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Modelling and Specification of Compliant Motions with Two and Three Contact Points

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Abstract

This paper describes the modelling and motion specification of compliant motion tasks with two or three contact points. These tasks cannot be done with Mason's classical "task frame" (TF) or "compliance frame" approach. Hence, a more flexible and versatile motion constraint model is introduced, that maintains most of the intuitiveness of the TF approach.

1 Introduction

The literature on force-controlled compliant motion most often uses (implicitly or explicitly) Mason's "Task Frame" (TF) approach, [5], to model the contact situation of the task, to specify the desired motion within this contact model, and to control the task execution. This approach is limited to tasks in which one single orthogonal reference frame suffices to model force-controlled and velocity-controlled directions of the motion constraint. (See [1] for more details.) Previous publications by the authors (e.g., [3]) have presented extensions to the TF approach that allow to tackle tasks that could not previously be executed successfully. These extensions use the concept of "virtual contact manipulators" to model the instantaneous motion freedom of the manipulated object: each contact is modelled by a kinematic chain that gives the manipulated object the same local motion freedom as the contact; if the total motion constraint consists of several contacts that act simultaneously on the manipulated object, this total motion constraint is hence modelled by a parallel manipulator. The kinetostatic properties of this parallel manipulator determine the force-controlled and velocity-controlled spaces at each instant. In the rest of the paper, these spaces will be called *wrench space* and *twist space*, respectively.

The advantage of this modelling approach is that it is completely general and independent of any coordinate representation. The resulting models, however, could lack the intuitiveness of the TF approach. Therefore, this paper gives ad hoc contact models for two frequently occurring contact situations (having two, respectively three, *vertex-surface* contacts between manipulated object and environment), in which the remaining degrees of freedom are defined in very intuitive and coordinate-independent ways. Coordinate expressions, however, are also given, such that implementation on a force-controlled robot system is straightforward. Section 2 describes the two-point contact situation, and Section 3 the three-point contact situation.

The contact models in this paper are basic to the motion specification and force control of every task that involves more than one single contact: *instantaneously* every contact is approximated by the position of the contact point and the direction of the contact normal, and this is exactly the situation where the presented models are valid. Moreover, *velocity-based* on-line identification of these contact parameters [1] (i.e., the errors in the current estimates of the contact point position and contact normal direction) can be done for each contact separately. Hence, these ad hoc models are very practical in two ways: (i) they allow to model and specify contact situations that the classical Task Frame formalism cannot cope with, and (ii) they simplify (without loss of functionality!) the general "virtual contact manipulator" approach in these particular cases.

2 Two-point contact

In this task, Fig. 1, the robot tool is in contact with two smoothly curved surfaces, which intersect each other in a "seam." Each contact is of the *vertex-surface* type, with five velocity-controlled and one force-controlled direction. Hence, the two contacts together reduce the dimension of the motion freedom space to four. Examples of such a task are: tracking pipes in chemical, nuclear or undersea plants (in this case, the "seam" exists in the model only); following a surface with a heavy tool that needs bracing on a support surface [6]; guidance of a welding torch or a glueing tool along a seam between two workpieces that have to be connected, etc.

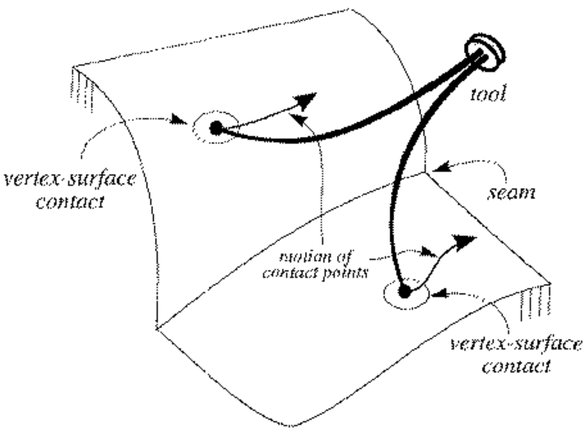

Figure 1: Seam following with two contact points.

2.1 Geometric parameters

The symbols $\{1\}$ and $\{2\}$ denote the two contact points, as well as their associated contact frames, Fig. 2. The *seam axis* is the intersection of the two tangent planes at $\{1\}$ and $\{2\}$. A *parallel plane* is each plane through a contact normal and parallel to the seam axis. A *perpendicular plane* is any plane perpendicular to the seam axis. The *seam angle* σ is the (free space) angle between the tangent planes. The contact points lie at distances d^1 and d^2, respectively, from the seam axis.

All these parameters can be calculated if the unit normal vectors e^1 and e^2 are known, as well as the vector p linking the two contact points. Expressed with respect to the reference frame $\{1\}$ of Fig. 3 this gives:

$$e^1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; e^2 = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}, \; p = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}. \qquad (1)$$

Then, the seam angle σ, the direction of the seam axis e^{sa}, the position vectors p_d^1 and p_d^2 of the points on the seam axis closest to $\{1\}$ and $\{2\}$, as well as the distances d^1 and d^2 from the contact points to the seam axis, are calculated as follows (the calculations are straightforward but rather tedious):

1. The seam angle σ:

$$\sigma = \arcsin\left(e^1 \cdot e^2\right) + \frac{\pi}{2} = \arcsin(e_z) + \frac{\pi}{2}.$$

2. The vector e^{sa} is the normalized cross product of e^1 and e^2:

$$e^{sa} = \frac{e^2 \times e^1}{|e^2 \times e^1|} = \frac{1}{\sqrt{e_x^2 + e_y^2}} \begin{bmatrix} e_y \\ -e_x \\ 0 \end{bmatrix}.$$

3. p_d^1 is the intersection of the tangent planes at $\{1\}$ and $\{2\}$, and the perpendicular plane through $\{1\}$:

$$p_d^1 = \frac{e_x p_x + e_y p_y + e_z p_z}{e_x^2 + e_y^2} \begin{bmatrix} e_x \\ e_y \\ 0 \end{bmatrix}.$$

Similarly, p_d^2 is the intersection of the tangent planes at $\{1\}$ and $\{2\}$, and the perpendicular plane through $\{2\}$:

$$p_d^2 = \begin{bmatrix} p_x \\ p_y \\ 0 \end{bmatrix} + \frac{e_z p_z}{e_x^2 + e_y^2} \begin{bmatrix} e_x \\ e_y \\ 0 \end{bmatrix}.$$

4. The distance d^1 is the length of p_d^1:

$$d^1 = \frac{|e_x p_x + e_y p_y + e_z p_z|}{\sqrt{e_x^2 + e_y^2}}.$$

The distance d^2 is the length of $p_d^2 - p$:

$$d^2 = |p_z| \sqrt{\frac{e_x^2 + e_y^2 + e_z^2}{e_x^2 + e_y^2}}.$$

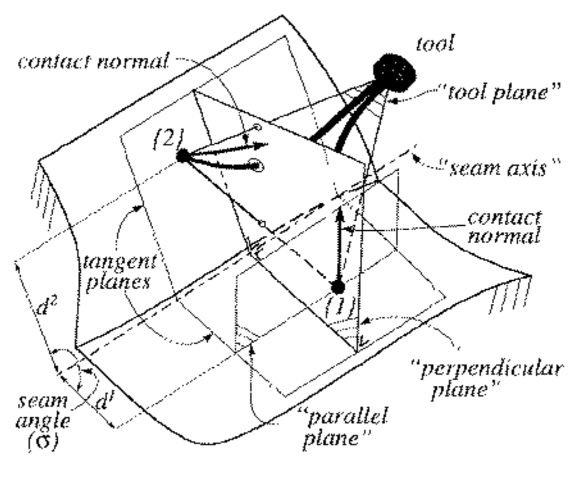

Figure 2: Two-point contact: geometric definitions.

The *tool plane* is defined as the plane through the two contact points and a third user-defined point p^t on the tool.

The tool plane has one coordinate-independent reference position when it lies in a perpendicular plane *and* $d^1 = d^2$. This situation is called the *symmetric* tool position. The tool looses one or two degrees of freedom if it is *parallel* to the seam, i.e., $d^1 = d^2 = 0$.

The contact frames at each of the two contact points have their Z axis along the contact normal. The X and Y axes are not uniquely determined geometrically; their direction can be freely chosen in the tangent plane. A contact frame is called *parallel* if its X axis is parallel to the seam axis, and the Y axis points towards the seam, Fig. 3.

2.2 Twist space basis

Each of the two contacts reduces the tool's motion freedom by one. With respect to the parallel reference frame in $\{1\}$, the bases for the five-dimensional twist space (i.e., the *Jacobian matrices* of the virtual manipulators at the two contacts) are:

$$J^1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (2)$$

$$J^2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -s_\sigma & 0 & -c_\sigma \\ 0 & 0 & -c_\sigma & 0 & s_\sigma \\ 1 & 0 & a & 0 & b \\ 0 & -c_\sigma & p_x c_\sigma & p_z & -p_x s_\sigma \\ 0 & s_\sigma & -p_x s_\sigma & -p_y & -p_x c_\sigma \end{bmatrix}. \qquad (3)$$

The first three rows represent angular velocity, the last three rows represent translational velocity. c_σ and s_σ are the cosine and sine of the seam angle σ; $a = -p_y c_\sigma + p_z s_\sigma$, and $b = p_y s_\sigma + p_z c_\sigma$. Column two of J^2 is used to simplify the other columns to:

$$J^2 \cong \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -s_\sigma & 0 & -c_\sigma \\ 0 & 0 & -c_\sigma & 0 & s_\sigma \\ 1 & 0 & a & 0 & b \\ 0 & -c_\sigma & 0 & p_z - p_y \frac{c_\sigma}{s_\sigma} & -\frac{p_x}{s_\sigma} \\ 0 & s_\sigma & 0 & 0 & 0 \end{bmatrix}. \qquad (4)$$

From this, it is clear that a basis for the twist space of the total constraint is found from either J^1 or J^2 with the second column removed. We define the total constraint's Jacobian matrix J as the matrix found from elementary column operations on J^2:

$$\begin{aligned} J &\cong \left[J_4^2 \;\; J_3^2 \;\; \left(s_\sigma J_5^2 - c_\sigma J_3^2 - p_y J_1^2\right) \;\; J_1^2\right] \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -s_\sigma & 0 & 0 \\ 0 & -c_\sigma & 1 & 0 \\ 0 & -p_y c_\sigma + p_z s_\sigma & 0 & 1 \\ p_z - p_y \frac{c_\sigma}{s_\sigma} & 0 & -p_x & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \qquad (5)$$

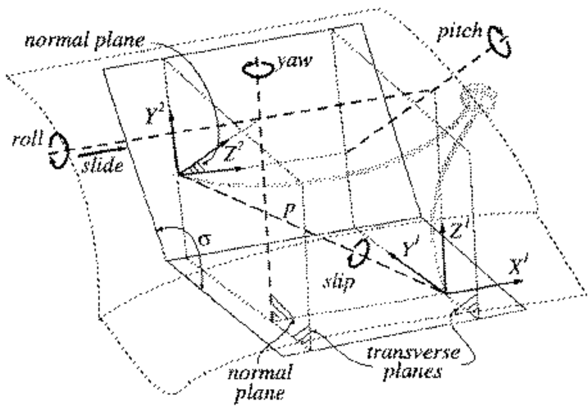

Figure 3: Two-point contact: *Roll, Pitch, Yaw, Slip,* and *Slide*.

These four remaining degrees-of-freedom represent rotations about, and translations along, geometrically defined lines, Fig. 3:

1. *Roll* is rotation about the intersection of the parallel planes through $\{1\}$ and $\{2\}$.

2. *Pitch* is rotation about the intersection of the perpendicular plane in $\{1\}$ and the parallel plane in $\{2\}$.

3. *Yaw* is the rotation about the intersection of the parallel plane in $\{1\}$ and the perpendicular plane in $\{2\}$.

4. *Slide* is translation in the direction of the seam axis. (It can also be considered as a rotation, i.e., about the intersection of the perpendicular planes in $\{1\}$ and $\{2\}$ which lies at infinity.)

Roll, Pitch and Yaw have (more or less) their original maritime interpretation if one looks at the tool as a "ship" travelling along the seam. Another coordinate-independent degree-of-freedom is the rotation about the line between the two contact points. This motion leaves the contact points unchanged on the environment; hence it is called *slip*. Slip is a linear combination of Roll, Pitch, Yaw and

Slide:

$$
\begin{aligned}
\boldsymbol{J}^{slip} &= [\boldsymbol{p}^T\ \boldsymbol{0}^T]^T \\
&= p_x\, \boldsymbol{J}^{roll} - \frac{p_y}{s_\sigma}\, \boldsymbol{J}^{pitch} + \left(p_z - p_y \frac{c_\sigma}{s_\sigma}\right)\, \boldsymbol{J}^{yaw} \\
&\quad - (p_z s_\sigma - p_y c_\sigma)\, \boldsymbol{J}^{slide}.
\end{aligned}
$$

2.3 Wrench space basis

The basis for the contact situation's wrench space (i.e., the force-controlled directions) is straightforward: it consists of unit forces along the contact normals in $\{1\}$ and $\{2\}$. The 6×2 matrix containing the coordinate expressions of these two forces is called the *wrench Jacobian matrix* and is denoted by $\boldsymbol{G}$.

2.4 Comparison to TF

The differences between a classical Task Frame model and the Roll-Pitch-Yaw-Slide/Slip model are:

1. The lines on which the basis twists and wrenches are defined do not intersect in one point.
2. The bases are time-varying, i.e., the relative positions of the lines changes during the motion, due to the curvature of the contact surfaces.

The similarities are that:

1. An intuitive, geometric and hence coordinate-independent twist and wrench space model exists.
2. Only zero or infinite pitch screws are needed. At least in the *geometric* model, since, due to the non-intersecting axes of the geometric model, no coordinate representation exists in which the Jacobian matrices $\boldsymbol{J}$ and $\boldsymbol{G}$ also contain only zero or infinite pitch screws.

2.5 Motion specification

The previous paragraphs describe bases allowing to specify unambiguously the instantaneous twist of the tool and the desired ideal wrench on the tool. However, a human user might like more intuitive ways of specifying the instantaneous twist or the desired position. The following paragraphs describe two possible approaches, a *local* one and a *global* one.

The local specification approach follows the classical TF intuition: each individual contact gets its own TF, as if it were the only contact occurring on the manipulated tool. However, the user should not specify more than four independent motions in both TFs together. It is then the controller's job to translate this local specification in an instantaneous twist that does not violate the contact constraints. This translation can, e.g., be done with "projection matrices" on the instantaneous twist space basis, [2].

The global approach relies on a model of the remaining four motion degrees of freedom, for example *Roll-Pitch-Yaw-Slide* as described above. Then, the well-known Jacobian equation, $\boldsymbol{t} = \boldsymbol{J}\dot{\boldsymbol{q}}$ applies, with $\boldsymbol{J}$ a basis of the twist space, and $\dot{\boldsymbol{q}}$ the magnitudes of the Roll, Pitch, Yaw and Slide basis twists. The advantage is that, *by construction*, any specified twist will be compatible with the modelled constraint. However, the resulting motion of each individual contact point might be less intuitive than in the local approach. If the user prefers to specify the desired *position* of the tool, instead of the desired *instantaneous twist*, he could for example specify desired values for the following four geometrically determined distances: d_1, p_z^t, p_x^2, and the desired position along the seam. Again, the controller is responsible for transforming these four numbers into a resultant motion that is instantaneously compatible with the contacts.

3 Three-point contact

The general contact situation with three contact points, Fig. 4, has three degrees of motion freedom. The following paragraphs present an intuitive and coordinate-independent way to model the instantaneous degrees of freedom in this contact situation.

3.1 Geometric parameters

Instantaneously, the contact situation is determined by the tangent planes at the three contact points. As in the two-point contact case, the unit normal vector at contact point i is denoted by $\boldsymbol{e}^i$. The geometric parameters defined in Sect. 2.1 exist in the three-point contact also, for each couple of contact points; the formulas to calculate these parameters remain unchanged. The notations, however, are slightly adapted, in order to discriminate the three possible combinations. For example, d_{13} denotes the distance between contact point 1 and the seam between the tangent planes in points 1 and 3; $\boldsymbol{e}^{13}$ is the unit vector parallel to this seam.

3.2 Twist space basis

A basis for the three-dimensional twist space can be chosen in many different ways. The following Jacobian

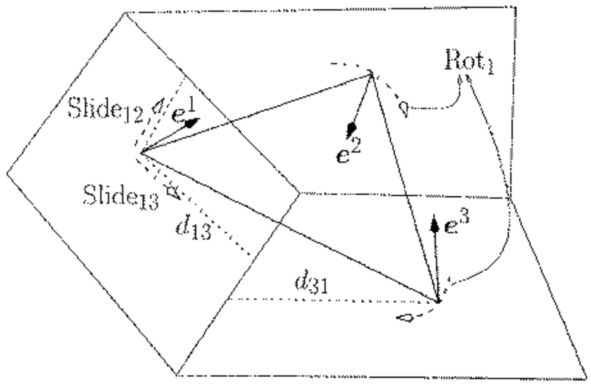

Figure 4: Three-point contact.

matrix has three basis twists that are an intuitively appealing extension to a classical Task Frame approach, Fig. 4:

$$J = [\text{Slide}_{ij}\ \text{Slide}_{ik}\ \text{Rot}_i]. \qquad (6)$$

Slide_{ij} is the translation of point i over its own tangent plane in the direction of the seam with point j; similarly for Slide_{ik}; Rot_i is the instantaneous pure rotation that leaves contact point i motionless, and moves the two other contact points in their local tangent planes. The basis in J can be used to specify the three available motion degrees of freedom by considering the motion of the contact point i only. Of course, all three basis motions must satisfy the instantaneous constraints. The following paragraphs explain how this is achieved:

1. **Slide**$_{ij}$. The seam between the contact points i and j is determined in exactly the same way as in the case of two-point contact. Hence, a corresponding "Roll" axis l_{ij} can be defined. Rotation about this axis makes the contact points i and j translate in their local tangent planes and perpendicular to the common seam axis. However, a pure rotation about this "Roll" axis is only possible if (i) the third tangent plane (i.e., the tangent plane at point $k, k \notin \{i, j\}$) is perpendicular to the two tangent planes that determine this "Roll" axis, or (ii) the third contact point k happens to lie on the "Roll" axis. Therefore, in general, a translational velocity v (Fig. 5) along the "Roll" axis should be added, in order to keep this third contact point k on its local tangent plane. Hence, the pure rotation "Roll" in the two-point contact case must be replaced by a *non-zero pitch screw* "Slide_{ij}" in the three-point contact case. The translational velocity component of this screw can be found as follows: $v_{ij,k}$ is the velocity of point k if it were to rotate about the "Roll" axis l_{ij}; v_k is the velocity in the tangent plane through k that makes the point k follow the rotation about the "Roll" axis without leaving its instantaneous tangent plane; v_k is perpendicular to e^k (since it lies in the tangent plane) and to $d_{ij,k}$ (i.e., the direction vector through k and perpendicular to the "Roll" axis l_{ij}); the translational component v of the "Slide_{ij}" screw is parallel to l_{ij}, and proportional to the tangent of the angle between the vector $v_{ij,k}$ and the unit vector along v_k. All these vectors and angles can be calculated with simple vector calculus.

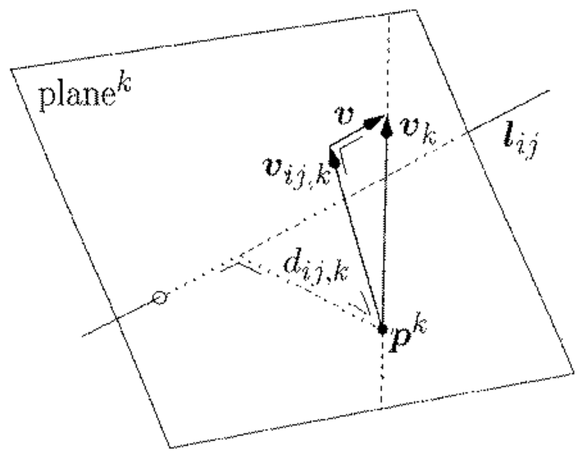

Figure 5: Three-point contact: velocity components due to slide.

2. **Rot**$_i$. Rotation about an axis through i moves the contact point j in the direction perpendicular to both $p^{i,j}$ (i.e., the vector from point i to point j, since point i remains motionless) and the normal direction e^j (since point j must move in its tangent plane). Hence, the axis of **Rot**$_i$ goes through i and has direction vector $(p^{i,j} \times e^j) \times (p^{i,k} \times e^k)$.

3.3 Wrench space basis

The basis for the contact situation's wrench space is equally straightforward as in the two-point contact case: it consists of unit forces along the contact normals in $\{1\}$, $\{2\}$ and $\{3\}$. The wrench Jacobian matrix G is now a 6×3 matrix, containing the coordinate expressions of these three forces.

3.4 Comparison to TF

The differences between a classical Task Frame model and the above-described three-point contact model are:

1. The lines on which the basis twists and wrenches are defined do not intersect in one point.

2. The bases are time-varying, i.e., the relative positions of the lines change during the motion, due to the curvature of the contact surfaces.

3. The twist space cannot be spanned anymore by pure translations and/or pure rotations.

3.5 Motion specification

As in the two-point contact case, both *local* and *global* motion specifications are possible.

In the local approach, (part of) the motion of each contact point individually is specified as if no constraints were acting on the object. At the user level, the constraints are only taken into account by the requirement that one should not specify more than three independent velocity set-points. The others get "don't care" values that the robot controller must fill in in such a way that the resulting motion is compatible with the constraints.

In the global approach, the user constructs a compatible instantaneous motion by using, for example, the Jacobian matrix in Eq. (6). Alternatively, he can specify three desired distances of the contact points to some of the seams; for example, the distances of one of the three contact points to its two neighbouring seams, together with the distance of one of the other contact points to the tangent plane of the first contact point. The controller must again take care of the instantaneous motion interpolation required to reach the specified goal without violating the contact constraints. To this end, he can use the instantaneous twist space basis in Eq. (6).

4 Conclusions

This paper has described how classical Task Frame motion constraint modelling and motion specification procedures are extended to contact situations with two or three contact points. The presented approach is completely coordinate independent and requires only the knowledge of the positions of the contact points as well as the contact normal directions in each of the points. The two presented contact models keep most of the intuitiveness of the Task Frame approach, but have nevertheless to compromise on two points: (i) some basis screws in the models are not pure translations or pure rotations, and (ii) the screw axes don't always intersect in one single point.

If the contact surfaces are curved, on-line "tracking" algorithms are required in order to be able to continuously update the contact normal directions during the motion of the contact points. This tracking can be done, for example, with the "velocity-based" tracking approach explained in [1, 4].

Acknowledgment

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