Analyzing Input Harmonic Currents of a Six-pulse AC/DC Converter by an Efficient Time-domain Approach

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Abstract

This paper presents an efficient approach for analyzing harmonic currents generated by a six-pulse ac/dc converter in steady state. The approach is carried out in time-domain and the interactions between the system and the converter are considered in the study. In calculating harmonic currents generated by the converter, the Poincaré map based approach is applied to increase the computational efficiency and solution accuracy. The computed ac-side harmonic components of the converter current are then extracted via FFT. Solutions obtained by the proposed method are compared with those obtained by using a brute force time-domain simulation tool, Simulink of Matlab. It is shown that the harmonic currents determined by the proposed approach well agree with those obtained by the simulation tool, but the solution time is significantly reduced. In addition, the proposed method also can be applied to analyze harmonic currents produced by other types of power-electronic devices operating periodically.

1 Introduction

The introduction of line-commutated converters has caused a significant increase in harmonic-generating loads. These devices are most usual operated as a six-pulse converter, as shown in Fig. 1, or configured in parallel arrangements for high-pulse operations. Major applications of these converters are to be used as a front-end in devices such as ac/dc adjustable speed drives (ASDs), HVDC links, and uninterruptible power devices (UPSs) [1]. For harmonic studies, usually the converter can be simply represented by a harmonic current source or a model that takes into account the interaction between the ac source network and the converter dc system. When the latter situation is considered, a more sophisticated converter analysis for the resulting harmonic currents as a function of system reactance, firing angle, and commutation angle is required [2]. The accuracy of converter model also needs to be considered to guarantee the convergence of the simulation.

In recent years, there is a trend to adopt time-domain approaches to analyze harmonic currents generated by a static converter due to the need of modeling the device in details [3]-[5]. Among various time-domain approaches, Poincaré map method is one way that leads to efficiently analyzing the ac- and dc-side currents of the converter because of the periodic operation nature of the circuit [6]-[8]. In this paper the authors propose an approach that adopts Poincaré map concept to solve the converter problem. The solutions obtained by the proposed approach are then compared with those obtained by the brute-force time-domain simulation tool, Simulink. Results indicate that the proposed approach is computationally efficient and the solution is relatively accurate. Therefore, the proposed approach is useful to quantify harmonic currents generated by the ac/dc converter and to facilitate the harmonic mitigation implementations for complying with harmonic standards such as IEC 61000-3-6 and IEEE-519.

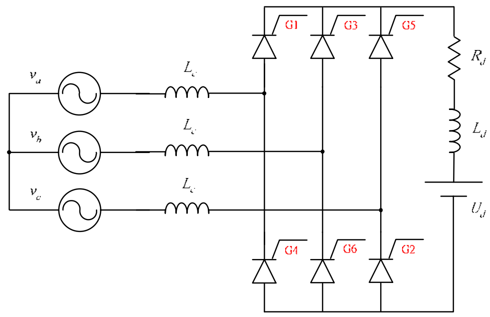

Fig. 1. A typical three-phase ac/dc six-pulse converter circuit

In the paper the operation principles of a six-pulse converter is firstly described. It is followed by a brief overview of the theoretical background and the illustration of applying Poincaré map concept to solve the converter problem. Next, simulation results obtained by the proposed solution method and by the use of Simulink are then compared to show the usefulness of the proposed method and the solution algorithm.

2 Review of Six-pulse AC/DC Converter Operation Principle

As indicated in the converter circuit of Fig. 1, the dc-side current, i_d , is a periodic function when the converter is operated in continuous conduction mode. At the ac source frequency, the dc-side current repeats every one-sixth period, *T/6*, under ideal operating conditions, or repeats every period, *T*, under asymmetrical operations. In the case of ignoring commutation, the ac-side source current at each phase, i_φ , $\varphi = a, b, c$ can be simply determined by the thyristor switching functions and the dc-side current, as given in (1).

$$i_\varphi = G_{\varphi,ac} i_d \;,\; \varphi = a,\, b,\, c, \tag{1}$$

where $G_{\varphi,ac}$, φ = a, b, c, are switching matrices that relate the dc and ac sides of the converter with values of 0 and 1 [4]. When considering commutation, the switching process is more conveniently expressed by differential equations. The following problem formulation describes the correlation between the dc- and ac-side currents for one commutation segment that occurs at thyristors T_1 and T_5.

During the commutation, two thyristors in (T_1, T_3, T_5) or in (T_2, T_4, T_6) conduct simultaneously. Fig. 2 shows the equivalent circuit during commutation from T_5 to T_1. By defining $u_e = v_a - v_c$ and $u_d = v_c - v_b$, two differential equations are obtained as follows.

$$\frac{di_d}{dt} = \frac{1}{3L_c + 2L_d}\left[u_e + 2u_d - 2i_d R_d - 2U_d\right], \tag{2}$$

$$\frac{di_e}{dt} = \frac{1}{3L_c + 2L_d}\left[\frac{(2L_c + L_d)}{L_c}u_e + u_d - i_d R_d - U_d\right]. \tag{3}$$

After solving (2) and (3) for i_d and i_e, the ac-side currents during commutation are $i_a = i_e$, $i_b = -i_d$, and $i_c = i_d - i_e$. The thyristor T_5 is turned off after commutation, the differential equation describing the circuit becomes

$$\frac{di_d}{dt} = \frac{1}{(2L_c + L_d)}\left[u_e + u_d - i_d R_d - U_d\right]. \tag{4}$$

Solving (4) for i_d yields $i_a = i_d$, $i_b = -i_d$, and $i_c = 0$.

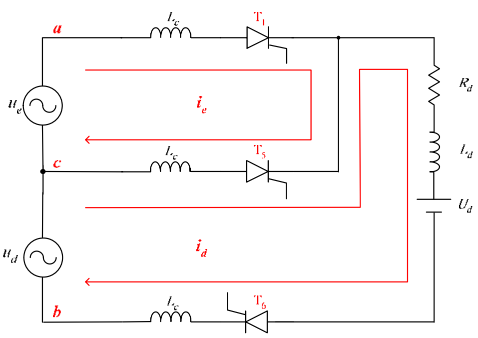

Fig. 2. Converter equivalent circuit during commutation (from T_5 to T_1).

Since there are six commutation segments in each period, each commutation segment is governed by two states (i.e. during and after commutation) including the three differential equations similar to (2)-(4), a total number of 18 equations are required to determine the dc-side current i_d . However, if the converter is symmetrically operated with ideal source voltages, only one commutation cycle (i.e. *T/6* period) is required to determine the dc-side current. The dc-side current over the remained segments can be obtained in the same way as solving (2)-(4). The ac-side current at each phase is then calculated from the determined dc-side current. Therefore, the required engineering effort for analyzing ac-side harmonic currents is greatly simplified. On the other hand, if the converter operation is asymmetrical, such as uneven loading conditions between converters or asymmetrical source-side reactances, it requires simulating a complete period for the twelve states created by the six commutations.

3 Poincaré Maps

The concept of a Poincaré map arises from the need of solving periodically forced linear constant-coefficient systems and systems with periodic-coefficient functions [9], it samples the response of a periodic system once every period. If there is a unique steady-state periodic solution for the given initial-valued differential equation, eventually two sampled values over two consecutive periods would be identical. The sampling process can be regarded as a mapping.

The definition of Poincaré map is expressed as: If $\dot{x}(t) = f(t, x)$ is a periodic function with a period of T and $\varphi(t, t_0, x_0)$ is a solution that passes through the initial value $x(t_0) = x_0$ in the function $f(t, x)$, then

$$P_{t_0}(x_0) = \varphi(t_0 + T, t_0, x_0) \tag{5}$$

is a Poincaré map of the given periodic differential equation. That is, the map moves $x(t_0) = x_0$ to the solution $\varphi(t, t_0, x_0)$ at $t = t_0 + T$. The k-th map of the initial value $x(t_0) = x_0$ is expressed as

$$P_{t0}^{(k)}(x_0) = \varphi(t_0 + kT, t_0, x_0) \tag{6}$$

If a steady-state solution is reached, it is concluded that the necessary and sufficient condition for a steady-state periodic solution passing through x_0 is that x_0 is a fixed point of the Poincaré map. A fixed point is defined as a steady-state solution, $\bar{x}$, that satisfies $\dot{x}(t) = f(t, \bar{x}) = 0$. If $\bar{x}$ is a fixed point of Poincaré map, then

$$P_{t_0}^{k}(\bar{x}) = P_{t_0}^{k+1}(\bar{x}) = \bar{x}, \qquad k = 1, 2, 3, \ldots \tag{7}$$

To give an effective illustration of how the Poincaré map is used to determine the steady-state solution of the dc-side current of the converter shown in Fig. 1, the progress of reaching the steady-state dc-side current is shown in Fig. 3. As indicated in Fig. 3, the first step is to determine the fixed point of the dc-side current; that is, the point where steady-state current starts. Given the initial guess of the fixed point at *t=0*, i_{d0} , one then checks if the dc current $i_{d1} = p_0(i_{d0})$ at $t=T$ is identical to i_{d0}, where i_{d1} is the Poincaré map of i_{d0}. If not, proceed to $t=2T$ and check if $i_{d2} = p_0(i_{d1})$. The same process repeats until $i_{d(n+1)} = p_0(i_{dn}) = i_{dn}$ at $t=(n+1)T$, where n is the number of maps when i_d reaches its steady state. Thus the fixed point is i_{dn}, which is the steady-state periodic solution of i_d. In Fig. 3, each current segment represents a pair of thyristors under continuous conduction mode and lasts *T/6* period. If the converter is in symmetrical operation under ideal source input, the mapping period can be adjusted to *T/6* instead of *T*. The solution time required to achieve the steady-state solution is considerably decreased.

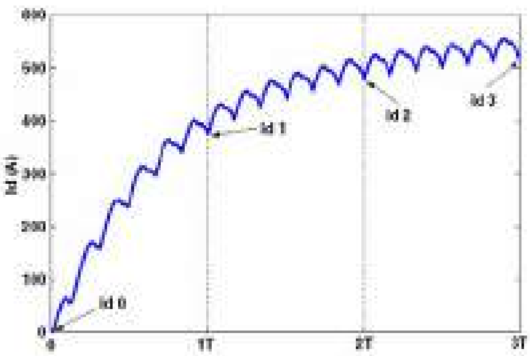

Fig. 3. Illustration of using Poincaré map to find the steady-state dc-side current of the six-pulse converter shown in Fig. 1.

In the proposed solution algorithm for calculating the converter dc-side current over one commutation segment by using Poincaré map based method, the mapping period is *T/6* when the converter is in symmetrical operation. If the converter is asymmetrically operated, the mapping period of *T* is chosen and the solution algorithm can be easily extended to include six commutation segments for a more detailed simulation. Major steps of the solution algorithm with the mapping period of *T/6* for the converter system are listed as follows.

1. Input converter system parameters including the thyristor firing angle and dc-side information indicated in Fig. 1.
2. Determine the source voltage zero-crossing time, t_z, and the thyristor firing angle time, t_f.
3. Assign the two initial values, i_{d0} and i_{e0} at $t_0 = t_z + t_f$ of (2) and (3) for the chosen commutation segment.
4. Solve i_d and i_e of (2) and (3) with the initial values of i_{d0} and i_{e0} until both currents are identical at the end of commutation, where the solution is $i_d = i_e = i_{df}$.
5. Solve i_d of (4) with assigning the initial value being i_{df} obtained at step 4 until $t = t_0 + T/6$.
6. Check if i_d equals i_{d0} of step 3. If yes, the fixed point of the converter dc-side current is found; stop and perform the ac-side current harmonics extraction by using FFT. Otherwise, let i_{d0} be $i_d(t_0 + T/6)$ obtained at step 5 and i_{e0} be i_{df} obtained at step 4. Return to step 4.

In the proposed converter solution algorithm, a commonly used ordinary differential equation (ODE) solver provided by Matlab is adopted to solve the corresponding differential equations with a fixed time step size and a convergence tolerance value. By taking advantage of the periodic solution speed of Poincaré map concept, the computational burden is greatly relieved and an accurate solution is achieved. It can be expected that, for the same studied time window, the Poincaré map based approach is more computationally efficient than other traditional brute-force time-domain simulation methods [9].

4 Simulation Results

To verify the usefulness of the proposed method for analyzing the steady-state harmonic currents generated by a six-pulse converter system, the CIGRE benchmark converter model is adopted for test purpose [10]. Though the high-voltage converter is used in the study, the proposed solution algorithm is suitable for converters at lower voltage levels of applications as well. Table 1 gives the benchmark converter system data with minor changes for simulation.

In the simulation, the proposed method for solving the converter ordinary differential equations is implemented by using Matlab along with its ODE23 solver. The ODE23 subroutine is provided with a fixed time step size of 1/60/3000 (i.e. 3000 points per period) and a tolerance value of 0.001 Ampere between two consecutive calculated values with a time span of *T/6* of the single-converter dc-side current. Results obtained by the proposed method are also compared with those obtained by using Simulink with variable time step [11].

	Converter Data
L_c	7.2e-2 H
R_d	5.0 Ω
L_d	8.3e-1 H
U_d	495.0 kV, dc
V_a, V_b, V_c	246.5 kV@60Hz
α	15°

Table 1: Parameters of the benchmark converter system

Under the assumptions that converter input voltages are ideal and the converter is with symmetrical operation, Fig. 4 through Fig. 8 show the dc-side current, the converter input current at *phase a*, converter input voltage at *phase a*, and corresponding harmonic spectra obtained by the proposed method for the converter in operation. The solutions obtained by Simulink, which models the converter circuit in details, are also shown at the bottom side of each figure for comparison.

Results show that the two solution sets have a good agreement. By comparing solutions obtained by the proposed method with Simulink solutions for Figs. 6 and 8, the largest error in the harmonic magnitude is less than 6% for Fig. 6, while the largest error in harmonics is about 7% for Fig. 8.

5 Conclusions

This paper presents an efficient approach for analyzing harmonic currents generated by a six-pulse converter system. The method used is categorized as a time-domain approach. As described in the paper, the theory of the proposed solution algorithm for analyzing harmonic currents generated by the converter in steady state is described in details. The usefulness of the approach is demonstrated through simulations for the converter. The results are then compared with those obtained by the use of Simulink. Both results show a close agreement. However, the solution time required by the proposed approach is much less than that of the brute force time-domain method used by Simulink. It is concluded that the proposed method is not only computationally efficient, but is also suitable for analyzing harmonic currents generated by other types of power-electronic devices with periodic operations.

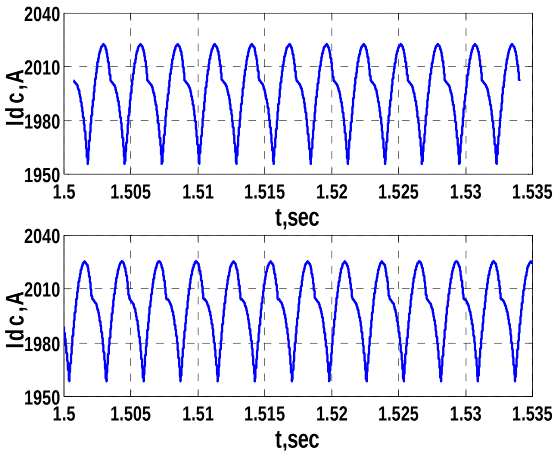

Fig. 4. Time-domain dc-side current of the converter (top waveform: proposed method; bottom waveform: Simulink)

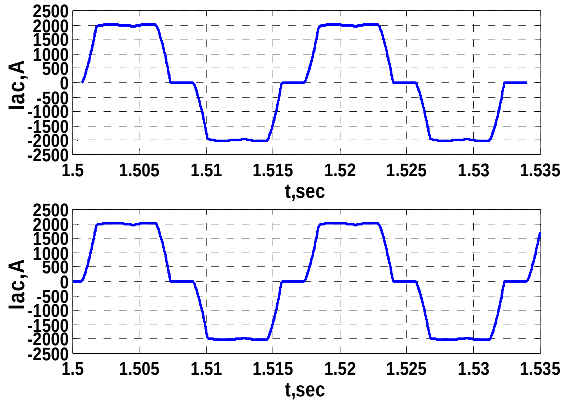

Fig. 5. Source-side input current drawn by the converter system at *phase a* (top: proposed method; bottom: Simulink)

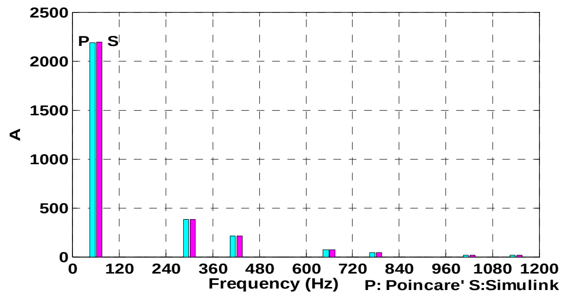

Fig. 6. Harmonic current spectra of Fig. 5 (left: proposed method; right: Simulink).

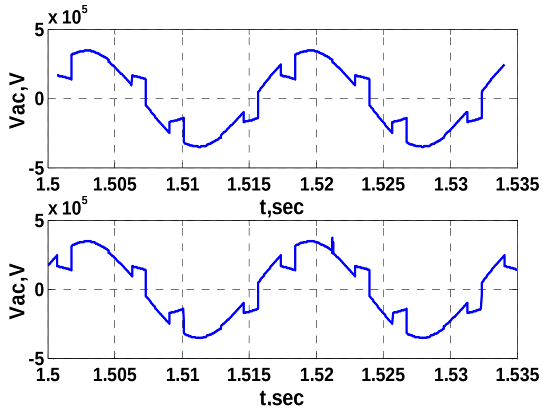

Fig. 7. Converter input voltage at *phase a* (top: proposed method; bottom: Simulink).

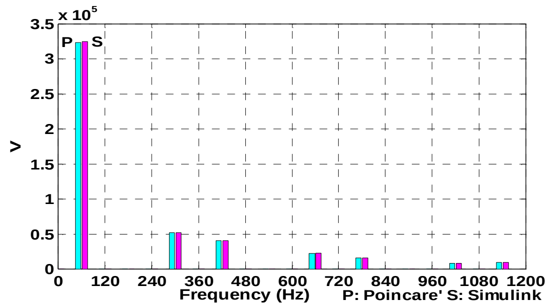

Fig. 8. Converter input voltage harmonics of Fig. 7 (left: proposed method; right: Simulink)

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