# PMSM Sensorless Speed Estimation Based on Sliding Mode Observers

Vasilios C. Ilioudis and Nikolaos I. Margaris

Aristotle University of Thessaloniki/Department of Electrical and Computer Engineering, Thessaloniki, Greece

Abstract-A new sensorless method of estimating the Permanent Magnet Synchronous Motor speed is presented. The method is based on sliding mode observer theory using Lyapunov stability criteria. The observation algorithm makes use of the machine model equations allowing the estimation of flux/current, stator resistance, rotor angle and rotor speed from the motor terminal measurement of currents and voltages. Speed/position observer is based on a modified back EMF state observer, which is considered in cascade with stator flux/current and resistance observers. The presented application has been implemented using an estimated  $\gamma$ - $\delta$  reference frame instead of  $\alpha$ - $\beta$  stationary reference frame. It is shown that the overall observation system gives exceptional estimation results at high and low speed ranges, without initial rotor angle knowledge. Simulation results of Permanent Magnet Synchronous Motor sensorless speed estimation are also presented.

*Index Terms*— permanent magnet synchronous motor (PMSM), Lyapunov function candidate, sliding mode observer (SMO), modified back EMF.

#### Notations

 $E_{\gamma} = \gamma$ -axis modified back EMF  $E_{\delta} = \delta$ - axis modified back EMF  $\theta$  = angular position  $i_{\gamma} = \gamma$ -axis current  $i\delta = \delta$ -axis current  $i_d$  = d-axis current  $i_q = q$ -axis current  $L_d$  = d-axis inductance  $L_q = q$ -axis inductance  $\lambda_{\gamma} = \gamma$ -axis magnetic flux  $\lambda \delta = \delta$ -axis magnetic flux  $\lambda_d$  = d-axis magnetic flux  $\lambda_q = q$ -axis magnetic flux  $\lambda_m$  = permanent magnet flux  $\lambda_{m\gamma} = \gamma$ -axis partial flux  $\lambda_{m\delta} = \delta$ -axis partial flux  $r_s = \text{stator resistance}$  $u\delta = \delta$ -axis voltage  $u_{\gamma} = \gamma$ -axis voltage  $u_d = d$ -axis voltage  $u_q = q$ -axis voltage

 $\omega$  = angular speed

## I. INTRODUCTION

In the sensorless control of a PMSM drive two main strategies are applied, the fundamental excitation method and the saliency and signal injection method [9], [14]. The fundamental excitation method estimates the rotor position and speed from the stator voltages and currents and it does not need any additional test signal. At the same time, it is hard to estimate position at the low-speed region. In the saliency and signal injection method, the inductance varies depending on the rotor position. This feature of the salient-pole PMSM is used to estimate rotor position even at low speeds and standstill. Some fundamental excitation method approaches are based on the estimation of the back electromotive force (EMF) or flux linkage due to permanent magnets by means of a state observer or an extended Kalman filter [12]. Also other simple methods are based on the voltage or current error between the detected variables and the calculated variables from the motor model using state observer techniques. Among different observation methods used. the sliding mode observer (SMO) is a promising approach and an effective technique due to its outstanding robustness properties against system parameter uncertainties and external disturbances [4]-[7]. The sensorless strategy proposed in this study is based on sliding modes using the fundamental excitation method with a modified back EMF. A mathematical model of PMSM in an estimated  $\gamma$ - $\delta$  rotating reference frame is considered to estimate both rotor speed and position. Fig. 1 shows the entire controlled system in block diagram.



Figure 1. Block diagram of PMSM controlled system using

## II. SLIDING MODE OBSERVER

## A. Flux/Current Observer of PMSM

In sensorless position/speed control, the rotor position cannot be detected, and therefore d-q axis mathematical model cannot be applied directly. Most approaches are based on the estimation of the back electromotive force (EMF) in the stationary reference frame  $\alpha$ - $\beta$ . The proposed PMSM mathematical model of sliding mode observer is reflected in an estimated reference frame  $\gamma$ - $\delta$ rotating at an estimated angular velocity  $\hat{\omega}$  and lagging behind the d-q reference frame by electrical angle error  $\overline{\theta}$ . Fig. 2 shows the relations between the synchronous reference model (dq-axis) and the estimated reference model ( $\gamma\delta$ -axis) used in this study. The mathematical model of PMSM in dq-axis synchronous rotating reference frame is presented by the following flux/current state equations.

$$\dot{\lambda}_d = -r_s \cdot \dot{i}_d + u_d + \omega \cdot \lambda_q \tag{1}$$

$$\dot{\lambda}_q = -r_s \cdot \dot{i}_q + u_q - \omega \cdot \lambda_d \tag{2}$$

$$\lambda_d = L_d \cdot i_d + \lambda_m \tag{3}$$

$$\lambda_q = L_q \cdot i_q \tag{4}$$

The conventional d-q axis model can be transformed to  $\gamma$ - $\delta$  axis as follows (see Appendix A).

$$\dot{\lambda}_{\gamma} = -r_s i_{\gamma} + u_{\gamma} + \left(\omega - \dot{\overline{\theta}}\right) \lambda_{\delta}$$
<sup>(5)</sup>

$$\dot{\lambda}_{\delta} = -r_{s}i_{\delta} + u_{\delta} - \left(\omega - \dot{\overline{\theta}}\right)\lambda_{\gamma} \tag{6}$$

$$\lambda_{\gamma} = L_d \cdot i_{\gamma} + \lambda_{m\gamma} \tag{7}$$

$$\lambda \delta = L_q \cdot i\delta + \lambda_m \delta \tag{8}$$

The partial fluxes  $\lambda_{m\gamma}$  and  $\lambda_{m\delta}$  are functions of  $\lambda_m$ ,  $\overline{\theta}$  and  $L_2$ , where  $L_2 = (L_q - L_d)/2$ . In the proposed sensorless scheme, this property of the partial fluxes is utilized to extract information of rotor speed and position error.

The flux/current state observer is expressed by the following equations using as sliding surfaces  $s_{\gamma} = L_d (i_{\gamma} - \hat{i}_{\gamma}) = L_d \overline{i}_{\gamma}$  and  $s_{\delta} = L_q (i_{\delta} - \hat{i}_{\delta}) = L_q \overline{i}_{\delta}$ .

$$\dot{\hat{\lambda}}_{\gamma} = -\hat{r}_{s}i_{\gamma} + u_{\gamma} + \left(\omega - \dot{\overline{\Theta}}\right)\hat{\lambda}_{\delta} + K_{\gamma}\operatorname{sgn}\left(L_{d}\overline{i}_{\gamma}\right) \qquad (9)$$

$$\dot{\hat{\lambda}}_{\delta} = -\hat{r}_{\delta}i_{\delta} + u_{\delta} - \left(\omega - \dot{\overline{\Theta}}\right)\hat{\lambda}_{\gamma} + K_{\delta}\operatorname{sgn}\left(L_{q}\overline{i}_{\delta}\right) \quad (10)$$

$$\hat{\lambda}_{\gamma} = L_d \cdot \hat{i}_{\gamma} + \hat{\lambda}_m \tag{11}$$

$$\hat{\lambda}\delta = L_q \cdot \hat{i}\delta \tag{12}$$



Figure 2. The stationary  $\alpha$ - $\beta$ , the synchronous rotating d-q and the estimated  $\gamma$ - $\delta$  reference frames.

Here  $K_{\gamma}$ ,  $K_{\delta}$  are adaptable positive gains. Using an appropriate Lyapunov function candidate  $V_{irs}$  determined by

$$V_{irs} = \frac{1}{2} \left[ \left( L_d \overline{i}_{\gamma} \right)^2 + \left( L_q \overline{i}_{\delta} \right)^2 + \overline{r_s}^2 \right]$$
(13)

and analyzing the flux/current error dynamics, the  $V_{irs}$  derivative is expressed by

$$\dot{V}_{irs} = \frac{1}{\gamma_r} \overline{r_s} \Big[ \dot{\overline{r}_s} - \gamma_r \big( i_\gamma L_d \overline{i}_\gamma + i_\delta L_q \overline{i}_\delta \big) \Big] - \dot{\overline{\lambda}}_{m\gamma} L_d \overline{i}_\gamma + \Big( \omega - \dot{\overline{\theta}} \Big) \overline{\lambda}_{m\delta} L_d \overline{i}_\gamma - K_\gamma \Big| L_d \overline{i}_\gamma \Big| - \dot{\overline{\lambda}}_{m\delta} L_q \overline{i}_\delta - \Big( \omega - \dot{\overline{\theta}} \Big) \overline{\lambda}_{m\gamma} L_q \overline{i}_\delta - K_\delta \Big| L_q \overline{i}_\delta \Big|$$
(14)

Here  $\gamma_r > 0$  represents the stator resistance observer gain.

The system is asymptotically stable, if the following conditions are valid.

$$\dot{\overline{r}}_{s} = \gamma_{r} \left( i_{\gamma} L_{d} i_{\gamma} + i_{\delta} L_{q} \overline{i}_{\delta} \right)$$
(15)

$$K_{\gamma} > \left| -\dot{\overline{\lambda}}_{m\gamma} + \left( \omega - \dot{\overline{\theta}} \right) \cdot \overline{\lambda}_{m\delta} \right|$$
(16)

$$K_{\delta} > \left| -\dot{\overline{\lambda}}_{m\delta} - \left( \omega - \dot{\overline{\theta}} \right) \cdot \overline{\lambda}_{m\gamma} \right|$$
(17)

Therefore the global current error convergence to the origin of the current observer is succeeded, that is  $(\overline{i}_{r}, \overline{i}_{r}) = 0$  and  $(\overline{i}_{s}, \overline{i}_{s}) = 0$  for  $t \ge t_n$  ( $t_n$  = time required to reach the sliding surface).

#### B. Stator Resistance Observer

The stator resistance observer is based on the condition expressed by Eq. (15). This is the adaptation law of stator resistance error leading to  $\overline{r_s} = 0$ ,  $\dot{\overline{r_s}} = 0$  and therefore the estimated stator resistance dynamics could be expressed by

$$\dot{\hat{r}}_s = -\gamma_r \left( i_\gamma L_d \overline{i}_\gamma + i_\delta L_q \overline{i}_\delta \right) \tag{18}$$

The stator resistance observer is embedded in the flux/current observer and efficiently improves the flux/current estimation.

## C. Modified Back EMF, Rotor Angle and Speed Observer

When sliding mode occurs in the flux/current and stator resistance observers, the error dynamics of the resulting system is expressed by means of partial fluxes.

$$-\overline{\lambda}_{m\gamma} + \left(\omega - \overline{\theta}\right)\overline{\lambda}_{m\delta} - \left(K_{\gamma}\operatorname{sgn}\overline{i}_{\gamma}\right)_{eq} = 0$$
(19)

$$-\overline{\lambda}_{m\delta} - \left(\omega - \dot{\overline{\theta}}\right)\overline{\lambda}_{m\gamma} - \left(K_{\delta}\operatorname{sgn}\overline{i}_{\delta}\right)_{eq} = 0$$
(20)

The last terms of the left-hand side of both equations represent the modified back EMF observer errors in  $\gamma$ - $\delta$  reference frame. These expressions are rearranged in such a way that the first right-hand terms are errors.

$$\overline{E}_{\gamma} = E_{\gamma} - \hat{E}_{\gamma} = -\left(K_{\gamma} \operatorname{sgn} \overline{i}_{\gamma}\right)_{eq}$$
(21)

$$\overline{E}_{\delta} = E_{\delta} - \hat{E}_{\delta} = -\left(K_{\delta} \operatorname{sgn} \overline{i}_{\delta}\right)_{eq}$$
(22)

Here  $E_{\gamma} = -\omega \lambda_m \sin \overline{\theta}$  and  $E_{\delta} = \omega \lambda_m \cos \overline{\theta}$ .

By defining

$$\dot{\widehat{E}}_{\gamma} = \left(\overline{\omega} - \dot{\overline{\theta}}\right) \overline{E}_{\delta} + c_1 \overline{E}_{\gamma}$$
(23)

$$\dot{\hat{E}}_{\delta} = -\left(\omega - \dot{\overline{\Theta}}\right)\overline{E}_{\gamma} + c_2\overline{E}_{\delta}$$
(24)

and using the Lyapunov function candidate  $V_{E\omega}$  determined by

$$V_{E\omega} = \frac{1}{2} \left[ \overline{E}_{\gamma}^{2} + \overline{E}_{\delta}^{2} + \frac{1}{\gamma_{\omega}} \overline{\omega}^{2} \right]$$
(25)

then

$$\dot{V}_{E\omega} = -c_1 \overline{E}_{\gamma}^2 + \frac{1}{\gamma_{\omega}} \overline{\omega} \Big[ \dot{\overline{\omega}} - \gamma_{\omega} \Big( \hat{E}_{\delta} \overline{E}_{\gamma} - \hat{E}_{\gamma} \overline{E}_{\delta} \Big) \Big] - c_2 \overline{E}_{\delta}^2$$
(26)

Here,  $C_1$ ,  $C_2$  and  $\gamma_{\omega} > 0$  are the modified back EMF and speed observer gains.

In order  $V_{E\omega}$  to be negative, the following speed adaptation law is assumed to be valid.

$$\dot{\overline{\omega}} = \gamma_{\omega} \left( \hat{E}_{\delta} \overline{E}_{\gamma} - \hat{E}_{\gamma} \overline{E}_{\delta} \right) \tag{27}$$

The above leads to the estimated speed dynamics expressed by



Figure 3. Block diagram of the entire sliding mode observer, in detailed form.

$$\dot{\hat{\omega}} = -\gamma_{\omega} \left( \hat{E}_{\delta} \overline{E}_{\gamma} - \hat{E}_{\gamma} \overline{E}_{\delta} \right) \tag{28}$$

From the previous analysis results that the rotor angle error could be calculated by

$$\overline{\theta} = \tan^{-1} \left( -\frac{E_{\gamma}}{E_{\delta}} \right) \simeq \tan^{-1} \left( -\frac{\hat{E}_{\gamma}}{\hat{E}_{\delta}} \right)$$
(29)

If rotor angle error is known and by considering  $\dot{\hat{\theta}} = \hat{\omega} + K_{\theta} \operatorname{sgn} \overline{\theta}$ , the error dynamics for  $\overline{\theta}$  is expressed as follows.

$$\overline{\theta} = \overline{\omega} - K_{\theta} \operatorname{sgn} \overline{\theta}$$
(30)
The angle error converges to zero for  $K_{\theta} > |\overline{\omega}|$ .

From (30) it is obvious that the term  $(\omega - \overline{\theta})$  used in the flux/error sliding mode observer is given by

$$\left(\omega - \dot{\overline{\theta}}\right) = \hat{\omega} + K_{\theta} \operatorname{sgn} \overline{\theta}$$
(31)

The above sliding mode observer analysis ensures the observer stability and the considerably fast tracking convergence. Fig. 3 shows that the modified back EMF observer is in cascade with the flux/current observer using as inputs the stator current errors.

#### III. SIMULATION RESULTS

The presented method was tested and verified using Matlab/Simulink facility on a PMSM voltage model. Simulation results are presented first without external torque disturbance. Afterwards it is considered that an external torque disturbance (1.0 p.u.) is applied at time  $t_1=1.5$ s and it is removed at time  $t_2=2.5$ s. Fig. 5 (b) shows the estimated ( $\hat{\omega}$ ) and real ( $\omega$ ) speed responses of PMSM with external disturbance applied. In the simulation, it is assumed that the stator resistance changes between 0.8 and 1.5 of the nominal value. It is found that the estimated stator resistance  $\hat{r}_s$  follows any variation of the real stator resistance  $r_s$ . This permits the overall observer to work at high and low speed as well, even close to zero.



Figure 4.  $\gamma$ -axis stator current error response for step change of speed reference 0-0.04pu with initial position error  $\pi/12$  and (b)  $\delta$ -axis stator current error response for step change of speed reference 0-0.04pu with initial position error  $\pi/12$ 



Figure 6. Position tracking experiments for step change of speed reference with initial position error  $\pi/12(a)$  0-0.04pu and (b) 0-0.14pu

(b)



Figure 5. Speed responses for step change of speed reference with initial position error  $\pi/12$  (a) 0-0.04pu and (b) 0-0.14pu



Figure 7. Stator resistance-tracking response for step change of resistance 1.0-1.5pu with initial position error  $\pi/12$  and speed reference 0.04pu



Figure 8. Position error-tracking response for step change of speed reference 0-0.04pu with initial position error  $\pi/12$ 

TABLE I           Parameters of Permanent Magnet Synchronous Machine		
Symbol	Quantity	Expressed in SI
$\lambda_m$	Permanent Magnent Flux	0.213 Wb
$V_{l-l}$	Voltage line to line	415 V
Р	Electric Power	2.2 kW
<i>r</i> s	Stator Resistance	3.01 Ω
$L_d$	D-axis Inductance	0.060 H
$L_q$	Q-axis Inductance	0.340 H
J	Moment of Inertia	0.089 Kgs
р	Magnetic Pole Pairs	2
ωn	Mechanical Angular Speed	3600 rpm

q  $\lambda_{d}$   $\lambda_{d}$ 

# IV. CONCLUSIONS

A new method is developed for speed estimation of a PMSM, witch uses sliding mode observer and gives an effective approach of rotor speed at high, low and almost zero speed. Moreover the developed sliding mode observer can effectively estimate the stator resistance variation, due to temperature change. The proposed scheme is based on the measured stator currents and voltages. Simulation results demonstrate the efficiency and the robustness of this sliding mode estimation method.

#### V. APPENDIX A

The sensorless approach considered in this study, is based on the property of d-q reference frame being transformed to an estimated  $\gamma$ - $\delta$  reference frame using the angle error  $\overline{\theta}$ . This is the angle difference between real and estimated reference frames and denoted as  $\overline{\theta} = \theta - \hat{\theta}$ . The transformation is described schematically in Fig. 9, where the vector  $\lambda_s$  represents the stator flux analyzed to its components in d-q and  $\gamma$ - $\delta$  reference frames. As it can be seen, these reference frames are rotating at different angular speeds  $\omega$  (synchronous speed) and  $\hat{\omega}$  (estimated speed).

The flux/current mathematical model of PMSM in d-q synchronous rotating frame is given by the system (A.1) and (A.2) in matrix form.

$$\dot{\lambda}_{dq} = \begin{bmatrix} \dot{\lambda}_d \\ \dot{\lambda}_q \end{bmatrix} = \begin{bmatrix} -r_s & +\omega L_q \\ -\omega L_d & -r_s \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} u_d \\ u_q \end{bmatrix} + \omega \lambda_m \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
(A.1)

$$\lambda_{dq} = \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \lambda_m \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
(A.2)

Figure 9. The relations between several reference frames used in PMSM Mathematical model analysis

It is obvious from Fig. 9, that

$$\lambda_{\gamma\delta} = \begin{bmatrix} \lambda_{\gamma} \\ \lambda_{\delta} \end{bmatrix} = \begin{bmatrix} \lambda_d \cos \overline{\theta} - \lambda_q \sin \overline{\theta} \\ \lambda_d \sin \overline{\theta} + \lambda_q \cos \overline{\theta} \end{bmatrix} = T\left(\overline{\theta}\right) \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix}$$
(A.3)

and

$$\dot{\lambda}_{\gamma\delta} = \begin{bmatrix} \dot{\lambda}_{\gamma} \\ \dot{\lambda}_{\delta} \end{bmatrix} = \dot{\overline{\Theta}} J \begin{bmatrix} \lambda_{\gamma} \\ \lambda_{\delta} \end{bmatrix} + T \left( \overline{\Theta} \right) \begin{bmatrix} \dot{\lambda}_{d} \\ \dot{\lambda}_{q} \end{bmatrix}$$
(A.4)

Since

$$\dot{T}\left(\bar{\theta}\right) = \dot{\overline{\theta}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\bar{\theta} & -\sin\bar{\theta} \\ \sin\bar{\theta} & \cos\bar{\theta} \end{bmatrix} = \dot{\overline{\theta}} JT\left(\bar{\theta}\right)$$

Here  $T(\overline{\theta})$  is the transformation matrix, used to transform any vector from d-q to  $\gamma$ - $\delta$  reference frame and it is defined by

$$T(\overline{\theta}) = \begin{bmatrix} \cos \overline{\theta} & -\sin \overline{\theta} \\ \sin \overline{\theta} & \cos \overline{\theta} \end{bmatrix}$$
  
and 
$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Equation (A.4) could be rewritten in the following form.

$$T\left(\overline{\theta}\right)\begin{bmatrix}\dot{\lambda}_{d}\\\dot{\lambda}_{q}\end{bmatrix} = \begin{bmatrix}\dot{\lambda}_{\gamma}\\\dot{\lambda}_{\delta}\end{bmatrix} - \dot{\overline{\theta}}JT\left(\overline{\theta}\right)\begin{bmatrix}\lambda_{d}\\\lambda_{q}\end{bmatrix}$$
(A.5)

Multiplying (A.1) by  $T(\overline{\theta})$  the following relation is evident.

$$T\left(\overline{\theta}\right)\begin{bmatrix}\dot{\lambda}_{d}\\\dot{\lambda}_{q}\end{bmatrix} = -r_{s}T\left(\overline{\theta}\right)\begin{bmatrix}i_{d}\\i_{q}\end{bmatrix} + T\left(\overline{\theta}\right)\begin{bmatrix}u_{d}\\u_{q}\end{bmatrix} - \omega JT\left(\overline{\theta}\right)\begin{bmatrix}\lambda_{d}\\\lambda_{q}\end{bmatrix}$$
(A.6)

The right-hand side parts of equations (A.5) and (A.6) are equals. Therefore by solving with respect to  $\lambda_{\gamma\delta}$ , a similar to equation (5.1) results.

$$\begin{bmatrix} \dot{\lambda}_{r} \\ \dot{\lambda}_{\sigma} \end{bmatrix} = -r_{s}T\left(\overline{\theta}\right) \begin{bmatrix} i_{d} \\ i_{q} \end{bmatrix} - \dot{\overline{\theta}}JT\left(\overline{\theta}\right) \begin{bmatrix} \lambda_{d} \\ \lambda_{q} \end{bmatrix} + T\left(\overline{\theta}\right) \begin{bmatrix} u_{d} \\ u_{q} \end{bmatrix} - \omega JT\left(\overline{\theta}\right) \begin{bmatrix} \lambda_{d} \\ \lambda_{q} \end{bmatrix}$$
(A.7)

Equations (A.1) and (A.7) can be rewritten in a more convenient form as follows.

$$\lambda_{dq} = -r_s \dot{i}_{dq} + u_{dq} - \omega J \lambda_{dq} \tag{A.8}$$

$$\dot{\lambda}_{\gamma\delta} = -r_s i_{\gamma\delta} + u_{\gamma\delta} - \left(\omega - \dot{\overline{\theta}}\right) J \lambda_{\gamma\delta} \tag{A.9}$$

Here

$$u_{dq} = \begin{bmatrix} u_d \\ u_q \end{bmatrix}, \ i_{dq} = \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \ u_{\gamma\delta} = \begin{bmatrix} u_{\gamma} \\ u_{\delta} \end{bmatrix} \text{ and } i_{\gamma\delta} = \begin{bmatrix} i_{\gamma} \\ i_{\delta} \end{bmatrix}$$

Equations (A.8) and (A.9) represent the flux/current mathematical model of PMSM in  $\gamma$ - $\delta$  reference frame.

It must be noted that  $(\omega - \overline{\theta})$  is used instead of  $\omega$  and the real and estimated reference frames coincide when  $\overline{\theta} = 0$  and  $\overline{\theta} = 0$ .

#### REFERENCES

- [1] P. C. Krause, "Analysis of Electric Machinery", McGraw-Hill International Editions, 1987
- V. I. Utkin, "Sliding Modes in Control Optimization", Springer-[2] Verlag Berlin, Heidelberg 1992 J-J. E. Slotine and W. Li "Applied Non linear Control," Prentice-
- [3] Hall International, Inc., 1991
- V. I. Utkin , "Variable Structure Systems with Sliding Mode: A Survey", IEEE Automat. Contr., vol AC-22 , no 2, pp212-222, April 1977.
- V. I. Utkin , "Sliding Mode Control Design Principles and [5] Applications to Electric Drives ", IEEE Trans. Ind. Electron. , vol 40, no. 1, pp. 23-36, Feb. 1993.
- Z. Yan, C. Jin, and V. I. Utkin, "Sensorless Sliding-Mode Control [6] of Induction Motors," IEEE Trans. Ind. Electron., vol 47, no. 6, pp. 1286-1297, Dec. 2000.
- Zhang Yan, and Utkin V., "Sliding Mode Observers for Electric [7] Machines – An Overwiew", *IECON0*,28<sup>th</sup> Annual Conference of the IEEE Industrial Electronics Society, vol 3, pp. 1842-1847, 5-8 Nov. 2002.
- [8] Lennart Harnefors, and Hans-Peter Nee, "A Genaral Algorithm for Speed and Position Estimation of AC Motors", IEEE Trans. Ind. *Electron.*, Vol. 47, no. 1, pp. 77-83, Feb 2000.
- Xie Yue, D. Mahinda Vilathgumuwa, and King-Jet Tseng, [9] "Observer-Based Robust Adaptive of PMSM With Initial Rotor Position Uncertainty", IEEE Trans. Ind. Applic., vol. 39, no. 3,pp. 645-656. May/June 2003.
- [10] Morimoto, S.; Kawamoto, K.; Sanada, M.; Takeda, Y., "Sensorless Control Strategy For Salient-Pole PMSM Based on Extended EMF in Rotating Reference Frame", Industry Applications Conference, 2001. Thirty-Sixth IAS Annual Meeting. Conference Record of the 2001 IEEE, vol. 4, pp. 2637 - 2644, 30 Sep-4 Oct 2001.
- [11] Z. Xu, and M. F. Rahman, "Encoder-less Operation of A Direct Torque Controlled IPM Motor Drive With A Novel Sliding Mode Observer", Proceedings of Australasian Universities Power Engineering Conference 2004, Brisbane, QLD, University of QLD, pp. 1131 – 1136, 26-29 Sep. 2004. [12] Changsheng Li, and Elbuluk M., "A Sliding Mode Observer for
- Sensorless Control of Permanent Magnet Synchronous Motors" Industry Applications Conference, 2001, Thirty-Sixth IAS Annual Meeting. Conference Record of the 2001 IEEE, vol. 2, Issue, pp. 1273 - 1278, 30 Sep-4 Oct 2001.
- [13] H. Kubota, K. Matsuse, and T. Nakano, "DSP-Based Speed Adaptive Flux Observer of Induction Motor", *IEEE Trans. Ind.* Applic., vol 29, no.2, pp. 344-348, March/April 1993.
- [14] Mathew J. Corley, and Robert D. Lorenz, "Rotor Position and Velocity Estimation For a Salient-Pole Permanent Magnet Synchronous Machine at Standstill and High Speeds", IEEE Trans. Ind. Applic., vol. 34, no. 4, pp. 784 - 789, July/August 1998.
- [15] Z. Yan, "Control and Observation of Electric Machines by Sliding Modes," PhD Dissertation, Ohio State University, 2002.