# Globally Stabilizing Switching Controllers for a Centrifugal Compressor Model with Spool Dynamics

Alexander Leonessa, Wassim M. Haddad, *Member, IEEE*, and Hua Li

***Abstract*—In this paper we develop a globally stabilizing stability-based switching controller for a three-state lumped parameter centrifugal compressor surge model. The proposed model involves pressure and mass flow compression system dynamics as well as spool dynamics to account for the influence of speed transients on the compression surge dynamics. The proposed nonlinear switching controller architecture involves throttle and compressor torque regulation and is directly applicable to compression systems with actuator amplitude and rate saturation constraints.**

***Index Terms*—Centrifugal compressors, surge, state-space model, globally stabilizing control, stability-based switching control.**

## I. Introduction

The desire for developing an integrated control system-design methodology for advanced propulsion systems has led to significant activity in modeling and control of flow compression systems in recent years (see, for example, [1]–[14] and the numerous references therein). While the literature on modeling and control of compression systems predominantly focuses on axial flow compression systems, the research literature on centrifugal flow compression systems is rather limited in comparison. Notable exceptions include [15]–[21] which address modeling and control of centrifugal compressors. In contrast to axial flow compression systems involving the aerodynamic instabilities of rotating stall and surge, a common feature of [15]–[21] is the realization that surge and deep surge is the predominant aerodynamic instability arising in centrifugal compression systems. Surge is a one-dimensional axisymmetric global compression system oscillation which involves radial flow oscillations and in some case even radial flow reversal (deep surge) which can damage engine components.

In this paper we address the problem of nonlinear stabilization for centrifugal compression systems. First, we present a three-state lumped parameter model for surge in centrifugal flow compression systems that is accessible to control-system designers requiring state-space models for modern nonlinear control. The low-order centrifugal compression system model presented here closely parallels the model developed in [21] and hence only salient portions of the model are presented which are relevant for the proposed control design framework. Specifically, the authors in [21] develop a centrifugal compression system model involving pressure and mass flow compression system dynamics using principles of conservation of mass and momentum. Furthermore, in order to account for the influence of speed transients on the compression surge dynamics, turbocharger spool dynamics are also considered.

Next, using a nonlinear stability-based switching framework, we develop a globally stabilizing control law for the lumped parameter centrifugal compressor surge model. The locus of equilibrium points on which the switching nonlinear controller is predicated on is characterized by the axisymmetric pressure-flow equilibria of the compression system. A similar controller design framework for *axial* flow compression systems was developed in [22], [23]. The proposed switching nonlinear state feedback controller is directly applicable to centrifugal compression systems with amplitude and rate saturation constraints. Finally, even though for simplicity of exposition we do not address system parametric uncertainty, the proposed controller can be extended as in [23] to provide robust stability guarantees in the face of system uncertainty.

## II. Governing Fluid Dynamic Equations for Centrifugal Compression Systems

In this section we develop a low-order three-state surge model for centrifugal compressors. Specifically, we consider the basic centrifugal compression system shown in Fig. 1, consisting of a short inlet duct, a compressor, an outlet duct, a plenum, an exit duct, and a control throttle. We assume that the plenum dimensions are large as compared to the compressor-duct dimensions so that the fluid velocity and acceleration in the plenum are negligible. In this case the pressure in the plenum is spatially uniform. Furthermore, we assume that the flow is controlled by a throttle at the plenum exit. In addition, we assume a low-speed compression system with oscillation frequencies much lower than the acoustic resonance frequencies so that the flow can be considered incompressible. However, we do assume that the gas in the plenum is compressible and acts as a gas spring. Finally, we assume isentropic process dynamics in the plenum and negligible gas angular momentum in the compressor passages as compared to the impeller angular momentum.

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The authors are with the School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150 USA (e-mail: wm.haddad@aerospace.gatech.edu).

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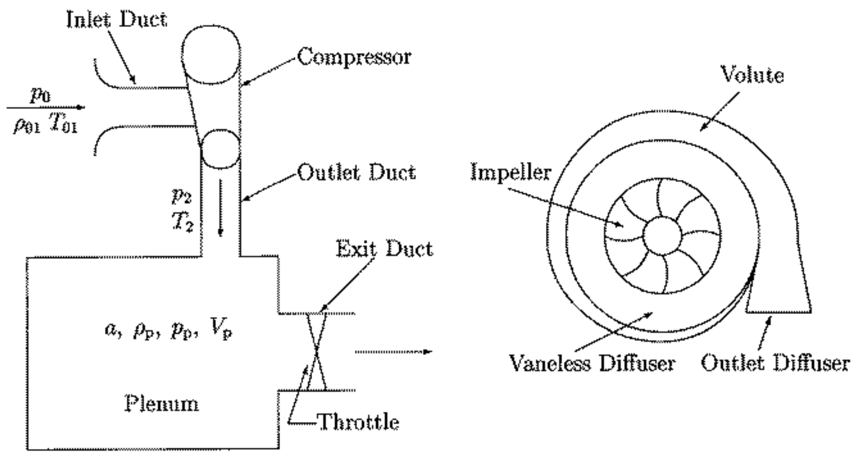


Fig. 1. Centrifugal compressor system geometry.

### A. Conservation of Mass in the Plenum

Using continuity and assuming the plenum is a rigid volume with isentropic flow dynamics, it follows that mass conservation in the plenum is given by [21]

$$\frac{dp_p}{dt} = \frac{a^2}{V_p}(m_c - m_t) \tag{1}$$

where $m_c$ is the mass flow rate at the plenum entrance, $m_t$ is the mass flow rate through the throttle, $V_p$ is the plenum volume, $p_p$ is the flow pressure inside the plenum, and $a$ is the ambient sonic velocity. Next, assuming that the throttle discharges to an infinite reservoir with pressure $p_0$ it follows that the pressure difference $p_p - p_0$ must balance both the throttle pressure loss and the net difference in pressure due to the flow acceleration through the throttle duct. Here we model the flow through the throttle by [3]

$$m_t = k_t\sqrt{p_p - p_0} \tag{2}$$

where the parameter $k_t$ is proportional to the throttle opening and $p_0$ is the downstream pressure. If the plenum exit duct is short, then $p_0$ can be regarded as the ambient pressure. Now substituting (2) into (1) and defining the nondimensional pressure, mass flow, and time, respectively, by

$$\psi \triangleq \frac{p_p - p_0}{p_0}, \quad \phi \triangleq \frac{a}{Ap_0}m_c, \quad \xi \triangleq \frac{Aa}{L^3}t \tag{3}$$

where $A$ is the cross sectional area of compressor exit duct and $L$ is the length of the compressor duct, it follows that

$$\dot{\psi} = \bar{a}(\phi - \gamma_{\rm th}\sqrt{\psi}) \tag{4}$$

where $\dot{(\,)}$ represents differentiation with respect to nondimensional time $\xi$ and

$$\bar{a} \triangleq \frac{L^3}{V_p}, \quad \gamma_{\rm th} \triangleq \frac{ak_t}{A\sqrt{p_0}}. \tag{5}$$

### B. Conservation of Momentum

Using a momentum balance with the assumption of incompressible flow, it follows that the pressure difference between the exit of the compressor and the plenum is proportional to the rate of change of the mass flow rate, that is,

$$A(p_2 - p_p) = L\frac{dm_c}{dt} \tag{6}$$

where $p_2$ is the pressure rise at the exit of the compressor. Next, assuming isentropic process dynamics with a constant specific heat $c_p$, it follows that [24]

$$\frac{p_2}{p_0} = \left(\frac{T_2}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \tag{7}$$

where $T_{01}$ is the compressor inlet temperature, $T_2$ is the fluid temperature at compressor rotor exit, and $\gamma$ is the specific heat ratio. Now, using $\Delta h_{\rm ideal} = c_p(T_2 - T_{01})$, where $\Delta h_{\rm ideal}$ is the ideal change in fluid specific enthalpy which, for a conservative system, is equal to the work done by the compressor rotor, it follows from (7) that

$$\frac{p_2}{p_0} = \left(1 + \frac{\Delta h_{\rm ideal}}{c_pT_{01}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{8}$$

The energy delivered to the fluid by the compressor is given by

$$\Delta h(m_c, \omega) = \Delta h_{\rm ideal}(\omega) + \Delta h_{\rm loss}(m_c, \omega) \tag{9}$$

where $\omega$ is the angular velocity of the compressor spool and

$$\begin{aligned}\Delta h_{\rm loss}(m_c, \omega) \triangleq\; & \Delta h_{\rm ii}(m_c, \omega) + \Delta h_{\rm di}(m_c, \omega) \\ & + \Delta h_{\rm if}(m_c) + \Delta h_{\rm df}(m_c)\end{aligned} \tag{10}$$

where $\Delta h_{\rm ii}(m_c, \omega)$, $\Delta h_{\rm di}(m_c, \omega)$, $\Delta h_{\rm if}(m_c)$, $\Delta h_{\rm df}(m_c)$, correspond to incidence and frictional losses at the inducer and diffuser, respectively. For details see [21].

In order to capture compressor efficiency, define the isentropic efficiency as [21]

$$\eta_c(m_c, \omega) \triangleq \frac{\Delta h_{\rm ideal}(\omega)}{\Delta h_{\rm loss}(m_c, \omega) + \Delta h_{\rm ideal}(\omega)}. \tag{11}$$

Substituting (8) and (11) into (6) yields

$$\frac{dm_c}{dt} = \frac{A}{L}\left[\left(1 + \eta_c(m_c, \omega)\frac{\Delta h(\omega)}{c_pT_{01}}\right)^{\frac{\gamma}{\gamma-1}} p_0 - p_p\right]. \tag{12}$$

Next, defining the nondimensional angular velocity of compressor spool by $\tilde{\omega} \triangleq r\omega/a$, where $r$ is the radius of the rotor tip, and using (3), it follows that

$$\dot{\phi} = \bar{b}(\psi_c(\phi, \tilde{\omega}) - \psi) \tag{13}$$

where $\bar{b} \triangleq L^2/A$ and $\psi_c(\phi, \tilde{\omega})$ is the compressor characteristic pressure-flow/angular velocity map given by

$$\psi_c(\phi, \tilde{\omega}) \triangleq \left(1 + \eta_c(\phi, \tilde{\omega})\sigma\bar{d}\tilde{\omega}^2\right)^{\frac{\gamma}{\gamma-1}} - 1 \tag{14}$$

where

$$\eta_c(\phi, \tilde{\omega}) = \frac{\sigma\tilde{\omega}^2}{\sigma\tilde{\omega}^2 + \frac{1}{2}(f_1\tilde{\omega} - f_2\phi)^2 + \frac{1}{2}(\sigma\tilde{\omega} - f_3\phi)^2 + f_4\phi^2 + f_5\phi^2} \tag{15}$$

$\sigma$ is the slip factor corresponding to the ratio between the tangential velocity of the fluid at the rotor outlet and the rotor tip velocity, and $\bar{d}$, $f_i$, $i = 1, \cdots, 5$, are nondimensional parameters related to the compressor dimensions, the ambient sonic velocity, the inducer and rotor geometry, and the friction coefficients. For details see [21].

It is important to note that the compressor characteristic map given by (14) holds for the case where the flow through the compressor is positive. In the case of deep surge involving negative mass flow, it is assumed that the pressure rise in the compressor is proportional to the square of the mass flow so that [17], [25]

$$\psi_c(\phi, \tilde{\omega}) = \mu\phi^2 + \psi_{c0}(\tilde{\omega}), \quad \phi < 0 \tag{16}$$

where $\mu$ is a constant and

$$\psi_{c0}(\tilde{\omega}) \triangleq \psi_c(\phi, \tilde{\omega})|_{\phi=0} = (1 + \sigma\eta_{c0}\bar{d}\tilde{\omega}^2)^{\frac{\gamma}{\gamma-1}} - 1 \tag{17}$$

where

$$\eta_{c0} \triangleq \eta_c(\phi, \tilde{\omega})|_{\phi=0} = \frac{2\sigma}{\sigma^2 + 2\sigma + f_1^2}. \tag{18}$$

Now, for a fixed $\tilde{\omega}$, taking the gradient of $\psi_c(\phi, \tilde{\omega})$ with respect to the nondimensional flow $\phi$ it follows that the flow corresponding to the maximum pressure point of the compressor characteristic map is directly proportional to the nondimensional angular velocity of the compressor spool and is given by $\phi_{\max} = k_f\tilde{\omega}$, where

$$k_f \triangleq \frac{f_1 f_2 + \sigma f_3}{f_2^2 + f_3^2 + 2f_4 + 2f_5}. \tag{19}$$

Similarly, for a fixed $\tilde{\omega}$ taking the gradient of $\eta_c(\phi, \tilde{\omega})$ with respect to the nondimensional flow $\phi$ we obtain that the maximum value for the isentropic efficiency is given by (20), shown at the bottom of the page. Note that $\eta_{c_{\max}}$ is constant for all spool speeds. This indicates that the compressor achieves the same maximum isentropic efficiency at each maximum pressure point for all spool speeds. However, since these points are critically stable, the need for active control is severe to guarantee stable compression system operation for peak compressor performance. Fig. 2 shows a typical family of compressor characteristic maps for different spool speeds along with the corresponding constant isentropic efficiency lines. The stone wall depicted in Fig. 2 corresponds to choked flow at a given cross-section of the compression system. For details see [21]

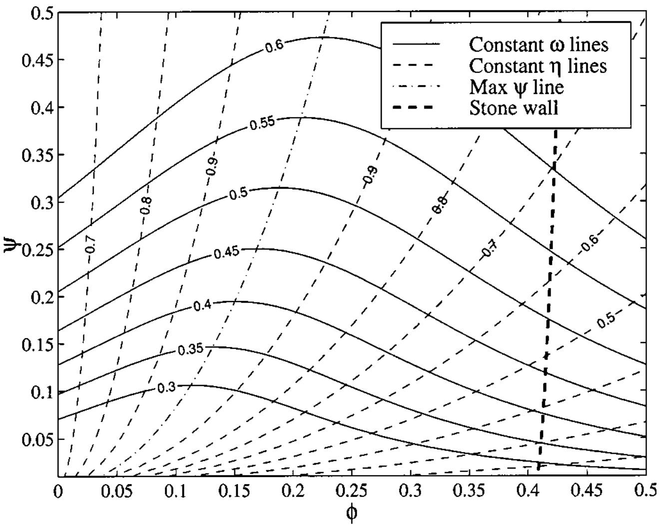


Fig. 2. Compressor characteristic maps and efficiency lines for different spool speeds.

## C. Turbocharger Spool Dynamics

Using conservation of angular momentum in the turbocharger spool it follows that the spool dynamics are given by

$$I_s\frac{d\omega}{dt} = \tau_d - \tau_c(m_c, \omega) \tag{21}$$

where $I_s$ is the spool mass moment of inertia, $\tau_d$ is the driving torque, and $\tau_c(m_c, \omega)$ is the compressor torque. Now, using the fact that the change in angular momentum of the fluid is equal to the compressor torque and assuming absence of prewhirl at the rotor inlet, it follows that [26], [27]

$$\tau_c(m_c, \omega) = \sigma r^2 m_c\omega. \tag{22}$$

Substituting (22) into (21) we obtain

$$I_s\frac{d\omega}{dt} = \tau_d - \sigma r^2 m_c\omega \tag{23}$$

$$\eta_{c_{\max}} = \frac{2\sigma\left(f_2^2 + f_3^2 + 2(f_4 + f_5)\right)}{\sigma(2+\sigma)f_2^2 - 2\sigma f_1 f_2 f_3 + (2\sigma + f_1^2)\,f_3^2 + 2\,(2\sigma + \sigma^2 + f_1^2)\,(f_4 + f_5)} \tag{20}$$

which, using (3) and $\tilde{\omega} = r\omega/a$, can be written in nondimensional form as

$$\dot{\tilde{\omega}} = \bar{c}(\tau - \sigma\phi\tilde{\omega}) \tag{24}$$

where

$$\bar{c} \triangleq \frac{L^3 r^2 p_0}{I_s a^2}, \quad \tau \triangleq \frac{\tau_d}{A p_0 r}. \tag{25}$$

## III. Parameterized System Equilibria and Local Set Point Designs

The stability-based switching nonlinear control strategy developed in this paper stabilizes a given nonlinear system by stabilizing a collection of nonlinear controlled subsystems over a parameterized set of system equilibria. In this section we develop Lyapunov-based subcontroller designs of the local set points parameterized by the axisymmetric stable pressure-flow equilibrium branch given by (14) for a fixed angular velocity of the compressor spool. It is important to note that even though a Lyapunov-based framework can be used to stabilize the compression system, the resulting controller may generate unnecessarily large control amplitude and rate signals that can amplitude and rate saturate the control actuators resulting in system performance degradation and even instability (see [28] and the references therein). To proceed with the local set point designs, first note that with control inputs $u_1 \triangleq \gamma_{\rm th}\sqrt{\psi}$ and $u_2 \triangleq \tau$ it follows from (4), (13), and (24), that a state-space model for the centrifugal compressor is given by

$$\dot{\psi} = \bar{a}(\phi - u_1) \tag{26}$$

$$\dot{\phi} = \bar{b}(\psi_c(\phi, \tilde{\omega}) - \psi) \tag{27}$$

$$\dot{\tilde{\omega}} = \bar{c}(u_2 - \sigma\phi\tilde{\omega}). \tag{28}$$

Next, note that for fixed values of the control inputs $u_1$ and $u_2$, (26), (27) and (28) give an equilibrium point $(\psi_{\rm eq}, \phi_{\rm eq}, \tilde{\omega}_{\rm eq})$, where $(\psi_{\rm eq}, \phi_{\rm eq}, \tilde{\omega}_{\rm eq})$ is given by

$$(\psi_{\rm eq}, \phi_{\rm eq}, \tilde{\omega}_{\rm eq}) = \left(\psi_c(\phi_{\rm eq}, \tilde{\omega}_{\rm eq}), u_{1_{\rm eq}}, \frac{u_{2_{\rm eq}}}{\sigma\phi_{\rm eq}}\right). \tag{29}$$

Defining the shifted variables $x_1 \triangleq \psi - \psi_{\rm eq}$, $x_2 \triangleq \phi - \phi_{\rm eq}$, and $x_3 \triangleq \tilde{\omega} - \tilde{\omega}_{\rm eq}$, so that the given equilibrium point is translated to the origin, along with the shifted controls $\tilde{u}_1 \triangleq u_1 - u_{1_{\rm eq}}$ and $\tilde{u}_2 \triangleq u_2 - u_{2_{\rm eq}}$, it follows that the parameterized translated nonlinear system is given by

$$\dot{x}_1 = \bar{a}(x_2 - \tilde{u}_1) \tag{30}$$

$$\dot{x}_2 = \bar{b}(\psi_{C_{\rm eq}}(x_2, x_3) - x_1) \tag{31}$$

$$\dot{x}_3 = \bar{c}(\tilde{u}_2 - f(x_2, x_3)) \tag{32}$$

where

$$\psi_{C_{\rm eq}}(x_2, x_3) \triangleq \psi_c(\phi_{\rm eq} + x_2, \tilde{\omega}_{\rm eq} + x_3) - \psi_c(\phi_{\rm eq}, \tilde{\omega}_{\rm eq}) \tag{33}$$

$$f(x_2, x_3) \triangleq \sigma(\phi_{\rm eq} x_3 + \tilde{\omega}_{\rm eq} x_2) + \sigma x_2 x_3. \tag{34}$$

Now, setting

$$\hat{u} = x_2 - \tilde{u}_1 \tag{35}$$

$$\tilde{u}_2 = -k_3 x_3 + f(x_2, x_3) \tag{36}$$

where $k_3 > 0$, and substituting (35) and (36) into (30)–(32) yields

$$\dot{x}_1 = \bar{a}\hat{u} \tag{37}$$

$$\dot{x}_2 = \bar{b}\left(\psi_{C_{\rm eq}}(x_2, x_3) - x_1\right) \tag{38}$$

$$\dot{x}_3 = -k_3 \bar{c} x_3. \tag{39}$$

Next, consider the equilibria-dependent Lyapunov function candidate

$$\begin{aligned} V_{\rm eq}(x_1, x_2, x_3) = \frac{\alpha_1}{2}(x_1 - \psi_{C_{\rm eq}}(x_2, x_3) - k_2 x_2)^2 \\ + \frac{\alpha_2}{2}x_2^2 + \frac{\alpha_3}{2}x_3^2 \end{aligned} \tag{40}$$

where $k_2 > 0$, $\alpha_i > 0$, $i = 1, 2, 3$. The corresponding Lyapunov derivative is given by

$$\begin{aligned} &\dot{V}_{\rm eq}(x_1, x_2, x_3) \\ &\quad = \alpha_3 x_3 \dot{x}_3 + \alpha_2 x_2 \dot{x}_2 + \alpha_1\left(x_1 - \psi_{C_{\rm eq}}(x_2, x_3) - k_2 x_2\right) \\ &\qquad \times \left(\dot{x}_1 - \dot{\psi}_{C_{\rm eq}}(x_2, x_3) - k_2 \dot{x}_2\right) \\ &\quad = -\left(x_1 - \psi_{C_{\rm eq}}(x_2, x_3) - k_2 x_2\right) \\ &\qquad \times \left(-\alpha_1 \bar{a}\hat{u} + \alpha_1 \psi_{C_{\rm eq}, x_2}(x_2, x_3)\dot{x}_2\right. \\ &\qquad \left. + \alpha_1 \psi_{C_{\rm eq}, x_3}(x_2, x_3)\dot{x}_3 + \alpha_1 k_2 \dot{x}_2 + \alpha_2 \bar{b} x_2\right) \\ &\qquad - \alpha_3 k_3 \bar{c} x_3^2 - \alpha_2 k_2 \bar{b} x_2^2 \end{aligned} \tag{41}$$

where

$$\begin{aligned} \psi_{C_{\rm eq}, x_2}(x_2, x_3) &\triangleq \frac{\partial \psi_{C_{\rm eq}}(x_2, x_3)}{\partial x_2} \\ &= \left.\frac{\partial \psi_c(\phi, \tilde{\omega})}{\partial \phi}\right|_{(\phi, \tilde{\omega}) = (\phi_{\rm eq} + x_2, \tilde{\omega}_{\rm eq} + x_3)} \\ &= \frac{\gamma\sigma\bar{d}\tilde{\omega}^2}{\gamma - 1}\left[\psi_c(\phi_{\rm eq} + x_2, \tilde{\omega}_{\rm eq} + x_3) + 1\right]^{\frac{1}{\gamma}} \\ &\quad \times \left.\frac{\partial \eta_c(\phi, \tilde{\omega})}{\partial \phi}\right|_{(\phi, \tilde{\omega}) = (\phi_{\rm eq} + x_2, \tilde{\omega}_{\rm eq} + x_3)} \end{aligned} \tag{42}$$

$$\begin{aligned}\psi_{C_{\rm eq},x_3}(x_2,x_3) &\triangleq \frac{\partial\psi_{C_{\rm eq}}(x_2,x_3)}{\partial x_3}\\ &= \left.\frac{\partial\psi_c(\phi,\tilde{\omega})}{\partial\tilde{\omega}}\right|_{(\phi,\tilde{\omega})=(\phi_{\rm eq}+x_2,\tilde{\omega}_{\rm eq}+x_3)}\\ &= \frac{\gamma\sigma\bar{d}}{\gamma-1}[\psi_c(\phi_{\rm eq}+x_2,\tilde{\omega}_{\rm eq}+x_3)+1]^{\frac{1}{\gamma}}\\ &\quad\times \left.\frac{\partial[\tilde{\omega}^2\eta_c(\phi,\tilde{\omega})}{\partial\tilde{\omega}}\right|_{(\phi,\tilde{\omega})=(\phi_{\rm eq}+x_2,\tilde{\omega}_{\rm eq}+x_3)} \end{aligned} \tag{43}$$

and where

$$\frac{\partial\eta_c(\phi,\tilde{\omega})}{\partial\phi} = \frac{\sigma\tilde{\omega}^2(f_2(\tilde{\omega}f_1-\phi f_2)+f_3(\sigma\tilde{\omega}-\phi f_3)-2\phi(f_4+f_5))}{\left(\sigma\tilde{\omega}^2+\frac{1}{2}(\tilde{\omega}f_1-\phi f_2)^2+\frac{1}{2}(\sigma\tilde{\omega}-\phi f_3)^2+\phi^2(f_4+f_5)\right)^2} \tag{44}$$

and we have (45), shown at the bottom of the page.

Now, choosing the nonlinear control law

$$\begin{aligned}\hat{u}_{\rm eq} &= -k_1\left(x_1-\psi_{C_{\rm eq}}(x_2,x_3)-k_2x_2\right)\\ &\quad+\frac{1}{\bar{a}}\left[\bar{b}\left(k_2+\psi_{C_{\rm eq},x_2}(x_2,x_3)\right)\left(\psi_{C_{\rm eq}}(x_2,x_3)-x_1\right)\right.\\ &\quad\left.-k_3\bar{c}\psi_{C_{\rm eq},x_3}(x_2,x_3)x_3+\frac{\alpha_2}{\alpha_1}\bar{b}x_2\right]\end{aligned} \tag{46}$$

where $k_1>0$, it follows that

$$\begin{aligned}\dot{V}_{\rm eq}(x_1,x_2,x_3) &= -\alpha_1k_1\bar{a}\left(x_1-\psi_{C_{\rm eq}}(x_2,x_3)-k_2x_2\right)^2\\ &\quad-\alpha_2k_2\bar{b}x_2^2-\alpha_3k_3\bar{c}x_3^2<0\end{aligned} \tag{47}$$

for $(x_1,x_2,x_3)\neq(0,0,0)$, so that the equilibria-dependent nonlinear controller

$$\tilde{u}_{\rm eq} \triangleq \begin{bmatrix}\tilde{u}_1\\ \tilde{u}_2\end{bmatrix} = \begin{bmatrix}x_2-\hat{u}_{\rm eq}\\ -k_3x_3+f(x_2,x_3)\end{bmatrix} \tag{48}$$

guarantees that the closed-loop system (30)–(32) is globally asymptotically stable for all parameterized system equilibria given by (29). As mentioned above, however, the nonlinear controller (48) may generate unnecessarily large control amplitude and rate signals leading to actuator amplitude and rate saturation. In the next section, we develop a globally stabilizing switching control strategy that directly addresses actuator amplitude and rate saturation constraints.

## IV. Stability-Based Switching Nonlinear Controller

In this section we develop a globally stabilizing switching control strategy for controlling the centrifugal compressor model (26)–(28). Specifically, using Lyapunov stability theory, a switching nonlinear globally stabilizing control law based on equilibria-dependent or, instantaneous (with respect to a given equilibrium) Lyapunov functions, with converging domain of attractions is developed. Since the desired maximum pressure operating point for a given angular velocity of the compressor spool can be captured by the parameterization given by (29), let the shifted variables $(\psi_s,\phi_s,\omega_s)$ correspond to a shifting of the desired maximum pressure point so that the translated system is given by

$$\dot{\psi}_s = \bar{a}(\phi_s-\tilde{u}_1) \tag{49}$$

$$\dot{\phi}_s = \bar{b}\left(\psi_{C_{\rm eq}}(\phi_s,\omega_s)-\psi_s\right) \tag{50}$$

$$\dot{\omega}_s = \bar{c}(\tilde{u}_2-f(\phi_s,\omega_s)). \tag{51}$$

Next, let the diffeomorphism $\sigma:[0,a]\to\mathbb{R}^3$, $a>0$, be such that $\sigma(s)$, $s\in[0,a]$, is an equilibrium point of (49)–(51) and $\sigma(0)=0$. Shifting the control law (48) and the Lyapunov function (40) to the shifted variables $(\psi_s,\phi_s,\omega_s)$, we obtain the shifted control law $\tilde{u}_s(\psi_s,\phi_s,\omega_s)$ which globally stabilizes the equilibrium point $\sigma(s)$, $s\in[0,a]$, with an associated Lyapunov function $V_s(\psi_s,\phi_s,\omega_s)$. Next, for the shifted control law $\tilde{u}_s(\psi_s,\phi_s,\omega_s)$, consider an estimate of the domain of attraction given by $\mathcal{D}_s \triangleq \{(\psi_s,\phi_s,\omega_s) : V_s(\psi_s,\phi_s,\omega_s)\leq c_s\}$, where $c_s>0$ is the finite value of $V_s(\cdot)$ on the boundary of $\mathcal{D}_s$. Furthermore, assume that $c_s$ is a $C^1$ function of $s\in[0,a]$ and note that since $c_s>0$ and $V_s(\cdot)$ is continuous and radially unbounded, $\mathcal{D}_s$ is a compact set for $s\in[0,a]$, which further implies that $\mathcal{D}_s$ is a positively invariant set of (49)–(51) with feedback control law $\tilde{u}_s(\psi_s,\phi_s,\omega_s)$, $s\in[0,a]$. Finally, since $\mathcal{D}_s$ is not empty for all $s>[0,a]$, there exists $s_1,s_2>0$ such that $s_2<s_1$ and $\sigma(s_1)\in\mathring{\mathcal{D}}_{s2}$, where $\mathring{\mathcal{D}}_s$ denotes the interior of $\mathcal{D}_s$, that is, $\mathring{\mathcal{D}}_s \triangleq \{(\psi_s,\phi_s,\omega_s) : V_s(\psi_s,\phi_s,\omega_s)<c_s\}$.

Using the properties of $\mathcal{D}_s$ we now present the following globally stabilizing switching nonlinear control strategy. Let $\{s_1,\cdots,s_q\}$ be such that $a=s_1>s_2>\cdots>s_q=0$, and $\sigma(s_i)\in\mathring{\mathcal{D}}_{s_i+1}$, $i\in\{1,\cdots,q-1\}$. Furthermore, define

$$s(\psi_s,\phi_s,\omega_s) \triangleq \min_{i=1,\cdots,q}\{s_i : (\psi_s,\phi_s,\omega_s)\in\mathcal{D}_{s_i}\}. \tag{52}$$

Since for each $i\in\{1,\cdots,q-1\}$, $\sigma(s_i)\in\mathring{\mathcal{D}}_{s_i+1}$ it follows that $s(\psi_s,\phi_s,\omega_s)$ is a nonincreasing function of time along the state trajectories of (49)–(51). Hence, with $u(\psi_s,\phi_s,\omega_s) \triangleq \tilde{u}_{s(\psi_s,\phi_s,\omega_s)}(\psi_s,\phi_s,\omega_s)$, where the notation $\tilde{u}_{s(\psi_s,\phi_s,\omega_s)}(\psi_s,\phi_s,\omega_s)$ denotes a switching nonlinear feedback controller with the switching function $s(\psi_s,\phi_s,\omega_s)$ defined as in (52), the solution $(\psi_s,\phi_s,\omega_s)$ of (49)–(51)

$$\frac{\partial[\tilde{\omega}^2\eta_c(\phi,\tilde{\omega})]}{\partial\tilde{\omega}} = \frac{\sigma\tilde{\omega}^3(2\sigma\tilde{\omega}^2+(\tilde{\omega}f_1-2\phi f_2)(\tilde{\omega}f_1-\phi f_2)+(\sigma\tilde{\omega}-2\phi f_3)(\sigma\tilde{\omega}-\phi f_3)+4\phi^2(f_4+f_5))}{\left(\sigma\tilde{\omega}^2+\frac{1}{2}(\tilde{\omega}f_1-\phi f_2)^2+\frac{1}{2}(\sigma\tilde{\omega}-\phi f_3)^2+\phi^2(f_4+f_5)\right)^2} \tag{45}$$

approaches $\mathcal{D}_{sq}$ in a finite time. However, since $\mathcal{D}_{sq}$ is a conservative estimate of the domain of attraction of the origin of (49)–(51) , it follows that $u(\psi_s,\phi_s,\omega_s)$ globally stabilizes the desired maximum pressure point.

Note that the control law $u(\psi_s,\phi_s,\omega_s)$ is a stability-based switching controller and hence discontinuous. Next, we present a modification to (52) so that the resulting control law is continuous modulo at most one discontinuity. For this development define the compact set $\mathcal{D} \triangleq \cup_{s\in[0,a]}\mathcal{D}_s$, consisting of the union of the compact sets $\mathcal{D}_s$, $s\in[0,a]$. Next, if $(\psi_s(0),\phi_s(0),\omega_s(0))\notin\mathcal{D}$, setting $u(\psi_s,\phi_s,\omega_s)=\tilde{u}_a(\psi_s,\phi_s,\omega_s)$ the state trajectories $(\psi_s(t), \phi_s(t),\omega_s(t))$, $t\geq 0$, will approach the globally asymptotically stable equilibrium point $\sigma(a)$. In particular, since $\sigma(a)\in\mathcal{D}$ then there exists $t>0$ such that $(\psi_s(t),\phi_s(t),\omega_s(t))\in\mathcal{D}$. Now, let $\bar{t}$ be such that $(\psi_s(\bar{t}),\phi_s(\bar{t}),\omega_s(\bar{t}))\in\mathcal{D}$ and define

$$s(\psi_s,\phi_s,\omega_s)\triangleq\min_{s\in[0,a]}\{s:(\psi_s,\phi_s,\omega_s)\in\mathcal{D}_s\} \tag{53}$$

and $\bar{s}\triangleq s(\psi_s(\bar{t}),\phi_s(\bar{t}),\omega_s(\bar{t}))$. From the definition of $s(\cdot)$ it follows that $(\psi_s(\bar{t}),\phi_s(\bar{t}),\omega_s(\bar{t}))$ is on $\partial\mathcal{D}_{\bar{s}}$, where $\partial\mathcal{D}_s$ denotes the boundary of $\mathcal{D}_s$, that is, $\partial\mathcal{D}_s\triangleq\{(\psi_s,\phi_s,\omega_s):V_s(\psi_s,\phi_s,\omega_s)=c_s\}$. Furthermore, since $\dot{V}_{\bar{s}}(\psi_s(\bar{t}),\phi_s(\bar{t}),\omega_s(\bar{t}))<0$, $(\psi_s,\phi_s,\omega_s)\in\mathcal{D}_{\bar{s}}\backslash\sigma(\bar{s})$, it follows that there exists $\delta>0$ such that $V_{\bar{s}}(\psi_s(t),\phi_s(t),\omega_s(t))<c_{\bar{s}}$, $t\in[\bar{t},\bar{t}+\delta)$. Hence, $s(\psi_s(t),\phi_s(t),\omega_s(t))<s(\psi_s(\bar{t}),\phi_s(\bar{t}),\omega_s(\bar{t}))$, $t\in[\bar{t},\bar{t}+\delta)$. Since $\bar{t}$ is chosen arbitrarily, it follows that if $(\psi_s(0),\phi_s(0), \omega_s(0))\in\mathcal{D}$ then $s(\psi_s(t),\phi_s(t),\omega_s(t)), t\geq 0$, is monotonically decreasing.

Now, with $u(\psi_s,\phi_s,\omega_s)=\tilde{u}_{s(\psi_s,\phi_s,\omega_s)}(\psi_s,\phi_s,\omega_s)$, $(\psi_s,\phi_s,\omega_s)\in\mathcal{D}$, define the Lyapunov function candidate

$$V(\psi_s,\phi_s,\omega_s)\triangleq V_{s(\psi_s,\phi_s,\omega_s)}(\psi_s,\phi_s,\omega_s)=c_{s(\psi_s,\phi_s,\omega_s)} \tag{54}$$

with Lyapunov derivative

$$\dot{V}(\psi_s,\phi_s,\omega_s)=\left.\frac{dc_s}{ds}\right|_{s=s(\psi_s,\phi_s,\omega_s)}\dot{s}(\psi_s,\phi_s,\omega_s). \tag{55}$$

Since $\dot{s}(\psi_s,\phi_s,\omega_s)<0$ for $(\psi_s,\phi_s,\omega_s)\in\mathcal{D}$, if we choose $c_s$, $s\in[0,a]$, such that $(dc_s/ds)>0$, it follows that $\dot{V}(\psi_s,\phi_s,\omega_s)<0$, for $(\psi_s,\phi_s,\omega_s)\in\mathcal{D}\backslash\{0,0,0\}$, which proves local asymptotic stability of the origin.

Now, to construct a globally stabilizing controller it need only be noted that $u(\psi_s,\phi_s,\omega_s)=\tilde{u}_a(\psi_s,\phi_s,\omega_s)$ if $(\psi_s,\phi_s,\omega_s)\notin\mathcal{D}$ and $u(\psi_s,\phi_s,\omega_s)=\tilde{u}_{s(\psi_s,\phi_s,\omega_s)}(\psi_s,\phi_s,\omega_s)$ otherwise, where $s(\psi_s,\phi_s,\omega_s)$ is given by (53). However, this control law may be discontinuous at the boundary of $\mathcal{D}$. Alternatively, a continuous control law which globally stabilizes the origin of the system can be obtained by setting $u(\psi_s,\phi_s,\omega_s)=\tilde{u}_a(\psi_s,\phi_s,\omega_s)$ if $(\psi_s(0),\phi_s(0),\omega_s(0))\notin\mathcal{D}$ and letting the state trajectories enter the domain $\mathcal{D}_a\subset\mathcal{D}$ before switching the control law to $u(\psi_s,\phi_s,\omega_s)=\tilde{u}_{s(\psi_s,\phi_s,\omega_s)}(\psi_s,\phi_s,\omega_s)$, where $s(\psi_s,\phi_s,\omega_s)$ is given by (53).

The equation $V_s(\psi_s,\phi_s,\omega_s)=c_s$, which implicitely defines $s(\psi_s,\phi_s,\omega_s)$, cannot be easily solved. An alternative approach for updating $s$ can be obtained by noting that the condition $V_s(\psi_s,\phi_s,\omega_s)=c_s$ must be satisfied for all $t\geq 0$, and hence its time derivative must also be satisfied for all $t\geq 0$. In particular, using (55) and noting that $\dot{V}(\psi_s,\phi_s,\omega_s)=\dot{V}_s(\psi_s,\phi_s,\omega_s)+(dV_s/ds)\dot{s}=(dc_s/ds)\dot{s}$ we obtain

$$\dot{s}=\frac{\dot{V}_s(\psi_s,\phi_s,\omega_s)}{\frac{d}{ds}(c_s-V_s(\psi_s,\phi_s,\omega_s))} \tag{56}$$

with $s(0)=s_0$ such that $V_{s_0}(x_{1_0},x_{2_0},x_{3_0},)=c_{s_0}$. Note that (56) along with $u(\psi_s,\phi_s,\omega_s)=\tilde{u}_s(\psi_s,\phi_s,\omega_s)$ provides a nonlinear first-order dynamic compensator equivalent to the original condition $V_s(\psi_s,\phi_s,\omega_s)=c_s$ which now needs to only be solved once to compute the initial condition $s_0$. Note that the compensator dynamics given by (56) characterize the admissible rate of the compensator state $s$ such that the switching nonlinear controller guarantees that $(\psi_s(t),\phi_s(t),\omega_s(t))\in\partial\mathcal{D}_{s(t)}$, $t\geq 0$.

Finally, since all control actuation devices are subject to amplitude and rate saturation constraints that lead to saturation nonlinearities [28], we discuss how the proposed switching nonlinear controller can be incorporated to address such practical limitations. Specifically, since the dynamic compensator state $s$ is related to the throttle opening (actuator) and since the dynamics given by (56) indirectly characterize the fastest admissible rate at which the control throttle can open while maintaining stability of the controlled system, it follows that by constraining the rate at which the dynamics of $s$ can evolve on the equilibrium branch effectively places a rate constraint on the throttle opening. Mathematically, this corresponds to the case where the switching rate of the nonlinear controller is decreased so that the trajectory $(\psi_s(t),\phi_s(t),\omega_s(t))$, $t\geq 0$, is allowed to enter $\mathcal{D}_{s(t)}$. Additionally, amplitude saturation constraints and state constraints can also be enforced by simply choosing $a_{\max}>0$ such that $\mathcal{D}_{\max}\triangleq\cup_{s\in[0,a_{\max}]}\mathcal{D}_s$ is contained in the region where the system is constrained to operate. In this case, the stability-based switching nonlinear controller provides a local stability guarantee with domain of attraction given by $\mathcal{D}_{\max}$. Of course, in practice it is sufficient to implement controllers with adequate domains of attraction and *a priori* saturation constraint guarantees rather than implementing global controllers without realistic actuator limitations. It is important to note that the proposed amplitude and rate saturation scheme is specific to the centrifugal compressor model developed herein. A related but different control scheme for an electrostatically shaped membrane with state and control constraints is given in [29].

## V. Switching Nonlinear Control for Centrifugal Compressors

In this section we apply the stability-based switching controller developed in Section IV to a centrifugal flow compressor problem. Specifically, we use the three-state centrifugal compressor model derived in Section II with $(\bar{a},\bar{b},\bar{c},\bar{d})=(9.37,310.81,6.97,0.38)$, $(f_1,f_2,f_3,f_4,f_5)=(0.44,1.07,2.18,0.17,0.12)$, $\gamma=1.4$,

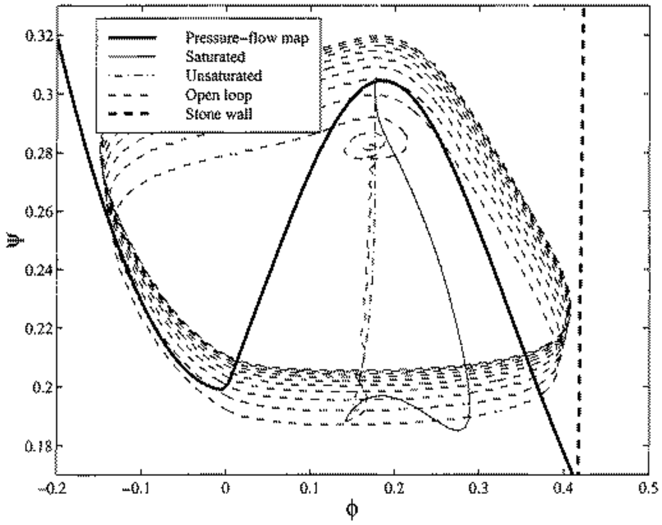


Fig. 3. Phase portrait of pressure-flow map.

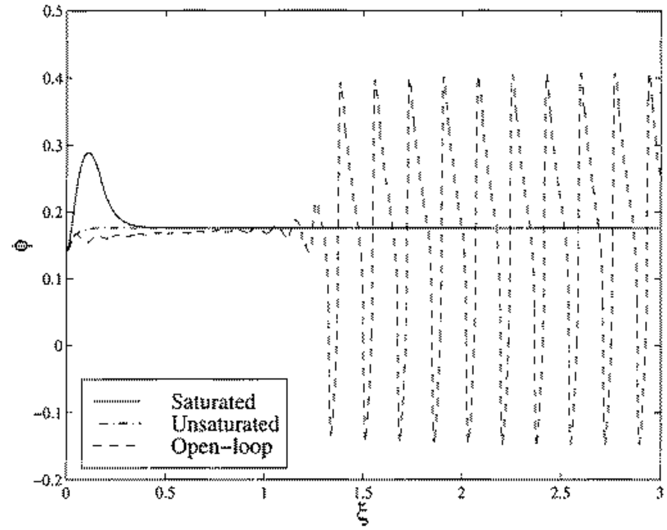


Fig. 5. Mass flow versus time.

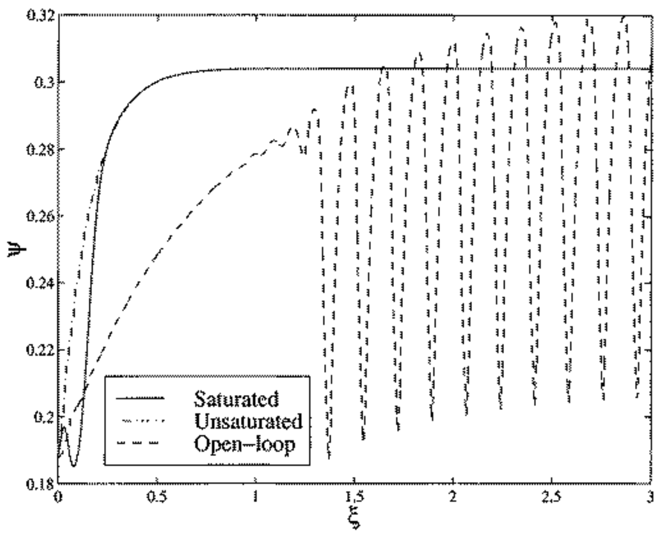


Fig. 4. Pressure rise versus time.

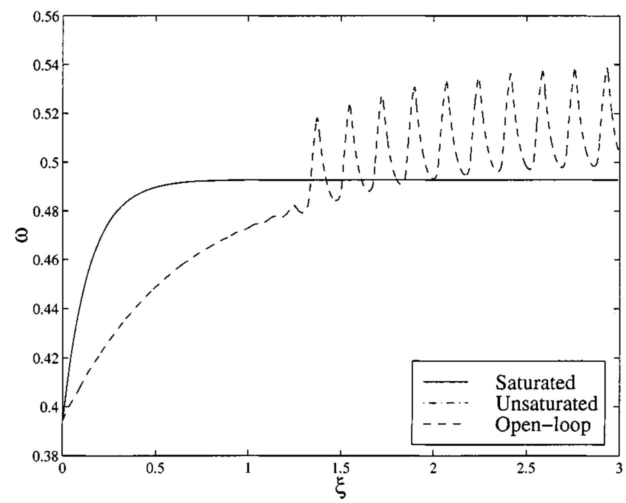


Fig. 6. Compressor spool speed versus time.

$\mu = 3$, and $\sigma = 0.9$. Since the torque calculated in (22) is for forward mass flow in the compressor, and since the compressor may enter deep surge, there is a need to derive an expression for the compressor torque for negative mass flow. Hence, assuming that a centrifugal compressor in reverse flow can be viewed as a throttling device and hence can be approximated as a turbine, it follows that in this case

$$\tau_c(m_c, \omega) = -\sigma r^2 m_c \omega. \tag{57}$$

Now, combining (22) and (57) gives [19]

$$\dot{\tilde{\omega}} = \bar{c}(u_2 - \sigma|\phi|\tilde{\omega}). \tag{58}$$

Using the initial conditions $(\psi_0, \phi_0, \tilde{\omega}_0) = (0.188, 0.141, 0.397)$, the design parameters $(\alpha_1, \alpha_2, \alpha_3) = (1, 0.1, 1)$ and $(k_1, k_2, k_3) = (1, 3, 1)$, the diffeomorphism $\sigma(s) = (\psi_{C_{\text{cq}}}(s, 0),\ s, 0)$, $s \in [0, 0.5]$, and $c_s = 0.01 + 2s$, the closed-loop system response with $(|\dot{\gamma}_{\text{th}}| \leq 5)$ and without a rate saturation constraint on the throttle opening is compared to the open-loop response when the compression system is taken from an operating speed of 20 000 r/min to 25 000 r/min. Fig. 3 shows the $\psi$-$\phi$ phase portrait of the state trajectories. The pressure rise, mass flow, and spool speed variations for the open-loop and controlled system are shown in Figs. 4, 5, and 6, respectively. Fig. 7 shows the control effort versus time. This comparison illustrates that open-loop control drives the compression system into deep surge while the proposed stability-based switching controller drives the system to the desired maximum pressure-flow equilibrium point $(\psi_{\text{eq}}, \phi_{\text{eq}}, \tilde{\omega}_{\text{eq}}) = (0.304, 0.176, 0.493)$. Note that the switching controller with a rate saturation constraint guarantees stability with minimal degradation is system performance.

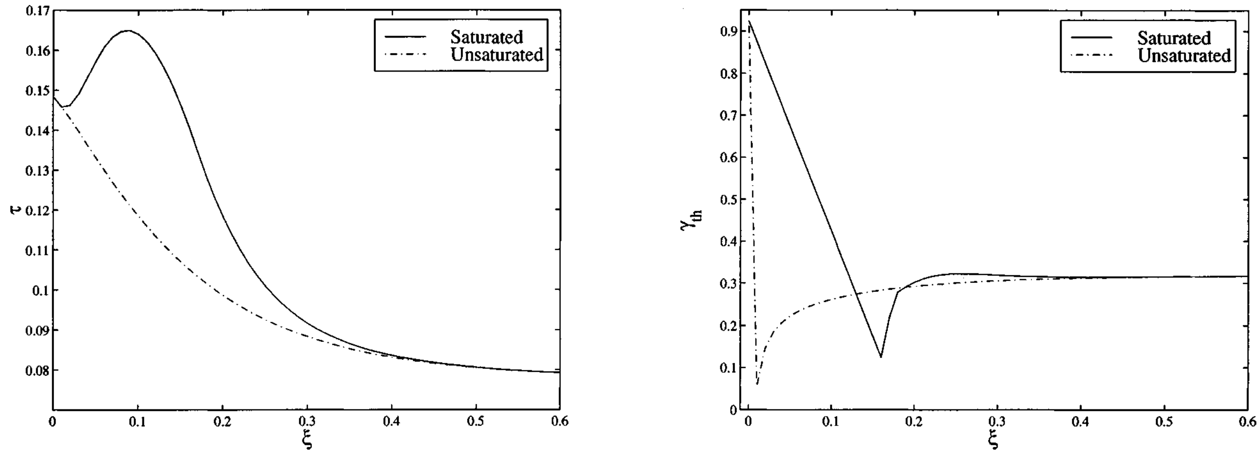


Fig. 7. Control effort versus time.

## VI. Conclusion

A three-state centrifugal compressor surge model involving pressure and mass flow compression system dynamics as well as spool dynamics to account for the influence of speed transients on the compression surge dynamics was developed. Using Lyapunov stability theory, a nonlinear globally stabilizing switching control law based on equilibria-dependent Lyapunov functions was developed. The proposed stability-based switching nonlinear control law was shown to be directly applicable to centrifugal compression systems with actuator amplitude and rate saturation constraints.

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**Alexander Leonessa** received the B.S. degree in aeronautical engineering from the University of Rome "La Sapienza," Rome, Italy, in 1993, the M.S. degree in aerospace engineering in 1997, and the M.S. degree in applied mathematics in 1999, both from Georgia Institute of Technology, Atlanta, GA, with specialization in stability theory for nonlinear dynamical switching systems.

From 1993 to 1994 he served as an officer in the Italian Navy and in 1995 he joined Alitalia Airlines as an Aircraft Maintenance Supervisor. Since September 1995 he has been a Graduate Research Assistant at Georgia Institute of Technology, where he is working toward completing the Ph.D. degree in aerospace engineering. His current research interests are in the areas of stability theory of nonlinear systems, robust control, adaptive control, saturation control, optimal control, Hamilton–Jacobi–Bellman theory, nonlinear dynamic compensation, gain scheduling, sliding mode control, linear parameter-varying control, and hierarchical switching control. Applied areas of interest include active control of propulsion and combustion systems, flight control systems, spacecraft attitude stability and control, and robot control.

**Wassim M. Haddad** (S'87–M'87) received the B.S., M.S., and Ph.D. degrees in mechanical engineering from Florida Institute of Technology, Melbourne, in 1983, 1984, and 1987, respectively, with specialization in dynamics and control.

From 1987 to 1994 he served as a consultant for the Structural Controls Group of the Government Aerospace Systems Division, Harris Corporation, Melbourne, FL. In 1988 he joined the faculty of the Mechanical and Aerospace Engineering Department at Florida Institute of Technology where he founded and developed the Systems and Control Option within the graduate program. Since 1994 he has been a member of the faculty in the School of Aerospace Engineering at Georgia Institute of Technology, Atlanta, GA, where he holds the rank of Professor. His current research interests are in the areas of linear and nonlinear robust multivariable control for aerospace systems; multiobjective mixed-norm $H_2/H_\infty$, $H_2/L_1$, and $H_\infty/L_1$ controller synthesis; optimization theory for fixed-structure dynamic compensation; fixed-architecture decentralized control; sampled-data systems; adaptive neural identification and control; stability theory for nonlinear systems; $\mu$-analysis and synthesis for linear and nonlinear systems; mixed $H_2/\mu$ robust controller analysis and synthesis with fixed-structure multipliers; saturation control; robust nonlinear feedback control (robust Hamilton–Jacobi–Bellman theory); nonlinear disturbance rejection control (nonlinear $H_\infty$ theory); adaptive control for nonlinear uncertain systems; nonlinear hierarchical switching control; hybrid systems and optimal switching control; and impulsive dynamical systems. Applied areas of interest include vibration control of large flexible structures; noise control; active vibration absorber and isolation technology; and active control of combustion and propulsion systems.

Dr. Haddad is recipient of the National Science Foundation Presidential Faculty Fellow Award and is a Member of Tau Beta Pi.

**Hua Li**, photograph and biography not available at the time of publication.