Mine ventilation optimization planning based on fuzzy-set pair theory

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Mathematical models for optimizing and evaluating mine ventilation systems

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ABSTRACT: In the initial mine planning or subsequent mine ventilation system upgrades, multiple viable ventilation plans for the given geological conditions and operation requirements could be proposed. Selecting the best one of them and evaluating its effectiveness and efficiency could be a complicated task. Based on former research and in-depth analysis of various mine ventilation systems, an integrated mathematical model is developed for such asks. The model itself is comprised of two sub models. One is the optimal selection ventilation system model which applies the fuzzy set pair analysis to assist the selection of the best ventilation plan from a set of various plans. The other is the comprehensive evaluation system model based on the comprehensive fuzzy-grey theory to evaluate the upgraded mine ventilation system. A test-case has been used to demonstrate the applicability of this integrated model.

1 Introduction

A mine's ventilation system is an important component of an underground mining system. It should provide a sufficient quantity of air to the underground mine workings, to dilute methane and other contaminants, and maintain suitable working environment and prevent accidents from happening. Very often, ventilation is a limiting factor for coal mine production.

In the planning stage for a new coal mine or for a change or upgrade to an old mine, multiple viable ventilation plans could be proposed to cope with varying geological conditions, production rates, mining laws and regulations, etc. Each of the plans may put emphasis on one or a number of different considerations. Therefore, how to select the best ventilation plan is a complicated task.

Many criteria have been used to assist the selection of ventilation plans including equivalent orifice (Murgue, 1873), total air quantity to working faces, ventilation efficiency, etc. Because of the complexity of coal mine ventilation systems, it would be difficult to select the most suitable plan if only one or two single or discrete criteria are used in the selection process since the individual criteria may not be able to reflect all characteristics of a mine ventilation system. In order to consider multiple variables, new comprehensive models have been developed. Huang (1994) picked seven critical variables or indices emphasizing on safety and economics to choose the ventilation plan. Sheng (1999) used twelve qualitative and quantitative indices to evaluate ventilation plans. Zhou & Wang (2002) tried to make the selection procedure a mathematical process using another twelve quantitative indices to evaluate plans.

In this paper, an integrated mathematical model based on the former findings is developed for both selecting and evaluating ventilation plans. It consists of two sub-models: the optimal selection mine ventilation system model; and the comprehensive evaluation system model. The optimal selection model establishes an index system which includes a set of individual criteria of technical, economical and safety aspects and then the fuzzy set pair analysis is applied to select the best ventilation plan. The comprehensive evaluation system model provides a comprehensive evaluation method based on the grey cluster analysis-fuzzy theory to evaluate the updated ventilation system.

2 Optimal Selection Mine Ventilation System Model

2.1 Optimal index system

Large variations in geological conditions and in the development and production requirements necessitates the need to consider multiple factors in the selection of mine ventilation system plans. In order to give full considerations to these factors, a well and synthetic index system including all aspects of ventilation system must be developed. This indexing system chooses sub-indexes from the economical, technical and safety aspects as shown in Figure 1. A set of proper weighting factors also should be given to the individual variables for generating a composite score of the ventilation system.

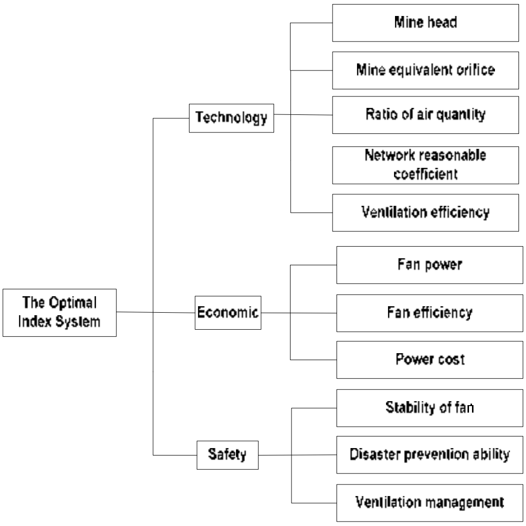

Figure 1 The optimal selection index system

2.2 Selection Method based on Fuzzy Set Pair Analysis

Set Pair Analysis (SPA) theory was developed by Zhao (2000). SPA is a kind of system analysis method and its main concept is to combine both the certain and uncertain phenomena into a system of certainty and uncertainty. Then the quantitative description and transition relationship could be represented by a mathematical model for finding reasonable results. In addition, by combining the fuzzy theory and SPA, the fuzzy set pair analysis could produce more accurate results. The fuzzy set pair analysis mixes certain and uncertain information in the form of a fuzzy set. Based on the set, the fuzzy theory is used again to analyze the problem. The procedure is shown in the following sections:

2.3 Establishing the decision plan matrix

Assuming the candidate plan is S_k ($k = 1, 2, \ldots, m$) and each of the sub-indices is e_r ($r = 1, 2, \ldots, n$), so d_{kr} is the property value of e_r with respect to S_k. Therefore the decision plan matrix D is:

$$D=\begin{pmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{m1} & \cdots & d_{mn} \end{pmatrix}=(d_{kr})_{m\times n} \tag{1}$$

2.3.1 *Determining the ideal plan*

In order to compare among the candidate plans in a certain range, the best and worst plan must be determined first. Assuming the best plan is $\boldsymbol{u} = (u_1, u_2, \ldots, u_n)$ and the worst plan is $\boldsymbol{v} = (v_1, v_2, \ldots, v_n)$. The terms u_r and v_r are the best and worst values for each e_r. So the best and worst plan are:

$$\boldsymbol{u} = (\max\{d_{k1}\},\ \max\{d_{k2}\}, \ldots, \max\{d_{kn}\}) \tag{2}$$

$$\boldsymbol{v} = (\min\{d_{k1}\},\ \min\{d_{k2}\}, \ldots, \min\{d_{kn}\}) \tag{3}$$

Thus, the comparative range of e_r is [u, v].

2.3.2 *Determining the degree of fuzzy correlation*

Identical subordinate degree

In the comparative range [u, v], the identical subordinate degree a_{kr} of the set pair $\{d_{kr}, u_r\}$ under the domain of discourse X_r is defined as

$$a_{kr}=\frac{d_{kr}}{u_r+v_r} \tag{4}$$

Where: $X_r = \{d_{kr}, u_r, v_r\}$, $k = 1, 2, \ldots, m$

Opposite subordinate degree

Under the aforementioned condition, the opposite subordinate degree c_{kr} is:

$$c_{kr}=\frac{\frac{1}{d_{kr}}}{\frac{1}{u_r}+\frac{1}{v_r}}=\frac{u_r v_r}{(u_r+v_r)\cdot d_{kr}} \tag{5}$$

The values of a_{kr} or c_{kr} represents how d_{kr} is close to u_r or v_r, respectively.

Differential subordinate degree

Since the definition of SPA states that $a + b + c = 1$, the differential subordinate degree b_{kr} of the set pair $\{d_{kr}, u_r\}$ is:

$$b_{kr}=1-a_{kr}-c_{kr}=\frac{(u_r-a_{kr})(a_{kr}-v_r)}{(u_r+v_r)\cdot d_{kr}} \tag{6}$$

With considering different weighting factor for each index, the identical, opposite and differential subordinate degrees can be expressed as:

$$a_k=\sum_{r=1}^{n} w_r\cdot a_{kr} \tag{7}$$

$$c_k=\sum_{r=1}^{n} w_r\cdot c_{kr} \tag{8}$$

$$b_k=\sum_{r=1}^{n} w_r\cdot b_{kr} \tag{9}$$

In these equations, W_r is the weighting factor and $\sum_{r=1}^{n} w_r=1$, finally, the degree of fuzzy correlation can be written as:

$$\mu(s_k,u)=a_k+b_k\bullet i+c_k\bullet j \tag{10}$$

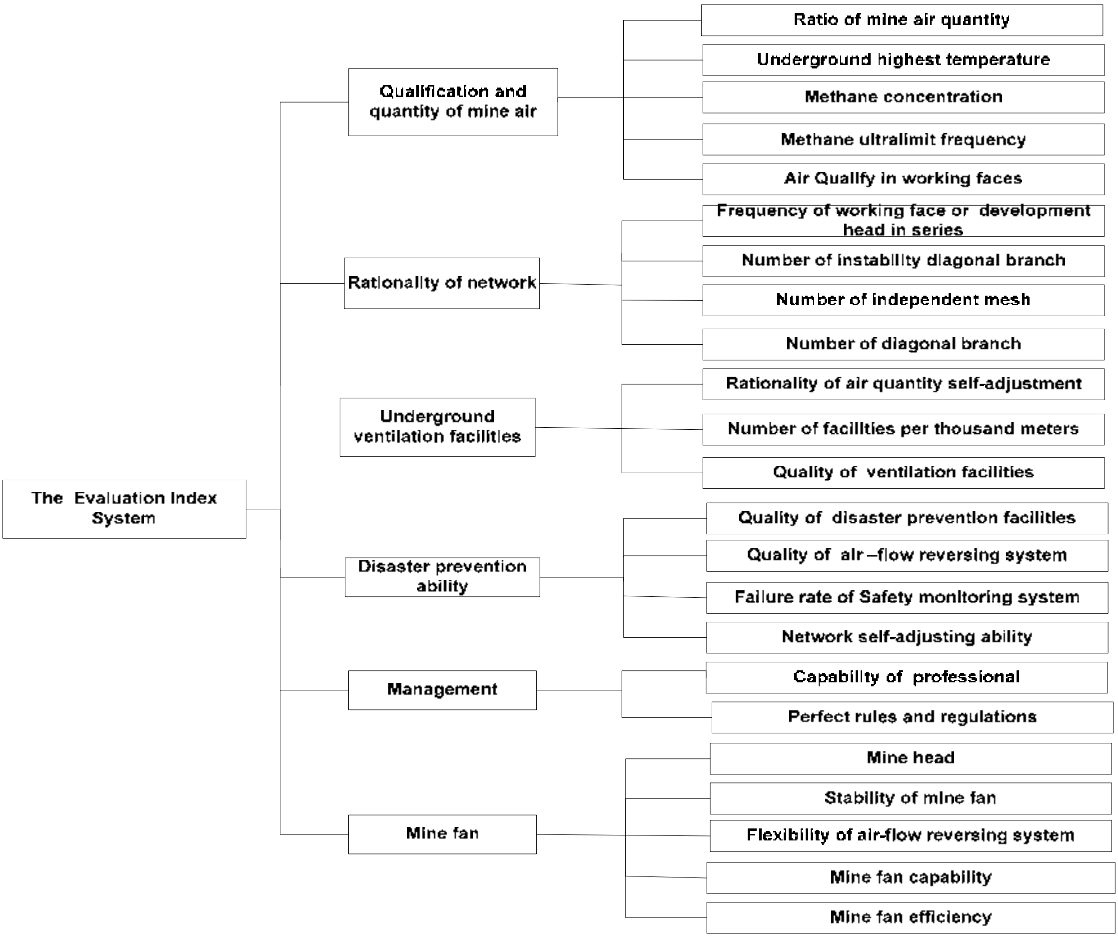

Figure 2 The evaluation index system

2.3.3 *Determining the relative degree of nearness and Selecting the optimal plan*

The meaning of a_{kr} or c_{kr} in the SPA analysis represents how close the candidate plan S_k is to the best or worst plan. It is a piece of information with certainty. So the relative nearness degree r_k is defined as: $r_k = \frac{a_k}{a_k + c_k}$. Thus, we can compute the value of r_k for a set of candidate plans. If $\max\limits_{1 \le i \le m}\{r_k\} = r_{ki}$, the plan S_{ki} can be considered to be the best one.

3 Comprehensive Evaluation Mine Ventilation System Model

3.1 The evaluation index system

Among the components of mine ventilation system, indexes are picked from the following six aspects: qualification and quantity of mine air, network rationality, underground ventilation facilities, mine fans, disaster prevention ability, and management as shown in Figure 2.

3.2 Evaluation model based on grey-cluster analysis fuzzy theory

As a mine ventilation system is a typical grey system, the author has developed a new evaluation method combining *grey cluster analysis* and *fuzzy evaluation* (Cheng, 2008). In general, grey cluster analysis (Liu & Guo, 1999) is used to build the evaluation matrix while fuzzy theory is applied to obtain the final score. The procedure is shown as follows:

a) If the number of indices is *m* and there are *s* different grey classes, the field measured value for a specified object *i* is X_{ij}(j=1, 2, …, *m*) with respect to a particular index *j*. Then the range of X_{ij} about each index is divided into *s* different types. For example, if the range of X_{ij} is $[a_1, a_{s+1}]$, then *s*.

Table 1 Recommended Values for Different a_k

First level	Second level	Units	Symbol	Weighting	Better Class	Medium Class	Worse Class
Qualification and quantity of mine air	Ratio of mine air quantity		X_1	0.0036	$1.1 \le x_1^1 < 1.2$	$1.2 \le x_1^2 < 1.3$	$1.3 \le x_1^3 < 1.5$
	Underground highest temperature	□	X_2	0.0016	$24 \le x_2^1 < 26$	$26 \le x_2^2 < 30$	$30 \le x_2^3 < 34$
	Methane concentration	%	X_3	0.0137	$0.1 \le x_3^1 < 0.5$	$0.5 \le x_3^2 < 0.75$	$0.75 \le x_3^3 < 1$
	Methane ultra limit frequency	time	X_4	0.0248	$1 \le x_4^1 < 2$	$2 \le x_4^2 < 3$	$3 \le x_4^3 < 4$
	Air Qualify in working faces	score	X_5	0.0036	$90 \le x_5^1 < 95$	$80 \le x_5^2 < 90$	$70 \le x_5^3 < 80$
Rationality of network	Frequency of working face or development head in series	%	X_6	0.0134	$5 \le x_6^1 < 10$	$10 \le x_6^2 < 20$	$20 \le x_6^3 < 25$
	Number of instability diagonal branch		X_7	0.0074	$1 \le x_7^1 < 2$	$2 \le x_7^2 < 3$	$3 \le x_7^3 < 4$
	Number of independent mesh		X_8	0.0364	$1 \le x_8^1 < 50$	$50 \le x_8^2 < 100$	$100 \le x_8^3 < 150$
	Number of diagonal branch		X_9	0.0134	$10 \le x_9^1 < 20$	$20 \le x_9^2 < 30$	$30 \le x_9^3 < 35$
Underground ventilation facilities	Rationality of air quantity self-adjustment		X_{10}	0.0828	$0.5 \le x_{10}^1 < 1$	$1 \le x_{10}^2 < 1.25$	$1.25 \le x_{10}^3 < 1.5$
	Number of facilities per thousand metes		X_{11}	0.0142	$1 \le x_{11}^1 < 3$	$3 \le x_{11}^2 < 7$	$7 \le x_{11}^3 < 10$
	Quality of Ventilation facilities	%	X_{12}	0.0484	$95 \le x_{12}^1 < 99$	$90 \le x_{12}^2 < 95$	$85 \le x_{12}^3 < 90$
Disaster prevention ability	Quality of disaster prevention facilities	score	X_{13}	0.0474	$95 \le x_{13}^1 < 98$	$93 \le x_{13}^2 < 95$	$91 \le x_{13}^3 < 93$
	Quality of air-flow reversing system		X_{14}	0.0474	1		0
	Failure rate of Safety monitoring system	%	X_{15}	0.0266	$1 \le x_{15}^1 < 2$	$2 \le x_{15}^2 < 3$	$3 \le x_{15}^3 < 4$
	Network self-adjusting ability		X_{16}	0.1685	$1 \le x_{16}^1 < 1.25$	$1.25 \le x_{16}^2 < 1.5$	$1.5 \le x_{16}^3 < 1.7$
Management	Capability of professional	score	X_{17}	0.0242	$90 \le x_{17}^1 < 95$	$85 \le x_{17}^2 < 90$	$80 \le x_{17}^3 < 85$
	Perfect rules and regulations	score	X_{18}	0.0081	$80 \le x_{18}^1 < 90$	$70 \le x_{18}^2 < 80$	$60 \le x_{18}^3 < 70$
Mine fan	Mine head	Pa	X_{19}	0.0367	$1000 \le x_{19}^1 < 2924$	$2924 \le x_{19}^2 < 3538$	$3538 \le x_{19}^3 < 4368$
	Stability of mine fan	score	X_{20}	0.1631	$80 \le x_{20}^1 < 95$	$50 \le x_{20}^2 < 80$	$20 \le x_{20}^3 < 50$
	Flexibility of air-flow reversing system	score	X_{21}	0.1140	$90 \le x_{21}^1 < 95$	$85 \le x_{21}^2 < 90$	$80 \le x_{21}^3 < 85$
Mine fan	Mine fan capability	%	X_{22}	0.0639	$1.2 \le x_{22}^1 < 1.3$	$1.1 \le x_{22}^2 < 1.2$	$1.05 \le x_{22}^3 < 1.1$
	Mine fan efficiency		X_{23}	0.0367	$80 \le x_{23}^1 < 85$	$70 \le x_{23}^2 < 80$	$60 \le x_{23}^3 < 70$

Table 2 Recommended values for a_0 and a_{s+2}

Symbol	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
x_j^0	1.05	23	0	0	100	0	0	0	0	0	0
x_j^5	1.70	42	2	5	60	30	5	200	60	3	25
Symbol	X_{12}	X_{13}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}
x_j^0	80	90	0	0	70	30	200	0	70	1	50
x_j^5	100	100	5	2	100	100	4410	100	100	1.2	90

Note: For X_{14}, Quality of air-flow reversing system is set to 1 or 0, so there are no short extra distances for X_{14}

different intervals are: $[a_1, a_2]$, … , $[a_{k-1}, a_k]$, …, $[a_{s-1}, a_s]$, $[a_s, a_{s+1}]$ where the values of a_k (k = 1, 2, …*s*, *s*+1) should be determined from the field experiences

b) The triangular Whitening weight (TWW) function is used. The value of TWW about the k^th grey class is set to 1 at $\lambda_k = (a_k + a_{k+1})/2$. Connecting the point ($\lambda_k$,1) with the starting point (a_{k+1}) of the (k-1)^th grey class and the ending point (a_{k+2}) of the (k+1)^th grey class, respectively, the TWW function (f_j^k) of the index j to the k^th grey class is obtained and can be expressed as:

$$f_j^k(x) = \begin{cases} 0 & x \notin [a_{k-1}, a_{k+2}] \\ \dfrac{x - a_{k-1}}{\lambda_k - a_{k-1}} & x \in [a_{k-1}, \lambda_k] \\ \dfrac{a_{k+2} - x}{a_{k+2} - \lambda_k} & x \in [\lambda_k, a_{k+2}] \end{cases} \qquad (11)$$

where j= 1, 2, …, *m* and k = 1, 2, …, *s*.

If the value of X_{ij} is obtained, the degree of membership $f_j^k(x)$ can be calculated by Equation 11. Especially for f_j^1 and f_j^s, a short extra distance is extended to a_0 and a_{s+2}. Figure 3 shows the TWW function.

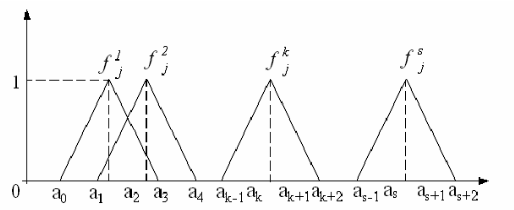

Figure 3 The TWW Function

c) σ_i^k strands for the integrated cluster coefficient about *k* grey class with respect to the object *i* and is defined as $\sigma_i^k = \sum_{j=1}^{m} f_j^k(x_{ij}) \cdot \eta_j$, where η_j is weighting factor. The values of σ_i^k (also considered as the membership degree) can be calculated for the second level index under different grey classes. Then σ_i^k could be used to built the matrix $\tilde{C}_i = (r_{i1}, r_{i2}, r_{i3})$, where different r_i equals to σ_i^k. Then $\tilde{C}_i$ (i = 1, …, 6) can also build a $i \times k$ matrix which is so called the fuzzy comprehensive matrix $\tilde{R} = (\tilde{C}_1, \tilde{C}_2, \cdots, \tilde{C}_n)^T$.

d) In this model, the assessment matrix (C) is C = {Best(C_1), medium(C_2), worse(C_3)} and the weighting factor matrix $\tilde{A}$ is also assigned. The equation $\tilde{A} \circ \tilde{R}$ is used to calculate the result matrix $\tilde{B} = (b_1, b_2, b_3)$.

e) To obtain the finial rating score, a preliminary score is first determined according to the maximum membership degree. Then the score is modified based on the following criteria(Cheng & Yang, 2007):
   - If the preliminary score is "Worse", the final score should be adjusted to "Medium" when $b_1 + b_2 > b_3/2$, or keep "Worse".
   - If the preliminary score is "Best", the final score should be adjusted to "Medium" when $b_2 + b_3 > b_1/2$, or keep "Best".
   - If the preliminary score is "Medium", the final score should be adjusted to "Worse" when $b_1 > b_2/2 > b_3$, or keep "Medium".

4 Case Demonstration

4.1 Optimal selection model demonstration

A coal mine located in the northern part of China has a 20-year production history. As a consolidation effort, it would be merged with four small coal mines in the immediate surrounding area to form a large mine. In order to meet the new production system requirements, the mine ventilation system must be upgraded. Three candidate ventilation plans were proposed and their ventilation parameters are showed as Table 3.

Table 4 Measured Values of the Mine Ventilation System Indices

Symbol	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
Value	1.2	24.6	0.62	3	94	3	1	6	12	1	2	93
Symbol	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}	
Value	96	1	2	1.6	92	75	2254	77	87	1.09	77.6	

Table 5 Values of TWW function for Each Index

Symbol	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
$f_j^1(x)$	0.67	0.80	0.29	0.00	0.80	0.40	0.67	0.24	0.80	0.50	1.00
$f_j^2(x)$	0.67	0.15	0.99	0.67	0.10	0.00	0.00	0.07	0.13	0.80	0.25
$f_j^3(x)$	0.00	0.00	0.32	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Symbol	X_{12}	X_{13}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}
$f_j^1(x)$	0.43	0.86	0.67	0.00	0.93	0.33	0.81	0.72	0.27	0.00	0.61
$f_j^2(x)$	0.92	0.50	0.67	0.31	0.40	1.00	0.56	0.60	0.93	0.40	0.74
$f_j^3(x)$	0.27	0.00	0.00	1.00	0.00	0.33	0.00	0.07	0.40	0.88	0.16

Table 3 Ventilation Parameters for Candidate Plans

Plan **Index**	**A**	**B**	**C**
Mine head (Pa)	2137.83	2014.02	1976.35
Mine equivalent orifice (m^2)	1.67	1.79	1.81
Ratio of air quantity	1.09	1.12	1.11
Network reasonable coefficient	0.98	0.92	0.87
Ventilation efficiency %	90.0	87.3	90.0
Fan power Kw	204.24	205.28	227.62
Fan efficiency %	80	76	78
Power cost Yuan/ton	1.17	1.18	1.31
Stability of fan	68.82	73.55	62.85
Disaster prevention ability	5.2	8.8	7.2
Ventilation management	6.86	8.28	6.09

The decision plan matrix is formed using the parameters in Table 3. The ideal plans are:

u = (5.06, 1.81, 1.12, 0.98, 90.0, 4.90, 80, 0.855, 73.55, 8.8, 8.28)

v = (4.68, 1.67, 1.09, 0.87, 87.3, 4.39, 76, 0.763, 62.85, 5.2, 6.09)

The degree of fuzzy correlation with consideration of the weighting factors for each plan can be calculated though Equations 4 – 10 as the following.

$$\mu(s_1,u)=0.480492+0.001345i+0.518063j$$

$$\mu(s_2,u)=0.534222+0.000516i+0.465162j$$

$$\mu(s_3,u)=0.487529+0.006111i+0.506197j$$

The relative degrees of nearness r_k for the three plans are:

$$r_1=\frac{a_1}{a_1+c_1}=\frac{0.480492}{0.480492+0.518063}=0.48$$

$$r_2=\frac{a_2}{a_2+c_2}=\frac{0.534222}{0.534222+0.465162}=0.53$$

$$r_3=\frac{a_3}{a_3+c_3}=\frac{0.487592}{0.487592+0.506197}=0.49$$

The resulting sequence is $r_2>r_3>r_1$, thus the second candidate plan is the best plan.

4.2 Comprehensive evaluation model demonstration

After upgrading this coal mine ventilation system, a thorough investigation has been carried out. Table 4 shows the measured field data for each index.

Following the aforesaid procedure; values of TWW function for each index are obtained.

The converted fuzzy comprehensive matrix is shown as:

$$\tilde{R}=\begin{vmatrix}0.011 & 0.033 & 0.021\\ 0.030 & 0.004 & 0.000\\ 0.076 & 0.114 & 0.013\\ 0.106 & 0.094 & 0.169\\ 0.025 & 0.018 & 0.003\\ 0.200 & 0.277 & 0.119\end{vmatrix}$$

The weighting factor matrix is:

$$\tilde{A}=\left|0.0473\ 0.0708\ 0.1453\ 0.2899\ 0.0322\ 0.4144\right|$$

After calculation and normalization, the result matrix becomes:

$$\tilde{B}=(b_1,b_2,b_3)=(0.3286,0.4127,0.2587)$$

The results indicate that the preliminary score is "Medium". However, since $b_1 > b_2/2 > b_3$, the final score is "Best".

5 Conclusions

An integrated comprehensive method for selecting and evaluating the most suitable mine ventilation system from various viable planes has been proposed. The method is comprised of an optimal selection and a comprehensive evaluation mine ventilation system models. Both of the models are established on index system. A set of mathematical tools have been used to derive solutions that are accurate and scientifically sound. Using this method, the severe influence caused by anthropogenic factors or single index can be avoided in the final result. Therefore, the selection and evaluation procedure of this method would ensure the selected mine ventilation system is the more rational and economical among the proposed viable plans. The case demonstration shows that this integrated model has better accuracy and reliability and could be applied in mine design practices.

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