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Mathematical Model of the Torque Characteristics for Hydraulic Motors*

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In the mathematical model described in this paper, the following friction components are considered: (1) friction torque proportional to operating pressure, (2) viscous friction torque proportional to rotational speed and viscosity of fluid and (3) constant friction torque independent of operating conditions. The friction component which is proportional to operating pressure decreases with an increase of the rotational speed. This effect is also considered in this model. Therefore this model can represent the whole torque performance from start to maximum speed.

1. Introduction

There are several mathematical models which describe torque characteristics of hydraulic motors.[(1)~(4)] In these models, a friction torque of motor is given by a function of operating conditions such as rotational speed, operating pressure and viscosity of fluid.

The objects of previous mathematical models are to describe the torque performance in high speed range where the viscous friction is dominant. Therefore, at start and in low speed range where the boundary friction is dominant, previous mathematical models are not applicable. Furthermore, to such motors in which the boundary friction is dominant in high speed range, the previous models are not applicable, either. Additionally, influences of outlet pressure on the friction torque of hydraulic motors are not considered in the previous models.

In this paper, based on the experimental results of torque characteristics of hydraulic motors,[(6)~(9)] a new mathematical model is constructed. The new model is deduced from Wilson's model with simple modifications and it is applicable to all operating conditions from start to maximum speed and the influences of outlet pressure are also considered.

2. Nomenclature

C_v :	Coefficient of viscous friction	(-)
C_{f0} :	Constant for each motor	(-)
N :	Revolutions per minute	(rpm)
P_1 :	Inlet pressure of motor	(Kg/cm^2)
P_2 :	Outlet pressure of motor	(Kg/cm^2)
ΔP :	Pressure difference across motor $=P_1-P_2$	(Kg/cm^2)
T :	Mean value of output torque	(Kg cm)
T_{th} :	Mean value of theoretical torque $=V_{th}\Delta P$	(Kg cm)
T_c :	Constant friction torque	(Kg cm)
T_f :	Friction torque related to P_1 and P_2	(Kg cm)
T_v :	Viscous friction torque	(Kg cm)
ΔT :	Loss torque of motor $=T_{th}-T$	(Kg cm)
V_{th} :	Mean value of displacement volume per radian	(cm^3/rad)
α :	Constant for each motor	(-)
ε :	Constant for each motor	(-)
μ :	Viscosity of fluid	(Kg s/cm^2)
η_T :	Mechanical efficiency $=T/T_{th}=1-\Delta T/\Delta T_{th}$	(-)
ω :	Angular velocity of motor	(rad/s)
ω_0 :	Constant for each motor	(rad/s)

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3. Derivation of mathematical model

The output torque of hydraulic motor is given by deducing the loss torque from the theoretical torque as follows.

$$T=T_{th}-\Delta T \quad \cdots\cdots(1)$$

Not only the theoretical torque but also the loss torque changes with rotational position of drive shaft of motor. However, only the mean values of them are discussed in this paper.

The loss torque is , as follows , the sum of the friction torque T_f which is proportional to pressure, the viscous friction torque T_v which is proportional to $\mu\omega$ and the constant friction torque T_c which is independent of operating conditions.

$$\Delta T=T_f+T_v+T_c \quad \cdots\cdots(2)$$

In addition to the above-mentioned components, it is also possible to consider a turbulent friction component which is proportional to fluid density and the square of rotational speed.[(3)]

The friction component T_f is proportional to the product of the load acting on the sliding surface in motor, the radial length from center of rotation to sliding surface and the friction coefficient.

The loads acting on the sliding surfaces in motors are classified into the following three types:

(1) Load proportional to inlet pressure

(2) Load proportional to outlet pressure
(3) Load proportional to pressure difference

Examples of type (1) are forces between pistons and cylinders or whose at slipper shoes and spherical joints in expanding stroke. Examples of type (2) are forces acting on pistons and other parts in compressing stroke. Examples of type (3) are loads of main bearings of gear motors or forces between vanes and a cam ring of vane motor.

Each hydraulic motor has all components of types (1), (2) and (3) in itself actually. Every component of types (1), (2) and (3) is a linear function of P_1 and P_2. Therefore, the total of these components is also a linear function of P_1 and P_2. Then the equivalent load at the sliding surfaces can be expressed by the product of $|P_1+\varepsilon P_2|$ and the equivalent area under pressure, where ε is a constant for each motor.

The equivalent area is proportional to the square of characteristic length R. And the radial length from center of rotation to sliding surface is proportional to R. So, T_f is proportional to R^3. R^3 is proportional to the displacement volume V_{th}. Therefore, T_f can be expressed as follows.

$$T_f=C_f V_{th}|P_1+\varepsilon P_2| \quad \cdots\cdots (3)$$

The value of C_f is proportional to the friction coefficient of sliding surface and it is a function of sliding speed. In the case of geometrically similar hydraulic motors, the value of C_f will be the same provided the sliding speed is the same.

According to many experimental works, the value of C_f is largest at start and it decreases with an increase in sliding speed. And in high speed range, it becomes nearly zero.[7)~(9)] These tendencies of C_f can be expressed by the following equation.

$$C_f=C_{f0}/\{1+(U/U_0)^\alpha\} \quad \cdots\cdots (4)$$

where U is sliding speed and it is given by $U=R\omega$. The values of C_{f0}, U_0 and α are constant for each motor and C_{f0} is the value of C_f at start.

Using the relation of $U/U_0=\omega/\omega_0$, one can rewrite Eq.(4) as follows.

$$C_f=C_{f0}/\{1+(\omega/\omega_0)^\alpha\} \quad \cdots\cdots (5)$$

ω_0 is the angular velocity when the value of C_f becomes one half of C_{f0} and it is a constant for each motor. Since $\omega_0=U_0\omega/U=U_0/R\propto U_0 V_{th}^{-1/3}$, ω_0 has a tendency to decrease with an increase in displacement volume.

The viscous friction torque T_v has also a tendency to increase with the displacement volume and it can be written as follows.[1)]

$$T_v=C_d V_{th}\mu\omega \quad \cdots\cdots (6)$$

Substituting Eqs.(3),(5) and (6) into Eq.(2), one can obtain the following equation.

$$\Delta T=\frac{C_{f0}}{1+(\omega/\omega_0)^\alpha}V_{th}|P_1+\varepsilon P_2|+C_d V_{th}\mu\omega+T_c \quad \cdots\cdots (7)$$

Dividing Eq.(7) by theoretical torque, one can obtain a nondimensional equation.

$$\frac{\Delta T}{T_{th}}=\frac{C_{f0}}{1+(\omega/\omega_0)^\alpha}\frac{|P_1+\varepsilon P_2|}{P_1-P_2}+C_d\frac{\mu\omega}{P_1-P_2}+\frac{T_c}{V_{th}(P_1-P_2)} \quad \cdots\cdots (8)$$

Then, mechanical efficiency becomes as follows.

$$\eta_T=1-\left[\frac{C_{f0}}{1+(\omega/\omega_0)^\alpha}\frac{|P_1+\varepsilon P_2|}{P_1-P_2}+C_d\frac{\mu\omega}{P_1-P_2}+\frac{T_c}{V_{th}(P_1-P_2)}\right] \quad \cdots\cdots (9)$$

In the special case of $\varepsilon=-1$ or $P_2=0$, Eq.(9) becomes as follows.

$$\eta_T=1-\left[\frac{C_{f0}}{1+(\omega/\omega_0)^\alpha}+C_d\frac{\mu\omega}{\Delta P}+\frac{T_c}{V_{th}\Delta P}\right] \quad \cdots\cdots (10)$$

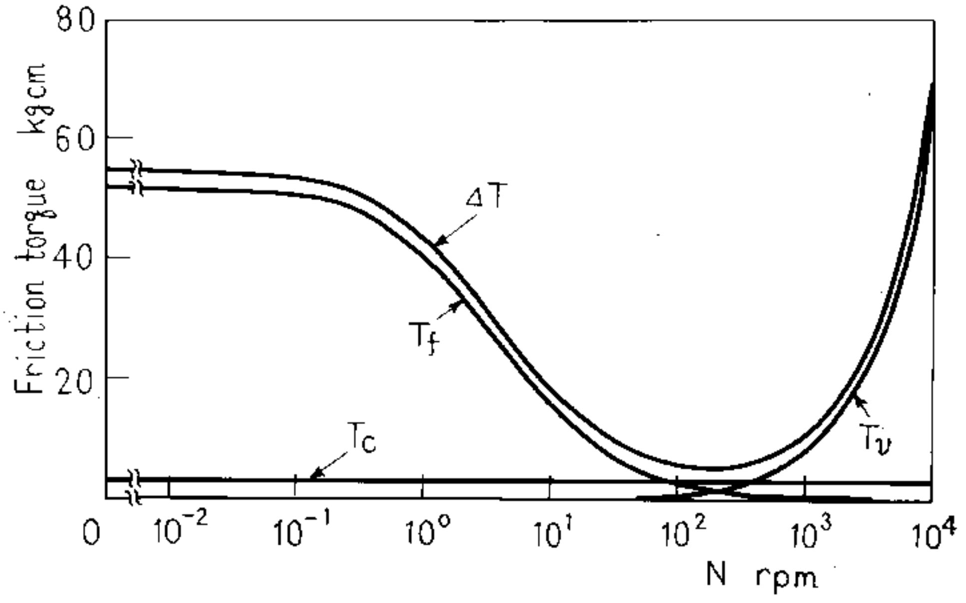

Fig.1 Changes of friction components with rotational speed

Figure 1 shows a typical example of changes of friction components. As Fig.1 indicates, T_f is dominant in low speed range and T_v is dominant in high speed range. Therefore, the friction torque in low speed range depends upon pressure and in high speed range it depends upon viscosity of fluid.

4. Comparison of experimental results and calculation results

Experiments were carried out with seven hydraulic motors shown in Table 1. Properties of hydraulic oil are given in Table 2. For the measurements of torque characteristics, an apparatus with a worm gear mechanism was used.[8]~[10]

4.1 Determination of coefficients in mathematical model

There are several ways to determine the values of coefficients from experimental results of torque characteristics of hydraulic motors. In this study, they are determined by the following procedures.

(1) Determination of C_{f0} and T_c

When $P_2=0$ and $\omega=0$ (i.e. at start), Eq.(7) becomes as follows.

$$\Delta T = C_{f0}V_{th}P_1+T_c \quad \cdots\cdots(11)$$

Therefore one can evaluate the values of C_{f0} and T_c from the relationship between friction torque at start and inlet pressure. This relationship can be obtained from starting torques measured at various inlet pressures. Generally, its linearity is very good. So one can determine the values of C_{f0} and T_c easily.

(2) Determination of C_d and T_c

In high speed range, as clearly seen from Fig.1, the first term on the right-hand side of Eq.(7) becomes nearly zero. Therefore the friction torque in high speed range is expressed by the following equation.

$$\Delta T = C_d V_{th}\mu\omega + T_c \quad \cdots\cdots(12)$$

Usually, in high speed range, the friction torque increases linearly with an increase in rotational speed. But, some hydraulic motors have a turbulent friction component and then the friction torque does not increase linearly but slightly in curves. However, in the present study, the relationship between ΔT and ω in high speed range is represented by a straight line approximately. Then the values of C_d can be obtained from the slope of the straight line and T_c can be obtained from the intersection on the ΔT axis.

The value of T_c obtained by the latter procedure does not coincide with that obtained by procedure (1). Therefore it is necessary to determine the value of T_c by comparing both results obtained by the procedures (1) and (2). Since the absolute value of T_c is relatively small, large errors do not occur.

In the case of some hydraulic motors, boundary lubrication is dominant even in high speed range and the friction characteristic which is represented by Eq.(12) does not appear within the experimental range. In that case, before the performance of procedure (2), one should practice the next procedure (3) and should determine the values of ω_0 and α. After that, the value of C_d is determined such that the evaluated result by the mathematical model can represent the measured value at the maximum speed.

(3) Determination of ω_0 and α

In low speed range, the second term on the right-hand side of Eq.(7) becomes nearly zero. Then, in low speed range and when $P_2=0$, the friction torque is expressed by the following equation.

$$\Delta T = \frac{C_{f0}}{1+(\omega/\omega_0)^{\alpha}} V_{th}P_1+T_c \quad \cdots\cdots(13)$$

From experimental relationship between ΔT and ω which is obtained in low speed range under the conditions of $P_1=$ constant and $P_2=0$, the values of ω_0 and α can be determined such that the calculated values can represent the experimental values as precisely as possible. In this case, the values of C_{f0} and T_c obtained by above-mentioned procedures are used.

(4) Determination of ε

The value of $\Delta T - T_c$ is defined as $(\Delta T - T_c)_{P_2=0}$ when $P_2=0$ or $P_1=\Delta P$. Under the same pressure difference, the value of $\Delta T - T_c$ is defined as $(\Delta T - T_c)_{P_2\neq 0}$ when P_2 is not equal to zero. From Eq.(7), the following equation is applicable in low speed range.

$$\frac{(\Delta T - T_c)_{P_2\neq 0}}{(\Delta T - T_c)_{P_2=0}} = 1+\frac{\varepsilon+1}{\Delta P}P_2 \quad \cdots\cdots(14)$$

Therefore, from the slope of experimental curve of $(\Delta T - T_c)_{P_2\neq 0}/(\Delta T - T_c)_{P_2=0}$ versus outlet pressure P_2 in low speed range where the second term on the right-hand side of

Table 1 Tested motors

Name	Type	V_{th} cm³ / rad
Motor A	Pressure plate type gear motor	3.23
Motor B	Fixed type vane motor	3.94
Motor C	Swash plate type axial piston motor	1.15
Motor D	Radial piston motor	1.67
Motor E	Rotary vane motor	2.68
Motor F	Fixed type gear motor	4.22
Motor G	Bent axis type axial piston motor	3.81

Table 2 Properties of oil used

Temp. °C	Viscosity Kg s / cm²	Specific gravity
40	0.467 10	0.864
45	0.365 10	0.860
50	0.291 10	0.856
55	0.236 10	0.851

petroleum of paraffin series

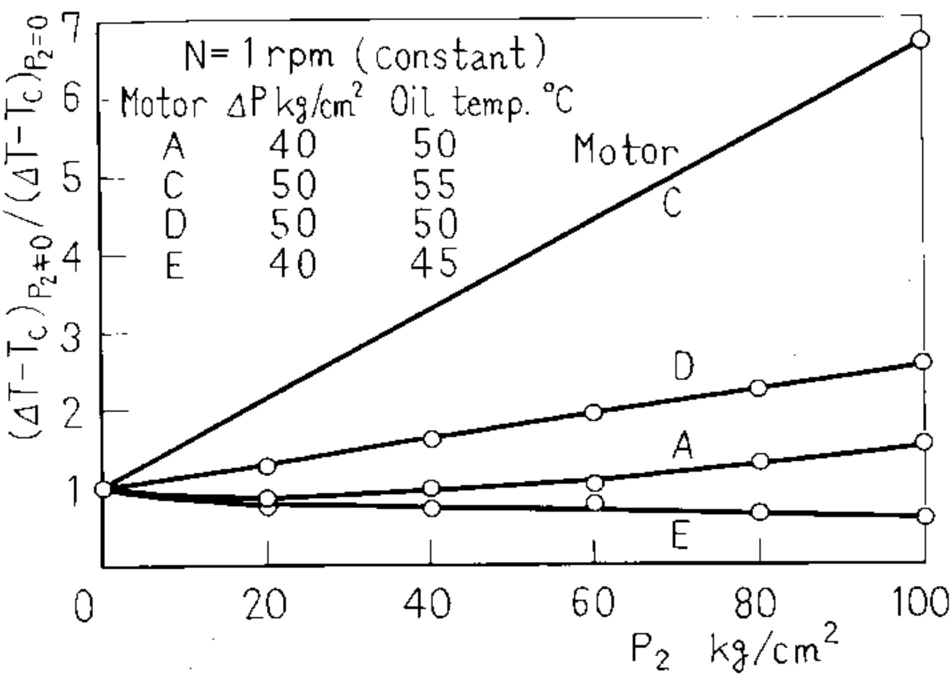

Fig.2 Influences of outlet pressure on friction torque

Table 3 Coefficients of tested motors

Name	C_{f0}	ω_0 rad/s	α	ε	C_d	Kg cm
Motor A	0.161	0.506	1.42	-0.786	2.42 $\times 10^5$	3.2
Motor B	0.448	0.796	0.425	-1.00	0.511	6.0
Motor C	0.299	0.321	0.835	1.85	2.03	3.0
Motor D	0.370	1.07	0.357	-0.210	0.964	1.0
Motor E	0.108	0.767	0.666	-1.16	1.58	0.3
Motor F	0.050	0.148	0.707	-1.00	1.75	0.0
Motor G	0.230	1.70	0.735	-	1.94	5.0

Eq.(7) is negligible, one can obtain the value of ε.

Figure 2 shows the experimental curves of $(\Delta T-T_c)_{P_2\neq 0}/(\Delta T-T_c)_{P_2=0}$ versus P_2 at N = 1 rpm and ΔP=40 or 50 kg/cm². The values of ε are obtained from the slopes of approximate straight lines for each motor. In the case of motors B and F, ΔT is independent of outlet pressure provided the pressure difference is constant. Consequently, the value of ε for motors B and F is -1. Motor G has such a construction what a positive outlet pressure is not permissible. So the value of ε for motor G could not be obtained.

In the cases of some gear- or vane motors, loads acting on the sliding surfaces are proportional to pressure difference and the friction torque is independent of the outlet pressure provided the pressure difference is constant. On the other hand, in the case of piston motors, because of the friction forces acting on the pistons in the withdrawing stroke, the friction torque increases with the outlet pressure even if the pressure difference is constant.

The values of coefficients which are determined by the above-mentioned procedures are shown in Table 3.

4.2 Comparisons of experimental results and calculation results

Experimental results and calculated results of torque characteristic are compared in Fig.3 for motor A and in Fig.4 for motor B. For the calculations the values in Table 3 were used.

Figure 3 shows a typical example in which viscous friction is observed in high speed range and Fig.4 shows another typical example in which viscous friction can not be observed within the experimental range. In both cases, the calculated results by Eq.(8) coincide with the experimental results. As Figs.3 and 4 indicate, the mathematical model in this paper can represent many different characteristics of torque performances by determining the values of coefficients properly.

Figure 3 and 4 are modified and plotted in Fig.5 using a logarithmic scale

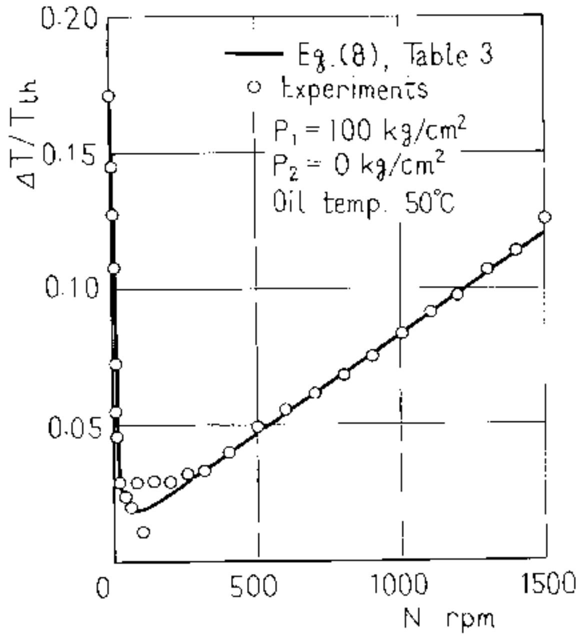

Fig.3 Comparison of experimental results and calculated result (Motor A)

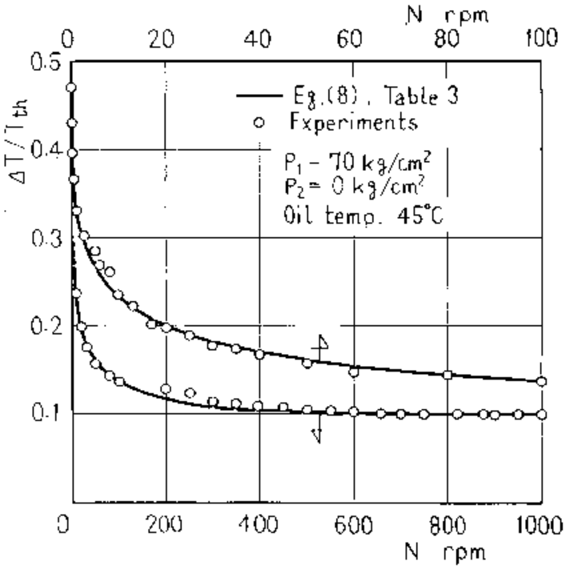

Fig.4 Comparison of experimental results and calculated result (Motor B)

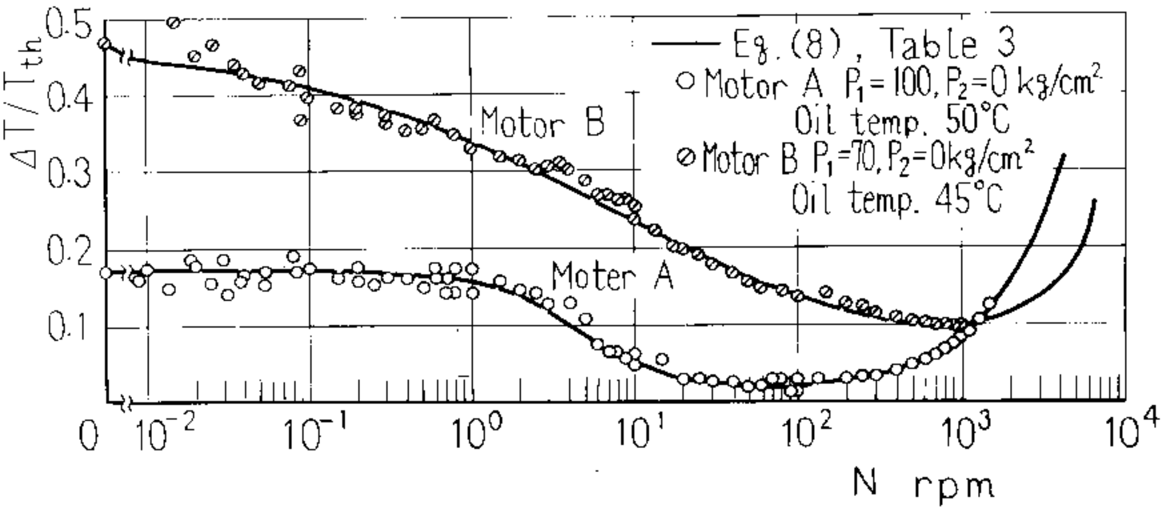

Fig.5 Comparisons of experimental results and calculated results (Motors A and B)

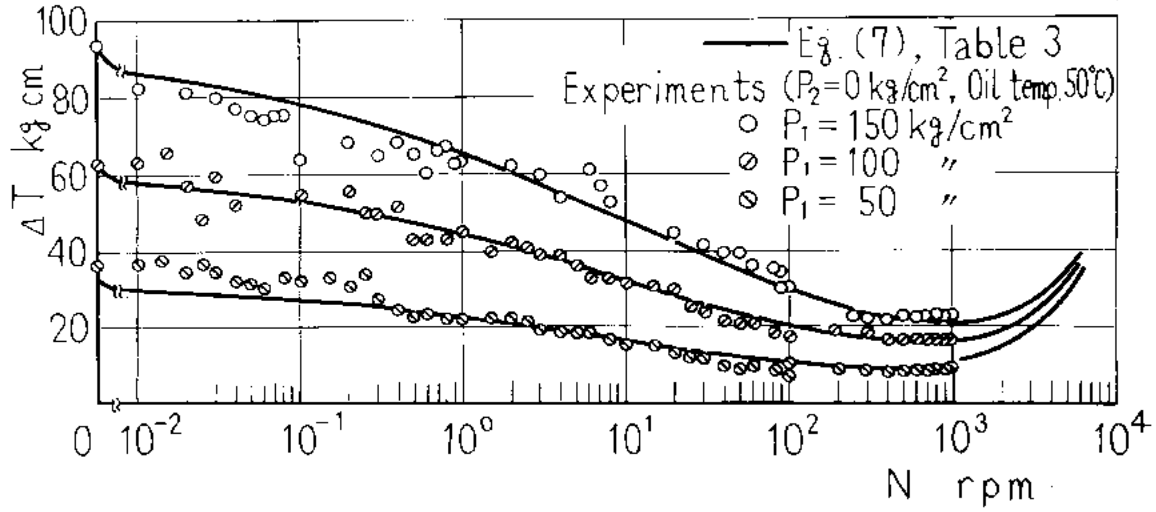

Fig.6 Comparisons of experimental results and calculated results (Motor D)

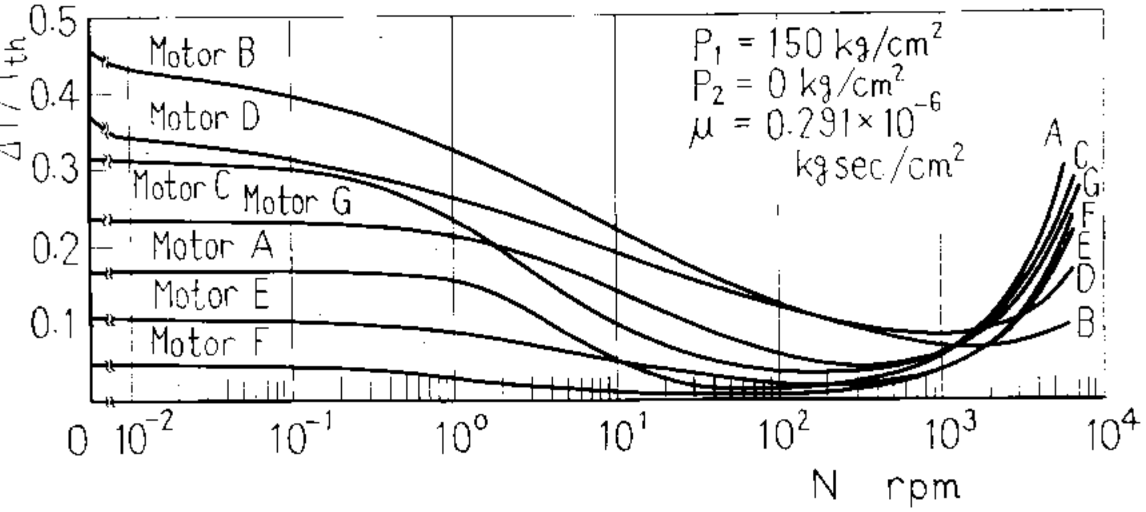

Fig.7 Comparisons of friction characteristics of various motors

in the abscissa. As Fig.5 indicates, the calculated results by the mathematical model coincide with the experimental results under all operating condition from start to maximum speed.

In Fig.6, the experimental relationship between ΔT and N at three different inlet pressures and the calculated results by Eq.(7) are compared. As Fig.6 indicates, the calculated results by the mathematical model coincide with the experimental results over a wide range of operating conditions of P_1 and N.

Figure 7 shows friction characteristics of various hydraulic motors in Table 1 that are evaluated under the same operating conditions by the mathematical model using the coefficients in Table 3. As Fig.7 indicates, seven curves take almost the same value in the resion of 1000 - 2000 rpm. In this region the values of $\Delta T/T_{th}$ become 0.05 - 0.1, namely, mechanical efficiencies are 95 - 90 %. The reason for these high efficiencies is that the rated or designed speed of all test motors used in this work is within the region of 1000 - 2000 rpm.

The mechanical efficiencies of test motors are almost equal in the region of 1000 - 2000 rpm. But in low speed range less than 1 rpm, they are very different.

In the case of motor B, the vanes are pushed against the cam ring by inlet pressure and then the mechanical efficiencies at start and in low speed range are low.[13] On the other hand, in the case of motor F which is a fixed type gear motor, the pushing force due to operating pressure does not act on sliding surfaces and then the mechanical efficiencies at start and in low speed range are superior.

5. Conclusions

A new mathematical model which describes the torque characteristics of hydraulic motors is proposed. In this model, the friction component proportional to operating pressure, the viscous friction component and the constant friction component independent of operating conditions are considered. It is also considered that the friction component which is related with operating pressure decreases with an increase in rotational speed. Furthermore the influence of outlet pressure on the friction torque is taken into consideration.

From experimental works for seven types of hydraulic motors, the values of coefficients in the mathematical model are determined and the experimental results are compared with the calculated values by the model.

As the result of this investigation, it becomes clear that the mathematical model in this paper can represent the torque characteristics which can not be described by previous mathematical models.

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