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The article on the subject  
Physical correlation of GPS measurements. The knowledge analysis of this  
problem

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## KEY WORDS

Satellite measurements – спутниковые измерения

Signal travel time – время распространения сигнала

Physical correlation – физическая корреляция

Ambiguity – неопределенность

Standard stochastic model – стандартная стохастическая модель

Residual-based stochastic model – остаточная стохастическая модель

Baselines – базовые линии

Zenith tropospheric delay – зенитная тропосферная задержка

## INTRODUCTION

The subject of this paper is “Physical correlation of GPS measurements. The knowledge analysis of this problem”.

The main purpose of this work is to analyze the existing researches of this problem and to determine the range of tasks of the master thesis.

To achieve the major aim of the paper the following tasks are to be solved:

1. To characterize the peculiarities of GPS measurements.
2. To explain the definition of physical correlation.
3. To give the reasons of physical correlation.
4. To examine the researches on this topic.
5. To analyze negative results of physical correlation on the basis of the found literature.
6. To designate a circle of tasks for further research.

The following works constitute the basis of this paper:

1. GPS Essentials of Satellite Navigation. Compendium [electronic resource]. – [www.u-blox.com](http://www.u-blox.com)
2. A. E-S. El Rabbany “The effect of physical correlations on the ambiguity resolution and accuracy estimation in GPS differential positioning”
3. S.G. Jin, O. Luo, C. Ren “Effects of physical correlations on long-distance GPS positioning and zenith tropospheric delay estimates”
4. M. Gerasimenko, N. Shestakov “Physical correlation? It may be dangerous!”
5. V. Schwieger “Time-dependent correlations using the GPS –a contribution to determine the uncertainty of GPS-measurements”

This work aims to show the relevance of the problem and to outline the range of tasks to solve it.

# 1 THE SPECIFICS OF GPS MEASUREMENTS

## 1.1 Basic principles of satellite measurements

Before starting the discussion of the problem of physical correlation in GPS measurements it should to consider the principle of satellite measurements to understand their specifics.

Satellite Navigation Systems all use the same basic principles to determine coordinates:

1. Satellites with a known position transmit a regular time signal.
2. Based on the measured travel time of the radio waves the position of the receiver is calculated [1, p. 13].

From the above it should be clear that to determine the coordinates of the point location we need to find the distance between satellite and receiver at first. The distance ( $D$ ) can be determined by the well-known formula:

$$D = \Delta\tau * c, \quad (1)$$

where

$\Delta\tau$  – the travel time from satellite to receiver;

$c$  – the speed of light.

According to the found distance the position can be determined with the help of special programs.

At first glance it seems very simple to determine coordinates by GPS. The way in is for ordinary users, like drivers of cars with GPS navigator, because the whole process of measurements and calculations is hidden from them; and also they do not need high precision measurements.

But the geodetic measurements are highly accurate. This complicates the measurements because of the need to consider many peculiarities and corrections.

The next paragraphs explain the specific of GPS measurements more detailed.

## 1.2 Signal travel time

Today the orbital group of GPS includes 30 working satellites. Each one of these satellites is equipped with onboard atomic clocks. Atomic clocks are the most precise time measurement instruments known, losing a maximum of one second every 30,000 to 1,000,000 years. In order to make them even more accurate, they are regularly adjusted or synchronized from various control points on Earth.

Satellites transmit their exact position to receiver on the ground. Signals are transmitted at the speed of light (300,000km/s) and therefore require approximately 67.3ms to reach a position on the Earth's surface directly below the satellite. The signals require a further 3.33 $\mu$ s for each additional kilometer of travel [1, p. 15].

But when measuring, the signal travel time from satellite to receiver is not fixed. The start and stop time are fixed (Figure 1). And the travel time is determined as:

$$\Delta\tau = \tau_2 - \tau_1, \quad (2)$$

where

$\tau_2$  – stop time;

$\tau_1$  – start time.

Thus to compute the travel time it is necessary the precise synchronization of satellite and receiver clock because even a slip of 1  $\mu$ s can lead to position error of 300 m.

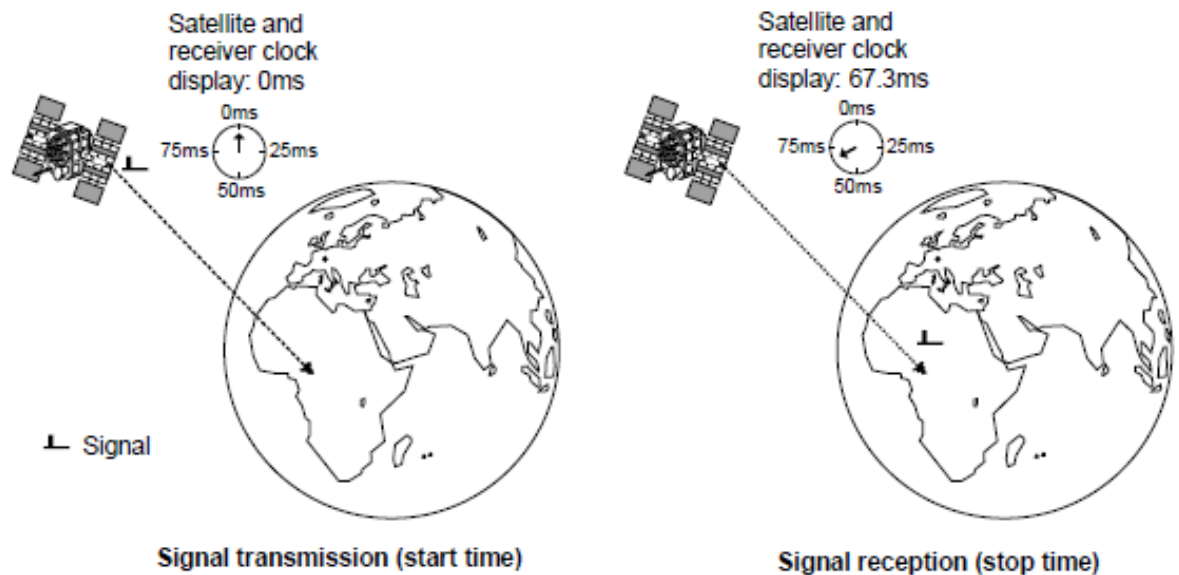


Figure 1 – Determining the signal travel time

As mentioned above the satellite clock is very precise. But the installation such clock in the receiver is unpractical because of their enormous cost (at about 1 000 000 \$). Therefore the precise synchronization is a very difficult task.

### 1.3 The complexity of determining the position

This paragraph explains how coordinates are determined using GPS.

Two satellites are orbiting far above the Earth transmitting their onboard clock times and positions. By using the signal travel time to both satellites we can draw two circles with the radius  $D_1$  and  $D_2$  around the satellites (Figure 2). Each radius corresponds to the calculated distance to the satellite. All possible positions relative to the satellites are located on these circles. If the position above the satellites is excluded, the location of the receiver is at the exact point where the two circles intersect beneath the satellites, therefore, two satellites are sufficient to determine a position on the X/Y plane.

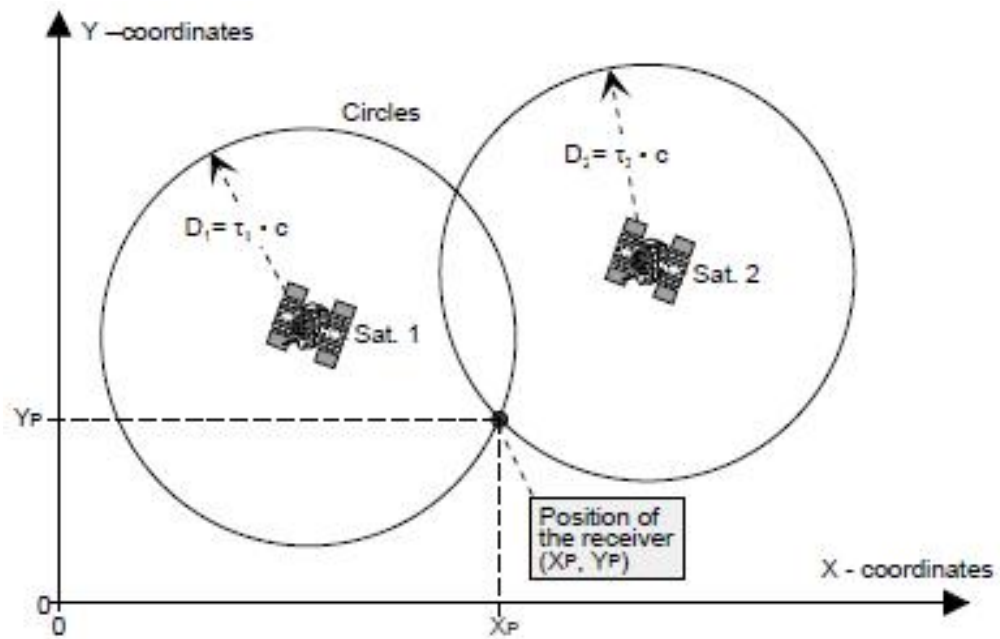


Figure 2 – The position of the receiver at the intersection of the two circles

But in the real world, a position has to be determined in three-dimensional space rather than on a plane. As the difference between a plane and three-dimensional space consists of an extra dimension (height  $Z$ ), an additional third satellite must be available to determine the true position. If the distance to the three satellites is known, all possible positions are located on the surface of three spheres whose radius corresponds to the distance calculated. The position is the point where all three of the spheres intersect (Figure 3).

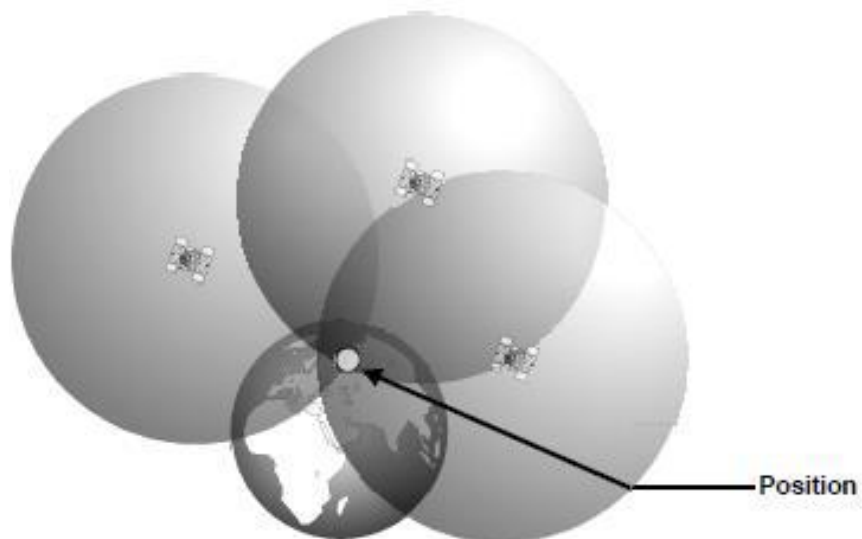




Figure 3 – The position is determined at the point where all three spheres intersect

Thus to determine the location it is needed to know 3 coordinates (X, Y, Z). From mathematics if  $N$  variables are unknown, it is needed  $N$  independent equations to identify them.

It would seem quite 3 equations to determine position, but it is one more unknown parameter: the time error ( $\Delta t$ ).

So in 3-dimensional space we will have four unknown variables:

- longitude (X)
- latitude (Y)
- height (Z)
- time error ( $\Delta t$ )

These four variables require four equations, which can be derived from four separate satellites (Figure 4).

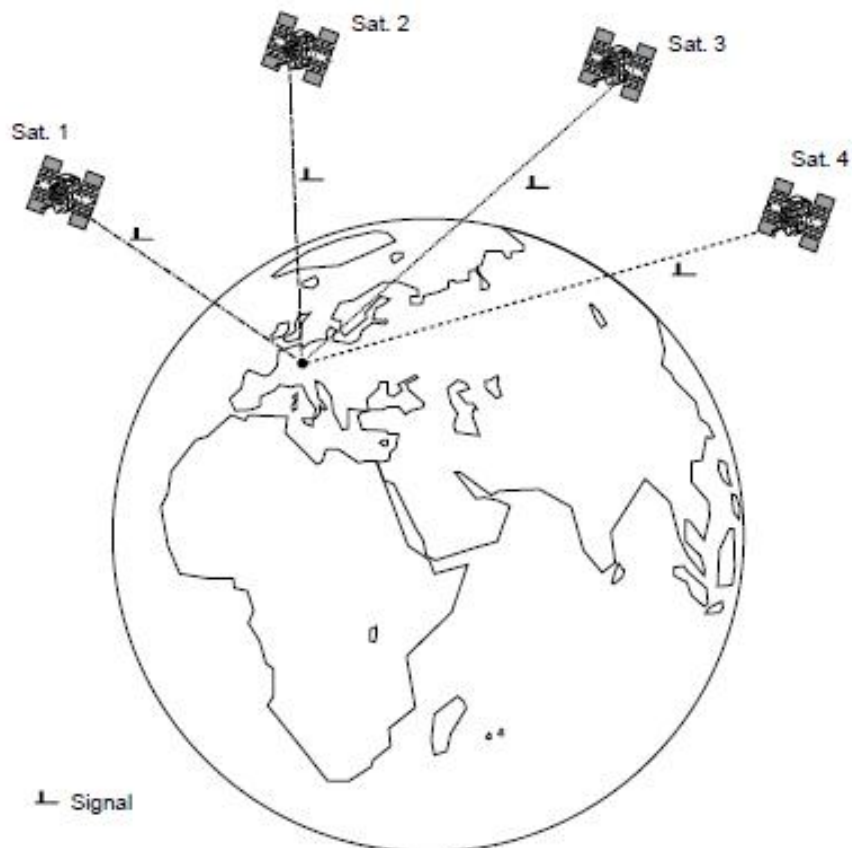


Figure 4 – Four satellites are required to determine a position in 3-D space

But do not forget about errors that may occur in real conditions. Primarily the accuracy of measurements depends on the terrain. Dense building increases the signals errors at times due to their reflection from buildings. The accuracy of the measurements is also influenced by:

- the weather phenomena (rain, snow);
- the leaves on the trees;
- household radios;
- and so on.

All of these factors require the use of not 4 satellites, which in theory are enough, but more of them, additionally using complex calculation algorithms of eliminate errors caused by noise.

So we can see that GPS measurements are very difficult. It creates a lot of problems, one of which is physical correlation. It is considered in the rest of this paper.

## 2 THE DEFINITION OF PHYSICAL CORRELATION AND THE REASONS OF IT

To begin with consideration of the problem of physical correlation in GPS measurements it is necessary to give the definition of physical correlation and the reasons of this phenomenon in GPS measurements.

Physical correlation in GPS is a dependence between measurements which is caused by physical measurements conditions and measuring technique.

This is because the same observations, made in the same time, contain similar influence of atmospheric parameters, error hour of satellite and receiver and refractive delay of microwave signals passing through the ionosphere and troposphere.

So the main reasons of physical correlation in GPS measurements are different errors which are difficult to account due to complexity and lack of knowledge of satellite measurements.

But physical correlation has a significant impact on the results of satellite measurements. So it should not be neglected.

The next part of this paper explains the influence of physical correlation and analyze negative results of this phenomenon.

## 3 THE PHYSICAL CORRELATION INFLUENCE OF SATELLITE MEASUREMENTS RESULTS

### 3.1 Study of the physical correlation influence of GPS measurements

The physical correlation influence of satellite measurements results was researched by foreign and native investigators. The following describes their researches and results.

#### 3.1.1 The research by A. E-S. El Rabbany

The Arabic investigator Ahmed El-Sayed El-Rabbany has made a significant contribution to study this phenomenon. He has shown the influence of physical correlation on the accuracy estimates of GPS measurements in an empirical way in his work “The effect of physical correlations on the ambiguity resolution and accuracy estimation in GPS differential positioning”.

In this work it was proposed to include the non-diagonal covariance matrix of ambiguities in post processing measurements to form a confidence region of a hyperellipsoid around their estimated values.

It will help to obtain the most likely integer values for the ambiguities at a certain probability level, which, in turn, will increase the accuracy of GPS measurements. The covariance matrix of the ambiguity parameters can be obtained using the method of the least squares [2, p.1].

For the nomination of such proposal this investigator has analyzed data of several baselines of different length observed under different ionospheric activities.

Two baselines of lengths 13 and 55 km were processed in different programs with including and neglecting physical correlation.

In all cases, neglecting the physical correlations leads to smaller standard deviations than those obtained with physical correlations included. In other words, neglecting the physical correlations yields an overly optimistic covariance matrix for the estimated parameters. For this reason it is very important to include the physical correlations in any software package and not to rely on using a scale factor [2, p. 2].

### 3.1.2 The research by S.G. Jin, O. Luo, C. Ren

This problem was also investigated by Chinese and Korean researches S.G. Jin, C. Ren and O. Luo in their work “Effects of physical correlations on long-distance GPS positioning and zenith tropospheric delay estimates”.

It is specially noted the necessity to research effects of physical correlations on long GPS baselines and zenith tropospheric delay estimates. Although physical correlations have an immense influence for long-distance GPS measurements, there are very few such investigations.

This work tests and analyses two easily realized stochastic modeling methods for GPS long baseline and ZTD estimates:

- A) standard stochastic model
- B) residual-based stochastic model.

It is appropriate to describe these models. Standard stochastic model is easy to be implemented in practice. In this model, it's usually assumed that all the carrier phases or pseudo-ranges have the same variance ( $\sigma^2$ ) and are statistically independent. Therefore, the observation  $\Phi$  is treated as independent and uncorrelated, and the covariance matrix of the observations  $\Phi$  can be formulated as:

$$Cov(\Phi) = \sigma^2 I, \quad (3)$$

where  $I$  is the unit matrix.

Through the error propagation law the time-invariant covariance matrix (called the stochastic model) of the double difference measurements can be determined.

But this simplified stochastic model may contain some mis-specifications which can lead to unreliable results.

The residual-based stochastic model is more difficult but also more precise. The residual-based stochastic model is based on the classic variation covariance. The classical variance covariance matrix is defined as following:

$$C_v = E[(v - \bar{v})(v - \bar{v})^T], \quad (4)$$

where

$v$  – the residual estimation from;

$\bar{v}$  – the mean value of residual  $v$ [4, p. 3].

The authors of this work treated measurements of long baselines (at about 1000-1500 km) and zenith tropospheric delay estimates using 2 stochastic models mentioned above.

They took measurements which had been performed in Australia (Figure 5).

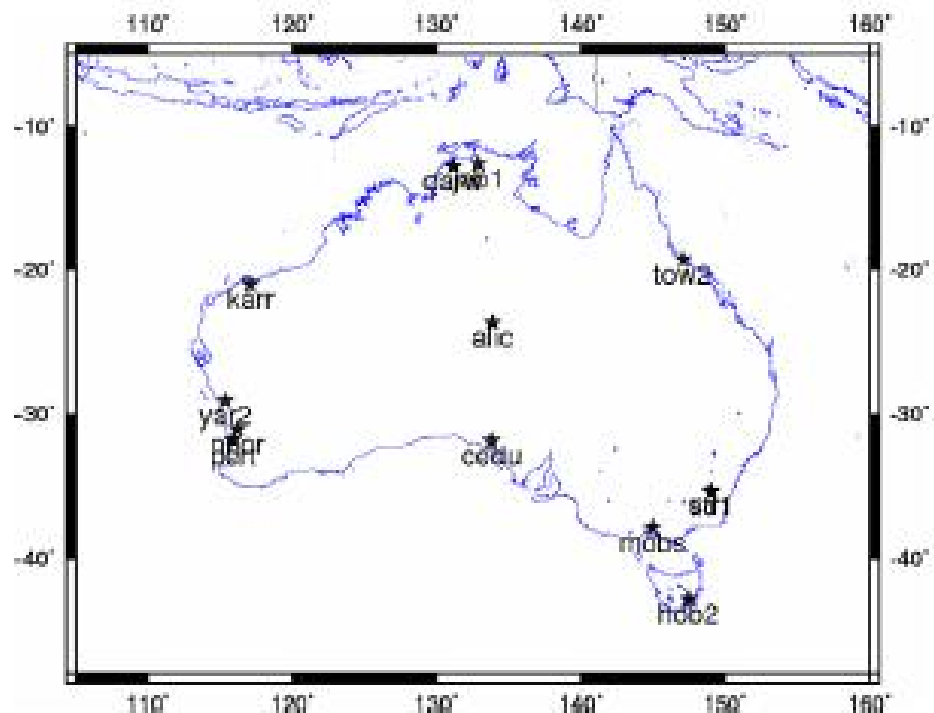


Figure 5 – IGS sites distribution in Australia

They found that using the residual-based stochastic model gives better results. It is illustrated on Figures 6 and 7.

Using model B the standard deviation of baseline component is obviously smaller than the method A (in the left panel of Figure 6), and the corresponding difference of baseline component is about 5.8 mm in the vertical component and 1–3 mm in the horizontal component (in the right panel of Figure 6) [4, p. 5].

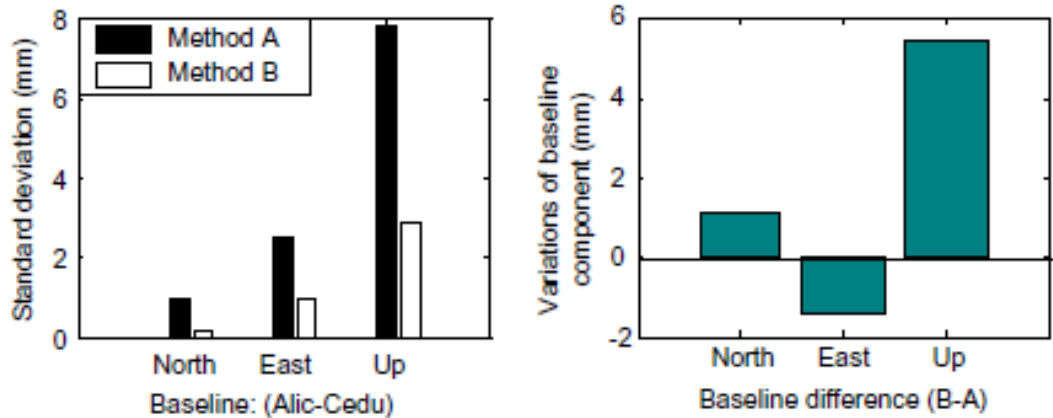


Figure 6 – Comparison of standard deviations and baseline differences

Using the residual-based stochastic model (method B), the better performance of ZTD estimate is obtained (Figure 7) [4, p. 6].

So it was established that stochastic model effects of results of GPS measurements. It is proposed to include errors caused by physical correlation in stochastic model to obtain more adequate results.

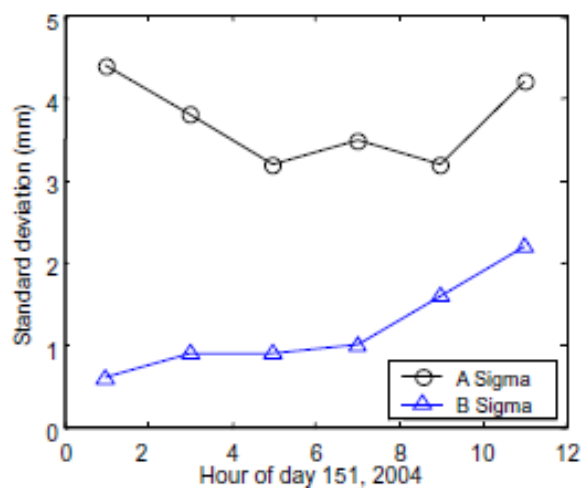


Figure 7 – Standard deviations of ZTD estimations using stochastic models A and B

### 3.1.3 The research by 1. M. Gerasimenko, N. Shestakov

The phenomenon of physical correlation is also studied by Russian researchers M. Gerasimenko and N. Shestakov. In their work “Physical correlation? It may be dangerous!” they describe the negative influence of physical correlation on GPS measurements. They report that the presence of physical correlation improves the accuracy of GPS measurements artificially. This is because the measurements are not introduced corrections for the correlation, i. e. not taken into account the covariance matrix of uncertainties. This situation can be explained by great dimension of the correlation matrix, significant processing volume and its complication. In this case many details of computation process are hidden from researchers, especially if all computations are made by using a computer. Artificial accuracy increasing of the final processing results is interpreted by them as a correct choice of the modeled function that were used for correlation coefficients. But in reality this accuracy increasing may be only a result of formal operations and doesn't reflect a real situation [3, p.1].

The work is considered 2 correlated observations. And even in this simplest case there are unexpected peculiarities and absurd results. For example, artificial increasing the weight  $P$  ( $P \rightarrow \infty$ ) of the final results while the weight even one observation  $p \rightarrow 0$ . It has no common physical sense. And what effects could be in enough spacious geodetic network?!

### 3.2 How the physical correlation effects on GPS results?

Having studied the works mentioned above we can say how the physical correlation impact on the satellite measurement and why is it so important to consider.



So physical correlation improves the accuracy of GPS measurement artificially. It could lead to extraordinary wrong results in determination coordinates. And this, in turn, is very dangerous for practice because of incorrect coordinates can lead to errors in subsequent calculations for the building that is facing economic losses and even death of people.

So a problem of investigation of correlated observations and their using in practice needs in solving. Considering these undesirable phenomena it should be noted that it not exclude that it is needed to re-estimate different methods of improving of GPS results that are involving in post processing measurements different non-diagonal covariance matrix of observations as well. The formal estimates of accuracy do not reflect the real situation with physical correlation of observations. It may be artificial improving of accuracy.

## CONCLUSION

This paper is not a complete study but it sheds light on the problem of physical correlation and shows how important it is.

Today GPS techniques are widely used in geodesy, because satellite measurements are the fastest and therefore the most productive today.

But GPS measurement is a very complex process. It is explained in the first paragraph.

Because of satellite measurements are correlated and need a very high accuracy, the influence of physical correlations should not be neglected.

If the physical correlation is not taken into account the measurement accuracy will be artificially increased. It could lead to extraordinary wrong results in determination coordinates. And this, in turn, is very dangerous for practice because of incorrect coordinates can lead to errors in subsequent calculations for the building that is facing economic losses and even death of people.

The main conclusion is that it is necessary to study in detail the physical correlation of satellite measurements. In this regard, the further implementation of the master thesis must be related to the search and analysis methods of accounting the physical correlation. And a main purpose of further research is to develop ways of decreasing the negative impact of the physical correlation on the satellite measurements.

## REFERENCES

1. GPS Essentials of Satellite Navigation. Compendium [electronic resource]. – [www.u-blox.com](http://www.u-blox.com)
2. El-Rabbany A.E.-S. The effect of physical correlations on the ambiguity resolution and accuracy estimation in GPS differential positioning, Ph.D. dissertation, University of New Brunswick, Department of Geodesy and Geomatics Engineering, Technical Report No.170, Fredericton, 1994.
3. Gerasimenko M.D., Shestakov N.V. Physical correlation? It may be dangerous! Institute of Applied Mathematics FEBRAS, Radio st., 7, Vladivostok 690041, Russia.
4. Jin S. G., Luo O., Ren C. Effects of physical correlations on long-distance GPS positioning and zenith tropospheric delay estimates. Article in Advances in Space Research - July 2010.