Comparison of Converter Transfer Function Models for Estimate Harmonic Currents Generated by AC/DC Converters under Unbalanced Conditions

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Abstract

This paper compares different transfer function models used for calculating ac and dc current harmonics produced by six-pulse ac/dc converter under unbalanced and or distorted supply conditions. The paper describes the functions and how they are used to calculate ac and dc current harmonics. Calculated and simulated results obtained from Power System Blockset (PSB) in the Simulink environment are presented for comparison purpose.

Keywords: *Harmonics, transfer function, ac/dc converter*

1. INTRODUCTION

The proliferation of variable-speed drives in many industrial and commercial applications is contributing significantly in decreasing electrical distributions power quality. It is well known that ac/dc converters, the most important sources of power system harmonics, generate harmonics both on the ac and dc side. In the last years, a great number of models have been proposed to evaluate harmonic currents injected in the power system networks by ac/dc converters. The assessment of the harmonic effects of ac/dc converters by means of converter transfer functions is an effective and proven method. The transfer function method is simple to use, fast and allows insight into how the harmonics are generated [1-6]. With this method, it is possible to simultaneously estimate voltage distortion on the dc side and current distortion on the ac side of the converter. However, authors have raised the problem that harmonic distortion estimates obtained by simplified transfer function models may result in unacceptable errors under some working conditions [5, 6]. Therefore, it is noteworthy to compare accuracy of the results obtained from available converter transfer function models with those obtained from time domain simulations for different working conditions. This papers presents a comparative study of two converter transfer function models used to evaluate current harmonics amplitudes injected in the ac network by six-pulse ac/dc converters working under distorted and/or unbalanced supply conditions and different values of plant parameters. Some calculated and simulated results obtained from Power System Blockset (PSB) in the Simulink environment are presented for comparison purpose.

2. CONVERTER DESCRIPTION

The simplified model of the three-phase thyristor converter analyzed in this paper is shown in Fig. 1. The ac and dc resistances are neglected since their effect on the harmonic spectrum is not important [7]. The mean value of the dc voltage drop on these resistances can be included in E_d, so that only the effect of ripple on them is neglected. The average value, E_d, of the dc side voltage, and the firing angle α are used to determine the converter operating point.

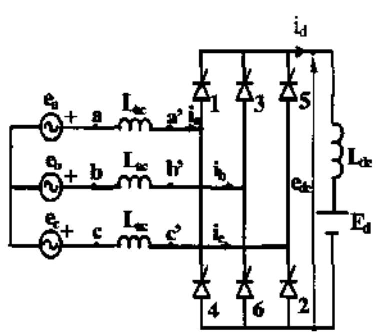

Fig. 1 : Three-phase thyristor converter model

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The three-phase supply voltages may be unbalanced and/or distorted. The phase voltages are then expressed as follows:

$$e_a=\sqrt{2}\sum_{n=1}^{N} E_{an}\sin\left(n\omega t+\varphi_{an}\right) \quad (1a)$$

$$e_b=\sqrt{2}\sum_{n=1}^{N} E_{bn}\sin\left(n\omega t+\varphi_{bn}\right) \quad (1b)$$

$$e_c=\sqrt{2}\sum_{n=1}^{N} E_{cn}\sin\left(n\omega t+\varphi_{cn}\right) \quad (1c)$$

where E_{an}, E_{bn} and E_{cn} are the rms values of each phase voltage harmonics; ϕ_{an}, ϕ_{bn} and ϕ_{cn} are the phase angles of each phase voltage harmonics.

The firing instants of firing of the thyristors are dependent on the selected control method. There are basically two types of firing methods namely individual and equidistant firing methods. Individual firing control requires that each thyristor gating pulse be referenced with respect to the zero crossing of its commutation voltage. The gating pulse is given to each thyristor after a delay α measured from the zero crossing of its commutation voltage. The commutation voltages of the six thyristors shown on Fig. 1 and numbered in firing sequences 1, 2, 3, 4, 5 and 6 are e_{ac}, e_{bc}, e_{ba}, e_{ca}, e_{cb} and e_{ab}, respectively. Equidistant firing control requires that gating pulses of one of the six Graëtz bridge thyristors be referenced with respect to its commutation voltage. The remaining gating pulses related to other thyristors are generated with a 60° delay. The individual firing control method which is a more general method is used in this paper. Thus, the angle of starting of conduction α_i of thyristor i is derived from the zero crossing γ_i of its commutation voltage and control angle α of the converter (i.e. $\alpha_i = \gamma_i + \alpha$). The six angles of end of commutation (δ_i) are deduced from the angles of starting of conduction (α_i) based on the approach presented in [7].

3. TRANSFER FUNCTION METHOD

Transfer function method is a powerful tool for the derivation of ac and dc current harmonics produced by ac/dc converters. From the application of the traditional transfer functions, the fictitious dc voltage and ac currents of the converter can be expressed as follows [1-5]:

$$e_{dc} = e_a S_{ua} + e_b S_{ub} + e_c S_{uc} \quad (2)$$

$$i_a = i_{dc} S_{ia} \quad (3a)$$

$$i_b = i_{dc} S_{ib} \quad (3b)$$

$$i_c = i_{dc} S_{ic} \quad (3c)$$

where, S_{ua}, S_{ub} and S_{uc} are the voltage transfer functions relating the fictitious dc voltage to the ac voltages and S_{ia}, S_{ib} and S_{ic} are the current transfer functions relating the dc current to the ac currents. The traditional voltage and current transfer functions of phase a are shown in Fig. 2.

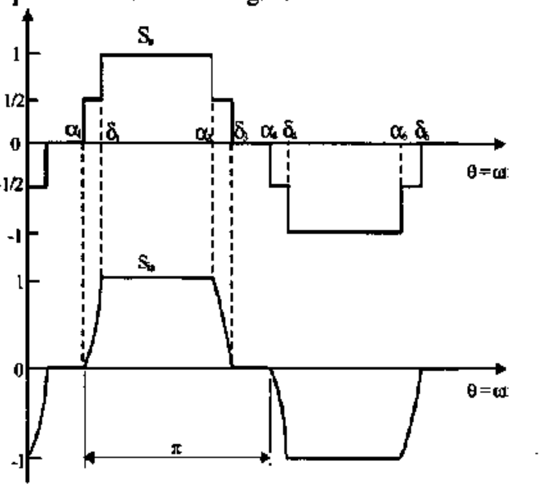

Fig. 2 : Traditional voltage and current transfer functions (for phase a)

The multiplication of any two functions f(t) and g(t) in the time domain is equivalent to a convolution of the related spectra $F(j\omega)$ and $G(j\omega)$, respectively, in the frequency domain. If the *n*th Fourier coefficients of f and g are assumed to be $C_{f,n}$ and $C_{g,n}$, respectively, their Fourier transforms can be expressed as follows:

$$F=\Im\{f(t)\}=2\pi\sum_{n=-\infty}^{\infty} C_{fn}\,\delta(\omega-n\,\omega_s) \quad (4)$$

$$G=\Im\{g(t)\}=2\pi\sum_{n=-\infty}^{\infty} C_{gn}\,\delta(\omega-n\,\omega_s) \quad (5)$$

$$\delta(\omega-n\,\omega_s)=\begin{cases}1 & \text{for } \omega=n\,\omega_s\\ 0 & \textit{otherwise}\end{cases}$$

where, ω_s is the fundamental frequency of f and g; f represents the ac voltage or the dc current; g represents the voltage or the current transfer function; their product h(t) = f(t)g(t) represents the dc voltage or the ac current.

The Fourier coefficient, C_{xn}, of a time domain function x is expressed as follows:

$$C_{xn}=\frac{1}{2\pi}\int_0^{2\pi} x(\omega t)\exp(-jn\,\omega t)\,d(\omega t) \quad (6)$$

where, x represents h, f or g.

In conformity with the convolution theorem [8], the Fourier transforms, H, of h(t)=f(t)g(t)), follows from the convolution of the Fourier transform of f and g:

$$H=F\otimes G=2\pi\sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} C_{fk}C_{gn}\delta(\omega-n\,\omega_s-k\,\omega_s) \quad (7)$$

The Fourier coefficients of h result from the following algebraic expression:

$$C_{hn}=\sum_{k=-\infty}^{\infty} C_{fk}\,C_{g(n-k)} \quad n\in \mathrm{N}. \quad (8)$$

where, C_{hn} is the nth Fourier coefficient of h.

It can be seen from (8) that the convolution is reduced to simple addition and multiplication. For good accuracy of the transfer function method, the evaluation of the converter current and voltage transfer functions is of great importance.

4. CALCULATION OF DC SIDE CURRENT HARMONICS

For accurate evaluation of dc current harmonics, care must be taken for proper evaluation of converter voltage transfer function [7]. The fact that the dc side equivalent circuit has variable topology (due to commutation and conduction modes) must be taken into account. Two approaches have been proposed for accurate evaluation of dc current harmonics. In the first approach, the traditional voltage transfer functions are used and the time varying equivalent ac reactance as seen from the dc side is represented as a Fourier series [4]. The equivalent ac reactance is expressed as a constant and a modulated part. In [1,2,3,5], the traditional transfer functions have been used and the ac reactance as seen from the dc side was approximated by the constant part. In the second approach, modified voltage transfer functions are used and the dc side equivalent circuit is modeled with constant reactance [7].

4.1 Time varying dc side reactance approach

With this approach, which has been proposed by Hu et al. [4], the dc voltage, e_d, is determined from (2) using the traditional voltage transfer functions S_{ua}, S_{ub}, and S_{uc}. Fig. 2 illustrates the transfer function for the phase a. Knowing the Fourier series coefficients of the input supply voltages and those of the voltage transfer functions S_{ua}, S_{ub}, and S_{uc}, the Fourier series coefficients of the dc voltage, e_d, hence its harmonic components, can be easily calculated by appropriatly applying (8).

Once the dc voltage harmonic components are evaluated, the dc side harmonic currents can be calculated if the dc networks and its parameters are known. The ac reactance, X_c, seen from the converter dc side varies in steps according to the phases in conduction or in commutation. Fig. 3 shows the variation of the equivalent reactance, X_c, with time.

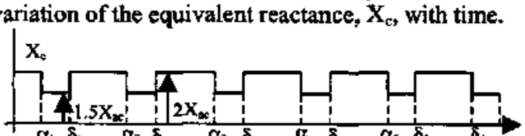

Fig. 3 : Variation of the dc side equivalent reactance

By applying Fourier series analysis, the dc side equivalent reactance, X_c, shown in Fig. 3, can be expressed as follows [4]:

$$X_c = X_{cav} + X_{ac} M_z \tag{9}$$

where M_z is the modulation function of the reactance; X_{cav} is the average value of X_c; $X_{ac} = L_{ac}\omega$.

It follows that the dc side current harmonics can be obtained from the dc side equivalent circuit shown on Fig. 4 where the dc voltage, e_d, the equivalent reactance, X_c, the reactance $X_{dc} = L_{dc}\omega$ and the average value, E_d, of the dc-side voltage are known.

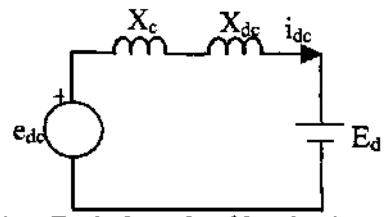

Fig. 4 : Equivalent dc side circuit with time varying reactance X_c

The dc side current harmonic component $\bar{I}^k_{dc}$, in Fig. 4, can be obtained from the following equation which can be solved by an iterative method:

$$\bar{I}^k_{dc} = \frac{\bar{E}^k_{dc} - X_{ac}\,(i_{dc}\otimes M_z)^k}{jkX_{dc} + jkX_{cav}} \tag{10}$$

where, $\bar{E}^k_{dc}$ and $(i_{dc}\otimes M_z)^k$ are the harmonic components of the dc side voltage, e_d, and the product $i_{dc} \times M_z$, respectively.

4.2. Constant dc side reactance approach

In [7], modified voltage transfer functions have been proposed to determine a new fictitious dc voltage that enables to model the dc side circuit using a constant reactance. The new fictitious dc voltage, e_{ndc}, is expressed as follows [7]:

$$e_{ndc} = e_a S_{mua} + e_b S_{mub} + e_c S_{muc} + E_d S_{cc} \tag{11}$$

where, S_{mua}, S_{mub} and S_{muc} are the modified voltage transfer functions applied to the ac voltages and S_{cc} is a new transfer function linked to the average dc value, E_d, of the dc-side voltage. Fig. 5 illustrates the transfer functions S_{mua} (for phase a) and S_{cc}.

It has been established in [7] that during commutation, the new fictitious dc voltage is given as follows:

$$e_{ndc} = \beta e_a - \kappa e_b - \kappa e_c - \theta E_d \qquad (12)$$

where,

$$\kappa = (2+f)/(3+2f) \qquad (13a)$$

$$\theta = (4+2f)/(3+2f) \qquad (13b)$$

$$\theta = 1/(3+2f) \qquad (13c)$$

where, $f = L_{dc}/L_{ac}$.

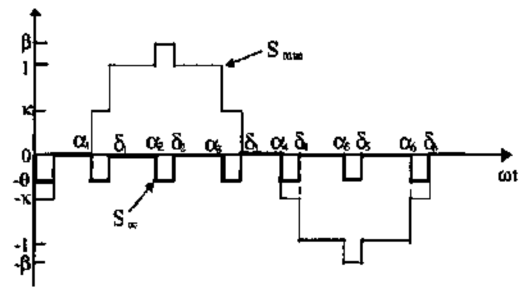

Fig. 5 : Modified voltage transfer function (for phase a)

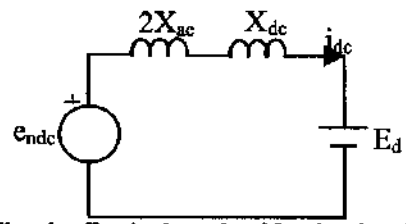

Fig. 6 : Equivalent dc side circuit with constant reactance

The dc side current harmonic component $\bar{I}^k_{dc}$, deduced from Fig. 6, is expressed as follows:

$$\bar{I}^k_{dc} = \frac{\bar{E}^k_{ndc}}{j2kX_{ac} + jkX_{dc}} \qquad (14)$$

where, $\bar{E}^k_{ndc}$ are the harmonic components of the new fictitious dc voltage, e_{ndc}, in Fig. 6.

The determination of the average value of the dc side current results from the calculations of the ac side current through ac side reactance and the ripple component of the dc side current deduced from (14).

5. CALCULATION OF AC SIDE CURRENT HARMONICS

The overlap period strongly influences the spectrum of the current converter functions.

Proposals were made to consider the commutation period accurately. In [3, 6], it was assumed that the converter current transfer functions are rising according to a linear function whereas in [1,2,4] it was assumed that they are rising according to a (1-cos(x)) function during the commutation periods. A more accurate current transfer function that takes into account dc current distortion during commutation has been used in [5]. In this paper, current transfer functions are approximated by (1-cos(x)) function during commutation (see Fig. 2) and equations (3) are used to compute ac current harmonics.

6. NUMERICAL EXAMPLES

In order to compare the transfer functions methods described above, a number of cases were tested. The parameters of table 1 are used. The circuit is supplied by unbalanced and distorted three-phase voltages given in table 2.

Table 1: Parameters for the thyristor converter case study (L_{dc} and L_{ac} in mH)

E_d (V)	α (°)	f = 1.33		f = 5		f = 15	
460	20	L_{dc}	L_{ac}	L_{dc}	L_{ac}	L_{dc}	L_{ac}
		0.4	0.3	1.5	0.3	4.5	0.3

Table 2: Supply voltages characteristics

Harmonic order	e_a		e_b		e_c	
	rms (V)	phase angle (°)	rms (V)	phase angle (°)	rms (V)	phase angle (°)
1	230.10	1.72	236	-120.84	224.05	119.12
5	11.5	0	11.5	120	11.5	-120

The values of the fundamental and the first eight odd harmonics of the ac side current are given in tables 3 for comparison purpose.

Table 3: AC side harmonic current (in % of Ia1=478.70 A for f = 1.33, with MTF*)

Harmonic order	1.33			f = 5			f = 15		
	MTF*	TTFM**	PSB	MTF	TTFM	PSB	MTF	TTFM	PSB
1	100.00	100.06	109.7143	99.75	99.80	104.27	99.59	99.62	100.75
3	2.44	2.49	4.40	0.79	0.86	2.07	0.47	0.49	0.99
5	18.71	18.95	19.50	17.63	17.73	18.65	17.11	17.14	18.03
7	9.55	8.95	9.80	9.96	9.80	10.04	10.37	10.34	10.35
9	0.38	0.32	0.86	0.52	0.51	0.49	0.63	0.63	0.40
11	3.21	3.29	3.12	3.53	3.56	3.88	3.70	3.71	4.27
13	2.68	2.53	2.50	2.40	2.36	2.45	2.25	2.25	2.47
15	0.82	0.82	0.89	0.62	0.63	0.49	0.52	0.52	0.32
17	0.23	0.31	0.89	0.53	0.57	0.82	0.69	0.70	0.83

*MTF: modified transfer function; **TTFM: traditional transfer function with modulation reactance

The simulation of the circuit was carried out using the Power System Blockset (PSB) in the Simulink environment and the results obtained are also given in table 3. There are some discrepancies between calculated and simulated results. The PSB, like any other time domain simulation approach is limited by the implicit error associated with the integration time step, the use of an approximated steady state and the fast Fourier transform to evaluate harmonics. On the other hand, it can be observed that the two approaches provide comparative results in the calculations of the ac side current harmonics.

7. CONCLUSION

In this paper, the comparison of two voltage transfer function models has been carried out, namely the traditional and the modified voltage transfer functions. With the traditional transfer function, time varying dc side impedance is used whereas with the modified transfer function, constant reactance is used. Both models provide comparative results in the calculations of the ac side current harmonics. However, the modified voltage transfer function is easier to implement than the traditional transfer function approach since the later requires an iterative process for the determination of the dc side current harmonics components.

Acknowledgements

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