monorail, rolling stock, mathematical model, swaying amplitude

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DYNAMIC MODEL OF MOVEMENT OF MINE SUSPENDED MONORAIL

Summary. In the article we have developed the dynamic model of interaction of rolling stock during the movement, on the suspended monorail, taking into account the side-sway. We have received the motion equations, carried out their analysis and determined the own oscillation frequencies of rolling stock of suspended monorail.

ДИНАМИЧЕСКАЯ МОДЕЛЬ ДВИЖЕНИЯ ШАХТНОЙ ПОДВЕСНОЙ МОНОРЕЛЬСОВОЙ ДОРОГИ

Аннотация. В статье разработана динамическая модель взаимодействия подвижного состава при движении по подвесному монорельсу с учетом бокового раскачивания. Получены уравнения движения, проведен их анализ и определены собственные частоты колебаний подвижного состава подвесной монорельсовой дороги.

1. INTRODUCTION

One of the problems of mine suspended monorails is side-sway of rolling stock which limits the speed of train motion. It involves increase of side clearance and excavation cross-sections therefore it leads to decrease of mine transport operating efficiency.

In the works [1, 2, 3] there have been given regularities of movements of suspended monorails and researched crew side-sway during the movement on monorail with irregularities. Works [4, 5] cover linear and nonlinear oscillations of structure units. Scientific substantiation of parameters for monorail suspension in the workings fixed by bolting is mentioned in the work [6], and for arched support it is in the work [7]. The estimation of operating parameters of suspended monorails for mines is mentioned in the work [8]. This article is the continuation of mentioned works. The objective of this research is the examination of process of interaction of rolling stock during the movement on suspended monorail taking into account side-sway.

2. RESERCH AND RESULTS

During the development of mathematical model of mine suspended monorail movement the following assumptions were taken into consideration. The rolling stock is represented as unsparing crew and in the form of one mass dynamic model (fig. 1). The motion speed is taken as constant. The monorail is regarded as solid having long plane of symmetry and double-point suspension system. Friction forces at the same time are not taken into consideration. Unbalance and hydroscope moments of gyrating mass of rolling stock are equal to zero. We examine only the single crew movement and, in such way, the coupler effect is removed.

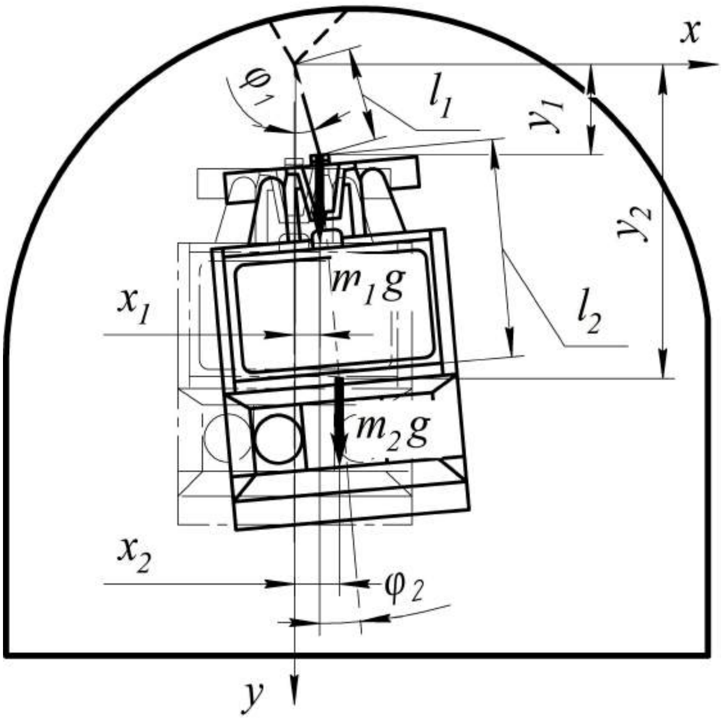

Fig. 1. The scheme of suspension of rolling stock of monorail
Рис. 1. Схема подвески подвижного состава монорельсовой дороги

For parameters of suspended monorail there have been introduced the following symbols: m_1 – reduced mass of suspended monorail; m_2 – reduced mass of rolling stock; l_1 – monorail suspension length; l_2 – distance from monorail suspension axes to gravity centre of rolling stock.

Let's choose as generalized coordinates the angles φ_1 and φ_2, which form with vertical the segments l_1 and l_2 respectively. Coordinates x_1 and y_1 can be represented as $x_1 = l_1 \sin\varphi_1$; $y_1 = l_1 \cos\varphi_1$.

Kinetic and potential energies of monorail m_1 are determined

$$T_1 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2); \tag{1}$$

$$U_1 = -m_1 g y_1, \tag{2}$$

where: g – gravitational acceleration.

Hence

$$T_1 = \frac{1}{2} m_1 l_1^2 \dot{\varphi}_1^2; \tag{3}$$

$$U_1 = -m_1 g l_1 \cos\varphi_1. \tag{4}$$

The same is for kinetic and potential energies of rolling stock

$$T_2 = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2); \tag{5}$$

$$U_2 = -m_2 g (l_1 \cos\varphi_1 + l_2 \cos\varphi_2), \tag{6}$$

where: $x_2 = l_1 \sin\varphi_1 + l_2 \sin\varphi_2, \quad y_2 = l_1 \cos\varphi_1 + l_2 \cos\varphi_2.$

Hence

$$T_2 = \frac{1}{2} m_2 \left(l_1^2 \dot{\varphi}_1^2 + l_2^2 \dot{\varphi}_2^2 + 2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2) \right). \tag{7}$$

Finally for Lagrange's function of given system we have:

$$\begin{aligned} L = T_1 + T_2 - (U_1 + U_2) = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\varphi}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\varphi}_2^2 + \\ + m_2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2) + (m_1 + m_2) g l_1 \cos\varphi_1 + m_2 g l_2 \cos\varphi_2. \end{aligned} \tag{8}$$

Then Lagrange's functions describing system will be

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\varphi}_1} - \frac{\partial L}{\partial \varphi_1} = 0; \tag{9}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\varphi}_2} - \frac{\partial L}{\partial \varphi_2} = 0. \tag{10}$$

After substitution and differentiation we will get Lagrange equations

$$\begin{cases} (m_1 + m_2)(l_1 \ddot{\varphi}_1 + g \sin\varphi_1) + m_2 l_2 \ddot{\varphi}_2 \cos(\varphi_1 - \varphi_1) + m_2 l_2 \dot{\varphi}_2^2 \sin(\varphi_1 - \varphi_2) = 0; \\ l_1 \ddot{\varphi}_1 \cos(\varphi_1 - \varphi_2) + l_2 \ddot{\varphi}_2 - l_1 \dot{\varphi}_1^2 \sin(\varphi_1 - \varphi_2) + g \sin\varphi_2 = 0 \end{cases} \tag{11}$$

or for small frequencies

$$\begin{cases} (m_1 + m_2)(l_1 \ddot{\varphi}_1) + m_2 l_2 \ddot{\varphi}_2 + (m_1 + m_2) g \varphi_1 = 0; \\ l_1 \ddot{\varphi}_1 + l_2 \ddot{\varphi}_2 + g \varphi_2 = 0. \end{cases} \tag{12}$$

Let's introduce coefficient symbols $\mu_m = \frac{m_2}{m_1}$ and $\mu_l = \frac{l_2}{l_1}$.

Then

$$\begin{cases} \ddot{\varphi}_1 + \omega_0^2 (1 + \mu_m) \varphi_1 - \omega_0^2 \mu_m \varphi_2 = 0; \\ \ddot{\varphi}_1 + \mu_l \ddot{\varphi}_2 + \omega_0^2 \varphi_2 = 0, \end{cases} \tag{13}$$

where $\omega_0 = \sqrt{\frac{g}{l_1}}$.

Let's find the answer in the form

$$\varphi_1 = A_1 \cos\omega t + B_1 \sin\omega t, \tag{14}$$

$$\varphi_2 = A_2 \cos\omega t + B_2 \sin\omega t. \tag{15}$$

For this system we will get characteristic equation

$$\begin{vmatrix} \omega_0(1+\mu_m)-\omega^2 & -\mu_m\omega_0^2 \\ -\omega^2 & \omega^2-\mu_l\omega^2 \end{vmatrix} = 0. \tag{16}$$

On this base the frequencies equation will be

$$\mu_l\omega^4 - \omega_0\omega^2(1+\mu_m)(1+\mu_l) + \omega_0^4(1+\mu_m) = 0. \tag{17}$$

The frequencies equation roots are

$$\omega_1 = \pm\omega_0\sqrt{\frac{1}{2\mu_l}\left((1+\mu_m)(1+\mu_l) - \sqrt{(1+\mu_m)^2(1+\mu_l)^2 - 4\mu_l(1+\mu_m)}\right)}, \tag{18}$$

$$\omega_2 = \pm\omega_0\sqrt{\frac{1}{2\mu_l}\left((1+\mu_m)(1+\mu_l) + \sqrt{(1+\mu_m)^2(1+\mu_l)^2 - 4\mu_l(1+\mu_m)}\right)}. \tag{19}$$

In the fig. 2 there have been given the graphic charts of frequencies dependences $\omega_1 = f(\mu_l)$ and $\omega_2 = f(\mu_l)$ for different values of monorail suspension length l_1 and coefficients μ_m. From these charts we can see that with increasing suspension length from 0.1 m till 0.4 m the first and the second oscillation frequencies decrease more than by twice.

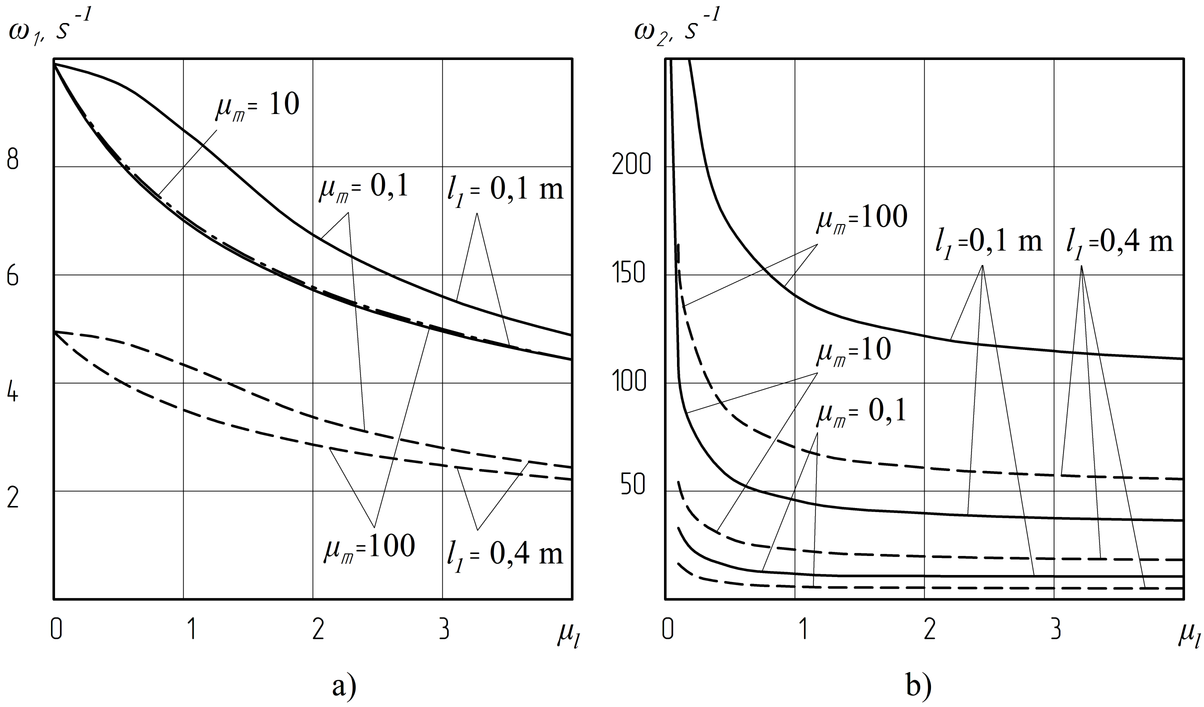

Fig. 2. Graphic dependences: a) – $\omega_1 = f(\mu_l)$; b) – $\omega_2 = f(\mu_l)$

Рис. 2. Графические зависимости: а) – $\omega_1 = f(\mu_l)$; b) – $\omega_2 = f(\mu_l)$

With increasing of values μ_l from 0 to 4 one can see the decreasing of frequency ω_1. With further increasing μ_l this frequency practically doesn't change. The same is with the frequency ω_2. Increasing the values μ_m in range from 0.1 till 100.0 frequencies decrease. But for the first frequency the decrease is not more than 10%, and for the second is more than by twice.

Dependences $\omega_1 = f(\mu_m)$ and $\omega_2 = f(\mu_m)$ at different values are given the length of monorail suspension l_1 in fig.3. From graphic charts it is followed that the length l_1 essentially influences the frequencies. For l_1=0.1 m and $\mu_m \le 100$ first frequencies change from 9,9 till 7,0 s^{-1}, for l_1=0.4 m – from 4.2 till 3,6 s^{-1}, and for l_1=1.0 m – from 2.8 till 2.2 s^{-1}.

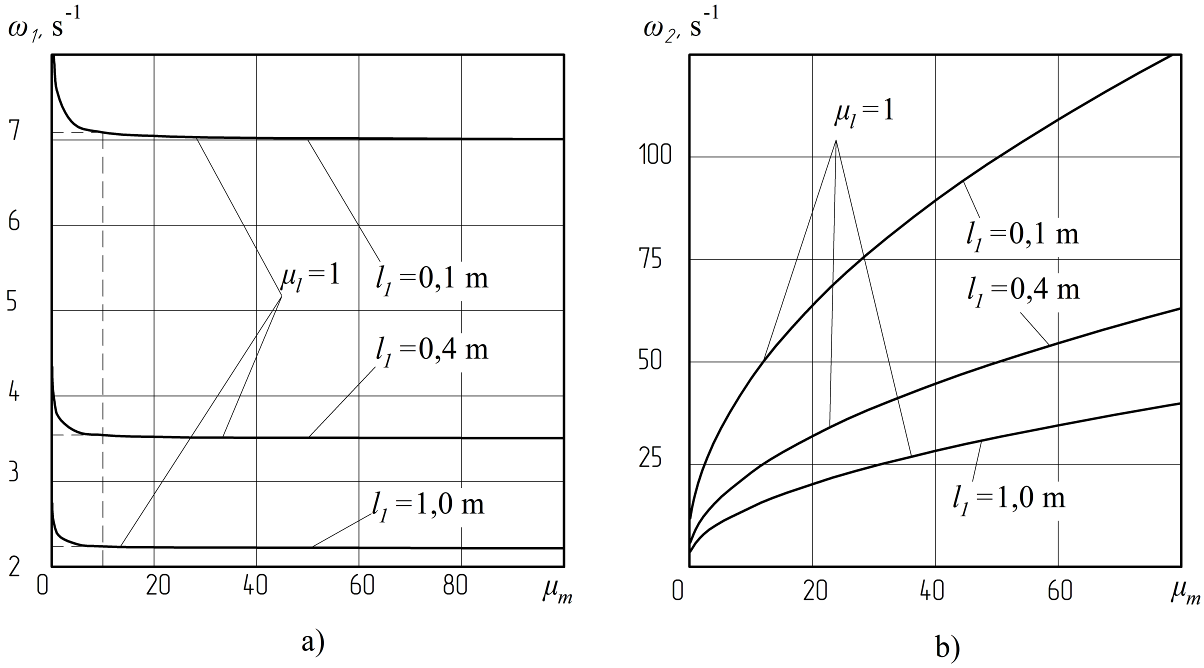

Fig. 3. Graphic dependences: a) – $\omega_1 = f(\mu_m)$; b) – $\omega_2 = f(\mu_m)$

Рис. 3. Графические зависимости: a) – $\omega_1 = f(\mu_m)$; b) – $\omega_2 = f(\mu_m)$

In fig.3 one can see that for the first frequencies with increase of coefficient μ_m it is occurred the decrease ω_1, and for the second ones it is occurred the increase ω_2. So with increase of the coefficient μ_m in the range from 0.1 till 10.0 the first frequencies decrease and at the further increase μ_m practically don't change. The second frequencies vice versa increase and reach their limit for values μ_m considerably more than 100.

3. CONCLUSIONS

The received mathematical model of process of interaction of rolling stock during the movement on suspended monorail taking into account side-sway will be used for improvement of already existing and anew projecting monorails. With the purpose of précising of received dependences in future it is planned to carry out the theoretical researches taking into account the forced oscillations caused by the effect of disturbances from horizontal and vertical irregularities of monorail.

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Received 19.10.2012; accepted in revised form 10.01.2014