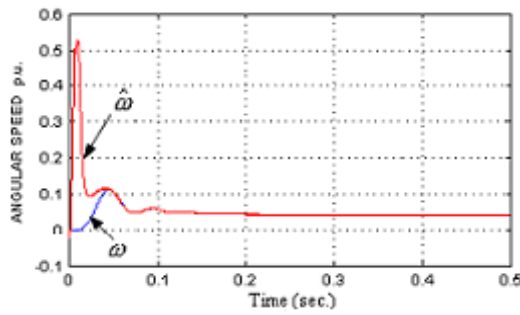


(a)

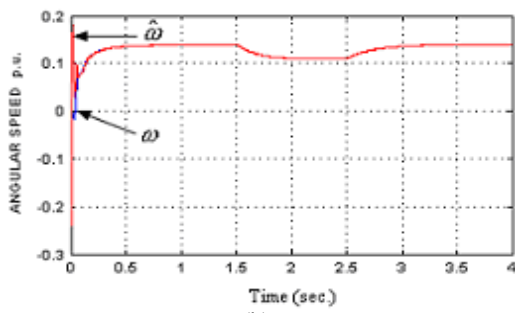


(b)

Figure 6. Position tracking experiments for step change of speed reference with initial position error $\pi/12$ (a) 0-0.04pu and (b) 0-0.14pu



(a)



(b)

Figure 5. Speed responses for step change of speed reference with initial position error $\pi/12$ (a) 0-0.04pu and (b) 0-0.14pu

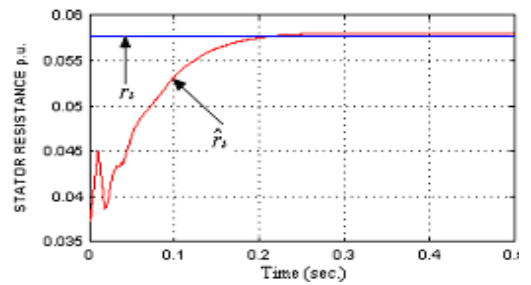


Figure 7. Stator resistance-tracking response for step change of resistance 1.0-1.5pu with initial position error $\pi/12$ and speed reference 0.04pu

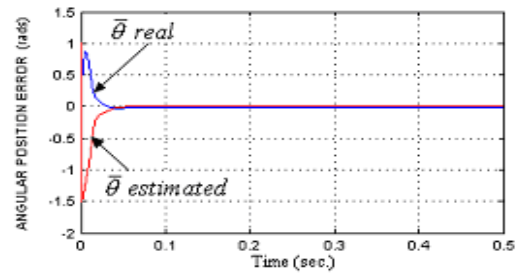


Figure 8. Position error-tracking response for step change of speed reference 0-0.04pu with initial position error $\pi/12$

TABLE I
PARAMETERS OF PERMANENT MAGNET SYNCHRONOUS MACHINE

Symbol	Quantity	Expressed in SI
λ_m	Permanent Magnet Flux	0.213 Wb
\bar{V}_{l-l}	Voltage line to line	415 V
P	Electric Power	2.2 kW
r_s	Stator Resistance	3.01 Ω
L_d	D-axis Inductance	0.060 H
L_q	Q-axis Inductance	0.340 H
J	Moment of Inertia	0.089 Kgs
p	Magnetic Pole Pairs	2
ω_n	Mechanical Angular Speed	3600 rpm

IV. CONCLUSIONS

A new method is developed for speed estimation of a PMSM, which uses sliding mode observer and gives an effective approach of rotor speed at high, low and almost zero speed. Moreover the developed sliding mode observer can effectively estimate the stator resistance variation, due to temperature change. The proposed scheme is based on the measured stator currents and voltages. Simulation results demonstrate the efficiency and the robustness of this sliding mode estimation method.

V. APPENDIX A

The sensorless approach considered in this study, is based on the property of d-q reference frame being transformed to an estimated γ - δ reference frame using the angle error $\bar{\theta}$. This is the angle difference between real and estimated reference frames and denoted as $\bar{\theta} = \theta - \hat{\theta}$. The transformation is described schematically in Fig. 9, where the vector λ_s represents the stator flux analyzed to its components in d-q and γ - δ reference frames. As it can be seen, these reference frames are rotating at different angular speeds ω (synchronous speed) and $\hat{\omega}$ (estimated speed).

The flux/current mathematical model of PMSM in d-q synchronous rotating frame is given by the system (A.1) and (A.2) in matrix form.

$$\dot{\lambda}_{dq} = \begin{bmatrix} \dot{\lambda}_d \\ \dot{\lambda}_q \end{bmatrix} = \begin{bmatrix} -r_s & +\omega L_q \\ -\omega L_d & -r_s \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} u_d \\ u_q \end{bmatrix} + \omega \lambda_m \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (\text{A.1})$$

$$\lambda_{dq} = \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \lambda_m \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (\text{A.2})$$

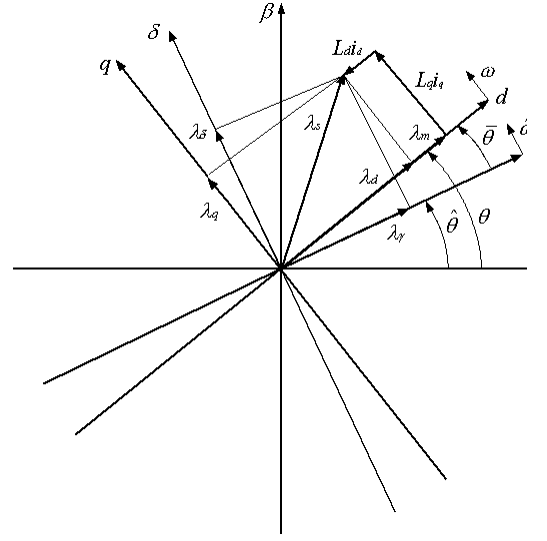


Figure 9. The relations between several reference frames used in PMSM Mathematical model analysis

It is obvious from Fig. 9, that

$$\lambda_{\gamma\delta} = \begin{bmatrix} \lambda_\gamma \\ \lambda_\delta \end{bmatrix} = \begin{bmatrix} \lambda_d \cos \bar{\theta} - \lambda_q \sin \bar{\theta} \\ \lambda_d \sin \bar{\theta} + \lambda_q \cos \bar{\theta} \end{bmatrix} = T(\bar{\theta}) \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} \quad (\text{A.3})$$

and

$$\dot{\lambda}_{\gamma\delta} = \begin{bmatrix} \dot{\lambda}_\gamma \\ \dot{\lambda}_\delta \end{bmatrix} = \dot{\bar{\theta}} J \begin{bmatrix} \lambda_\gamma \\ \lambda_\delta \end{bmatrix} + T(\bar{\theta}) \begin{bmatrix} \dot{\lambda}_d \\ \dot{\lambda}_q \end{bmatrix} \quad (\text{A.4})$$

Since

$$\dot{T}(\bar{\theta}) = \dot{\bar{\theta}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \cos \bar{\theta} & -\sin \bar{\theta} \\ \sin \bar{\theta} & \cos \bar{\theta} \end{bmatrix} = \dot{\bar{\theta}} J T(\bar{\theta})$$

Here $T(\bar{\theta})$ is the transformation matrix, used to transform any vector from d-q to γ - δ reference frame and it is defined by

$$T(\bar{\theta}) = \begin{bmatrix} \cos \bar{\theta} & -\sin \bar{\theta} \\ \sin \bar{\theta} & \cos \bar{\theta} \end{bmatrix}$$

$$\text{and } J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Equation (A.4) could be rewritten in the following form.

$$T(\bar{\theta}) \begin{bmatrix} \dot{\lambda}_d \\ \dot{\lambda}_q \end{bmatrix} = \begin{bmatrix} \dot{\lambda}_\gamma \\ \dot{\lambda}_\delta \end{bmatrix} - \dot{\bar{\theta}} J T(\bar{\theta}) \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} \quad (\text{A.5})$$

Multiplying (A.1) by $T(\bar{\theta})$ the following relation is evident.

$$T(\bar{\theta}) \begin{bmatrix} \dot{\lambda}_d \\ \dot{\lambda}_q \end{bmatrix} = -r_s T(\bar{\theta}) \begin{bmatrix} i_d \\ i_q \end{bmatrix} + T(\bar{\theta}) \begin{bmatrix} u_d \\ u_q \end{bmatrix} - \omega J T(\bar{\theta}) \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} \quad (\text{A.6})$$

The right-hand side parts of equations (A.5) and (A.6) are equals. Therefore by solving with respect to $\dot{\lambda}_{\gamma\delta}$, a similar to equation (5.1) results.

$$\begin{bmatrix} \dot{\lambda}_\gamma \\ \dot{\lambda}_\delta \end{bmatrix} = -r_s T(\bar{\theta}) \begin{bmatrix} i_d \\ i_q \end{bmatrix} - \dot{\bar{\theta}} J T(\bar{\theta}) \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} + T(\bar{\theta}) \begin{bmatrix} u_d \\ u_q \end{bmatrix} - \omega J T(\bar{\theta}) \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} \quad (\text{A.7})$$

Equations (A.1) and (A.7) can be rewritten in a more convenient form as follows.

$$\dot{\lambda}_{dq} = -r_s i_{dq} + u_{dq} - \omega J \lambda_{dq} \quad (\text{A.8})$$

$$\dot{\lambda}_{\gamma\delta} = -r_s i_{\gamma\delta} + u_{\gamma\delta} - (\omega - \dot{\bar{\theta}}) J \lambda_{\gamma\delta} \quad (\text{A.9})$$

Here

$$u_{dq} = \begin{bmatrix} u_d \\ u_q \end{bmatrix}, i_{dq} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}, u_{\gamma\delta} = \begin{bmatrix} u_\gamma \\ u_\delta \end{bmatrix} \text{ and } i_{\gamma\delta} = \begin{bmatrix} i_\gamma \\ i_\delta \end{bmatrix}$$

Equations (A.8) and (A.9) represent the flux/current mathematical model of PMSM in γ - δ reference frame.

It must be noted that $(\omega - \dot{\bar{\theta}})$ is used instead of ω and the real and estimated reference frames coincide when $\bar{\theta} = 0$ and $\dot{\bar{\theta}} = 0$.

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