A systematic graph-based method for the kinematic synthesis of non-anthropomorphic wearable robots

Fabrizio Sergi*, Dino Accoto*, Nevio Luigi Tagliamonte*, Giorgio Carpino*,
Lakshmi Pathiyil † and Eugenio Guglielmelli*
*CIR, Laboratory of Biomedical Robotics and Biomicrosystems
Università Campus Bio-Medico di Roma, Rome, Italy
† School of Mechanical and Aerospace Engineering
Nanyang Technological University, Singapore

***Abstract*—The choice of non-anthropomorphic kinematic solutions for wearable robots is motivated both by the necessity of improving the ergonomics of physical Human-Robot Interaction and by the chance of exploiting the intrinsic dynamical properties of the robotic structure for an optimal interaction with the human body. Under these aspects, this new class of robotic solutions is potentially advantageous over the one of anthropomorphic robotic orthoses. However, the process of kinematic synthesis of non-antrhopomorphic wearable robots is very complex and difficult to be tackled by human intuition and engineering insight alone. A systematic approach is more useful for this purpose, since it allows to obtain the number of independent kinematic solutions with desired properties. In this perspective, this paper presents a method which enables to list the possible kinematic solutions for wearable robotic orthoses, which generalize the set of solutions of the problem of kinematic synthesis of a non-anthropomorphic wearable robot. This method has been implemented to derive the atlas of topologies of robotic kinematic chains which can be employed to support a 1-DOF human joint.**

***Index Terms*—Non-anthropomorphic wearable robots, topology, graph theory.**

I. INTRODUCTION

Physical Human-Robot Interaction (pHRI) is a very important attribute in the design of robots operating in human environments. This attribute becomes crucial in the case of wearable robots, which are person-oriented systems worn by human operators to extend, complement, substitute or enhance human function and capability [1]. In the design of robotic orthoses, both for human performance augmentation and for functional restoring, the most followed route has been that of designing the robot so to replicate as much as possible the kinematic structure of the human limbs [2], [3], [4]. Robots belonging to this class were thus named exoskeletons, according to the definition given in [5], where it is reported that an exoskeleton is *"an active mechanical device that is essentially anthropomorphic in nature, is "worn" by an operator and fits closely to his or her body, and works in concert with the operator's movements"*. Robot kinematic chain is not a free design parameter for robotic exoskeletons, while wearable robots can be designed to have a possibly non-anthropomorphic kinematic structure, according to the classification introduced in [6].

A. Advantages of non-anthropomorphic robot kinematic structures

Studies stemming from the biological considerations of the locomotion of lower animal forms highlighted how morphology and control are intimately coupled [7]. Robotic researchers have afterwards demonstrated that these findings could be replicated on artificial agents [8], [9]. These studies demonstrated that the design of robot structure which aims at achieving a successful intrinsic dynamical interaction with the environment can reduce the complexity of the control. This enabled to conclude that there exists a *morphology and control trade-off* [10] in the design of robots, which can be tuned to exploit the so-called *extradimensional bypass* [11] in the concurrent design of both robot morphology and control. These considerations allow to conclude that robot performances can enormously benefit from a careful design of robot morphology, which is open in the case of non-anthropomorphic wearable robots, and can allow to achieve a better dynamical interaction with the human body.

Furthermore, the problem of kinematic compatibility is very relevant in the ergonomics of pHRI. In most cases, a kinematic

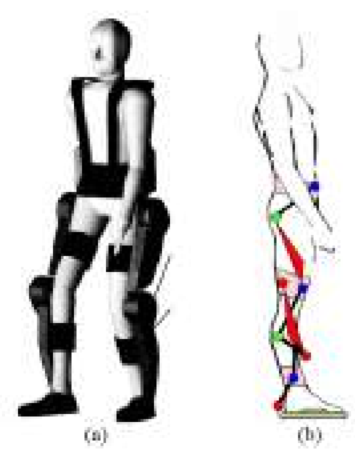

Fig. 1. Example of an anthropomorphic wearable robot for the lower limbs (a), from [12]. Concept of a non-anthropomorphic wearable robot for the lower limbs (b).

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model of a human limb is used for testing virtual concepts of wearable robots using 3D design software. Limbs models may not replicate accurately the biomechanical properties of the real human limbs for several reasons (e.g. inter-subject variability of parameters, or oversimplification of the kinematic model of human joints). Hence, kinematic incompatibilities between the real human limbs and the robot may occur, as described by [13]. These incompatibilities may be classified into: *macro-misalignments*, induced by a mismatch between the degrees of freedom of the human limb and the exoskeleton; *micro-misalignments*, induced by the non-coincidence of joints axes, when the exoskeleton kinematic structure aims at replicating human kinematics or by slippage of the exoskeleton attachments on the skin during motion. Both kinds of misalignments cause the exchange of unwanted interaction forces at the sites of contact between the human and the robot. Some studies have also demonstrated that they are source of discomfort and even pain for the user [14], [15].

On one hand, macro-misalignments are unavoidable since the robot designer has the necessity of simplifying the structure of the robot thus restricting the number of degrees of freedom of the mechanism. On the other hand micro-misalignements can be avoided if there is no need to align the rotation axes of the robot with those of the human limbs, as it is the case of non-anthropomorphic wearable robots. In [13], a new paradigm for the design of kinematically compatible wearable robots for rehabilitation was proposed, postulating that a wearable robot must explicitly not copy the kinematic structure of the adjacent human limbs, and should provide a moving system acting in parallel with the human degrees of freedom.

Non-anthropomorphic wearable robots promise positive performances from both standpoints of the dynamical interaction with the human body and of ergonomics. Despite of that, the process of optimal kinematic synthesis of non-anthropomorphic wearable robots is difficult to be solved by human intuition and engineering insight alone. In order to facilitate this task, a systematic approach is useful, since it allows to obtain the number of independent kinematic solutions with desired kinematic properties, which generalize the set of solutions of the problem of kinematic synthesis of a non-anthropomorphic wearable robot with given requirements.

This paper describes a method for the systematic enumeration of the topologies of kinematic solutions of non-anthropomorphic wearable robotic orthoses and provides the atlas of all possible independent kinematic structures for the case of an orthosis for a 1 DOF human joint.

II. KINEMATIC STRUCTURE ENCODING

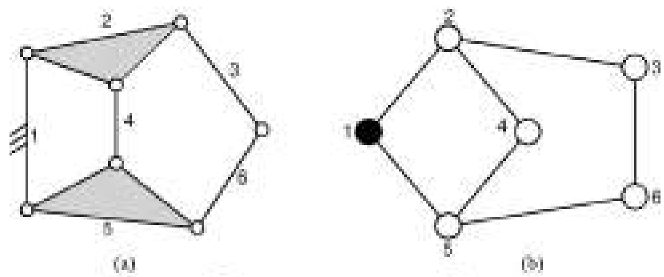
As a first step, en encoding needs to be defined, in order to represent the kinematic structure of robotic orthoses. Since the aim is to evaluate also the mobility of the human limbs connected to a robotic orthosis, the whole parallel kinematic chain consisting of both robot links and human limbs is considered. The description of this kinematic chain can be performed at two levels of abstraction: *i) topology*, which defines the number of links and the connections among them and *ii) morphology*, which instantiates a given topology, adding the geometrical properties of links and of joints.

A. Topology

Under some reasonable hypotheses [16], many properties of mechanisms kinematics, such as the number of degrees of freedom, are entirely determined only by the topology of the kinematic chain and unaltered by the geometric properties of its links. Employing for this purpose the representation introduced by Dobrjanskyj and Freudenstein in 1967 [17], a kinematic chain can be uniquely represented by a graph, whose vertices correspond to the links of the chain and whose edges correspond to the joints of the chain. A graph can then be encoded in the most simple way through the Topology vertex-vertex Adjacency Matrix (TAM), which is a binary symmetric matrix of order n (where n corresponds to the number of links) where the element a_{ij} equals to 1 if link i and link j are connected through a joint, and to 0 otherwise. This representation is complete in the description of kinematic chains topology and allows converting the problem of kinematic synthesis into a problem of graphs enumeration.

B. Morphology

The previously described representation for the topology of the kinematic chain can be adapted also for describing robot morphology, which gives details on the geometrical properties of links and is necessary to evaluate the possible kinematic configurations of the chain. We restrict our focus on planar kinematic chains containing only revolute joints. Then, the only necessary information to define robot morphology is the position of joints in a particular configuration. In such a way a binary link is represented by a bar jointed at its extremities and a ternary link by a triangle jointed at its vertices and so on. Under these assumptions we only need to modify the



$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Fig. 2. Structural representation (a), graph representation (b) and TAM (c) of a six links kinematic chain. Link 1 is filled in black since it is mechanical ground.

adjacency matrix by adding the information concerning joints coordinates. This can be done by introducing the Morphology Adjacency Matrix (MAM), which is a square matrix of order n containing x-coordinates of joints (if any) in its strictly upper-triangular part and y-coordinates of joints in its strictly lower-triangular part. A “null” value is inserted in the indexes where the TAM contains zeroes, to avoid confusion between the absence of a joint and the superposition of one joint to the x or y axis of the reference frame.

III. ENUMERATION OF KINEMATIC CHAINS

By employing the described encoding for representing the topology of kinematic chains, the problem of structural synthesis is converted into a problem of enumeration of the corresponding graphs, which can be solved in a systematic and computationally efficient way. The enumeration is applied to a particular case, which is that of the structural design of a robotic orthosis which can be applied in parallel to a 1-DOF human joint. The described method can be easily generalized to more complex assemblies of human body segments, as described in the following sections.

The problem is graphically represented in Fig. 3, where the structural representation, the graph representation and the corresponding TAM are reported. The process of enumerating kinematic chains consists of three successive steps: *(A) matrix generation and connectivity evalutation*, *(B) mobility evaluation*, *(C) isomorphism detection*. These steps will be described separately in the following paragraphs.

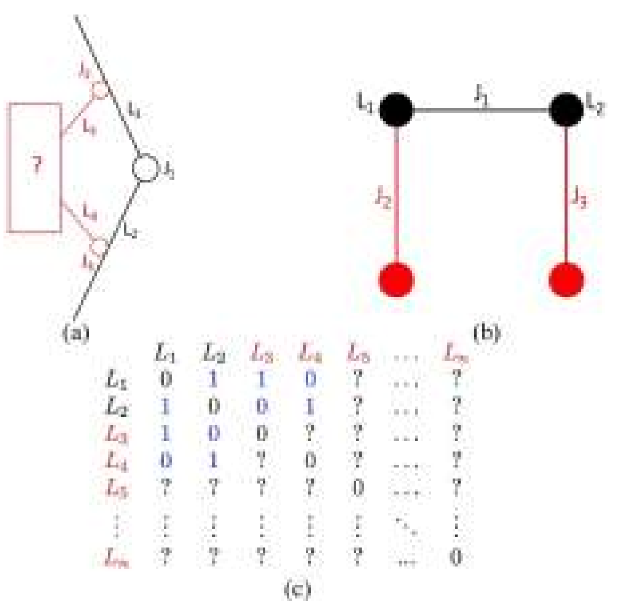


Fig. 3. Structural representation (a), graph representation (b) and TAM (c) of the problem of structural synthesis of robotic orthoses for a 1 DOF biological joint. Human joint and limbs are drawn in black, while robot links and joints are drawn in red. Blue color is used to represent the entries of the TAM which are determined by conditions (1), (2), (3) listed in paragraph IIIA.

A. Matrix generation and connectivity evaluation

Without loss of generality, each kinematic solution will be represented by a graph with n vertices, but which statisfy the following constitutive conditions:

1) links 1 and 2 are human limbs which are directly connected through the biological articulation;
2) link 3 is by construction connected to link 1 and cannot be connected to link 2 (otherwise the subgraph containing links 1, 2 and 3 would constitute a locked subchain impairing movements of the biological joint);
3) link 4 is by construction connected to link 2 and cannot be connected to link 1;

The complete list of independent kinematic solutions consisting of a total of *n* links can be obtained from the basic TAM shown in Fig. 3(c). Question marks in the upper triangular part of the matrix are substituted by the bits of a binary counter of length $n(n-1)/2-5$ (as it can be obtained using Gauss' formula of summations). It can be noted that imposing the above-defined conditions concerning links connectivity implies that five values of the TAM are fixed and are shown in blue in Fig. 3(C). Each value of the counter corresponds to a different matrix of adjacencies and finally to a kinematic configuration of the ensemble of human+robot links. A number of $2^{\frac{n(n-1)}{2}-5}$ TAMs are enumerated at this stage.

After matrices enumeration, links connectivity is evaluated. The *order* of link *i* is defined as the number of joints in which link *i* participates. It can be easily calculated from the TAMs by summing elements in the i^{th} row:

$$order_i = \sum_{k=1}^{n} a_{ik} \tag{1}$$

Since only closed kinematic chain are addressed by this enumeration, solutions in which any of the links has order lower than 2 are excluded.

B. Mobility evaluation and degeneracy testing

A selection over the list of enumerated topologies is performed, in order to consider only those kinematic chains which:

- do not impair the movements of the biological joint;
- do not contain over-constrained subchains;
- correspond to connected graphs (i.e. there is a path in the graph connecting each couple of vertices).

For what concerns the mobility analysis, the Kutzbach mobility criterion [19] is used to determine the number of degrees of freedom of the resulting kinematic chain, consisting of both human and robot links. For a given planar kinematic chain with n links and with f joints with 1 DOF, the total number of degrees of freedom (*DOFs*) is given by Kutzbach's formula:

$$DOFs = 3 \cdot (n-1) - 2 \cdot f \tag{2}$$

With the above-described method to represent matrices of adjacencies of graphs corresponding to kinematic chain, the number of DOFs of the structure is easily calculated since the number of joints in the structure is given by

$$f = \frac{1}{2}\sum_{i=1}^{n} order_i. \qquad (3)$$

Degenerate kinematic chains (i.e. those which contain at least one subchain with zero or negative DOFs according to Kutzbach formula, such as triangular subchains with 3 binary links and 3 joints) are then eliminated. To this aim, a degeneracy testing algorithm was implemented, which cyclically analyzes loops of 3 to n links and calculates the mobility of the corresponding subchain.

With the so-far listed steps, the method would also provide solutions representing disconnected mechanisms (i.e. such that there is not a path connecting each couple of vertices of the corresponding graph). Solutions of this kind are simply eliminated with a purposively developed algorithm which verifies the existence of a path between each couple of vertices.

C. Isomorphism detection

Once we have the complete list of the valid TAMs, we need to obtain those which correspond to non-isomorphic graphs.

Two kinematic chains K_1 and K_2 are said to be isomorphic if there exists a one-to-one correspondence between links of K_1 and K_2 such that a pair of links of K_1 are jointed if and only if the corresponding pair of links of K_2 are jointed. This means that from the graph corresponding to K_1 one can obtain the graph corresponding to K_2 by only relabelling link numbers.

Many attempts have been proposed in literature to find an accurate and computationally efficient test for isomorphism. Most computationally efficient methods (as that described in [21]) move around the obstacle of explicit isomorphism detection between pairs of kinematic chains. They are based on group theory applied to graphs and exhaustively generate isomorph-free classes of graphs, but are not easily applicable to the described problem, since it has been conditioned with the constraints indicated in section IIIA. These methods would give the exhaustive list of all non-isomorphic graphs with n vertices respecting the above described conditions and give as output only one graph for each homomorphic group of graphs. In this way it is not granted that a valid graph (i.e. a graph which respects the fixed entries in the matrix of adjacencies shown in Fig. 3 (c)) would be obtained. For these reasons the method of progressive enumeration of kinematic chains has been adopted, which is not computationally efficient, but involves only the open parameters in our schematization producing always valid kinematic chains (i.e. those which respect the constraint imposed in section IIIA). However, the chosen method implies the need of an isomorphism detection algorithm to avoid producing two kinematically identical solutions.

A function defined on a kinematic chain is called an *index of isomorphism* if any given pair of kinematic chains is isomorphic if and only if the corresponding values of the function are identical. The methods for isomorphism detection can be classified into spectral methods and canonical code-based methods [18]. Spectral methods are based on the evaluation of the characteristic polynomial, eigenvalues and eigenvectors of TAM (i.e. properties of their spectrum). Since algorithms used for finding spectral properties can be solved in polynomial time, finding an index of isomorphism based on the spectral properties of matrices of adjacencies implies finding a polynomial-time algorithm for solving the isomorphism problem. Conversely, in canonical code-based methods a unique code is assigned to a kinematic chain, hence there is a unique way of representing it through a TAM. Checking two given kinematic chains for isomorphism reduces to checking the corresponding canonical codes for equality and the canonical code can be an index of isomorphism. Methods belonging to this family are more reliable but may require an exponential time for creating the canonical code. In the present work an isomorphism detection method belonging to the family of spectral methods has been used. The index of isomorphism which has been evaluated is the characteristic polynomial of the Extended Adjacency Matrix (EAM) of order d, $A^{(d)}$, as also suggested in [20], which can be obtained from a TAM (A, of elements a_{ij}), by employing the following formula:

$$A^{(d)} = \begin{pmatrix} s_d(-\mathbf{a_1}) & a_{12} & \dots & a_{1n} \\ a_{21} & s_d(-\mathbf{a_2}) & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & s_d(-\mathbf{a_n}) \end{pmatrix} \qquad (4)$$

where the vector $\mathbf{a_i} = (a_{i1}, a_{i2}, ..., a_{in})$ contains elements of the i^{th} row and s_d is the elementary symmetric polinomial of order d in the n variables of vector $\mathbf{a}$, defined as:

$$\begin{array}{l} s_0(\mathbf{a}) = 0 \\ s_1(\mathbf{a}) = \sum_i a_i \\ s_2(\mathbf{a}) = a_1a_2 + a_1a_3 + ... + a_{n-1}a_n = \sum_{i<j} a_ia_j \\ \vdots \\ s_k(\mathbf{a}) = \sum_{j_1<j_2<\dots<j_k} a_{j_1}a_{j_2}\cdots a_{j_k} \end{array} \qquad (5)$$

It can be shown [20] that given two matrices of adjacencies A_1 and A_2, if A_1 and A_2 are isomorphic, also $A_1^{(d)}$ and $A_2^{(d)}$ are such that $A_1^{(d)} = PA_2^{(d)}P^T$ for every d (where P is a permutation matrix. Despite of that, the equality of the coefficients of the characteristic polynomial of $A_1^{(d)}$ and $A_2^{(d)}$ is a necessary but not sufficient condition for isomorphism. This means that for a number of pairs of kinematic chains this test of isomorphism concludes that they are isomorphic while they are actually not. This is a situation of *isospectral* but not *isomorphic* kinematic chains. Some studies quantifying the reliability of this method of isomorphism detection are reported in literature; [18] demonstrates that the simultaneous evaluation of the characteristic polynomial of both $A^{(0)}$, $A^{(1)}$ and $A^{(2)}$ has a reliability of 100% for kinematic chains with 1 and 3 DOFs consisting of up to 11 links.

For this reason this technique for isomorphism detection has been employed, which shows to be a very good compromise between reliability and computational efficiency (since it requires a polynomial time for evaluating isomorphism). Said $\mathbf{poly}_i^{(j)}$ the vector containing the normalized coefficients of the characteristic polynomial of matrix $A_i^{(j)}$, our test of isomorphism is such that two matrices A_1 and A_2 are isomorphic if each of the three equations in (6) are verified.

$$\mathbf{poly}_1^{(j)} = \mathbf{poly}_2^{(j)}, \text{ for } j = 0, 1, 2 \tag{6}$$

IV. RESULTS

The described algorithm of enumeration has been implemented in MATLAB (The MathWorks, Inc.) and was executed in order to obtain the complete list of topologies describing the kinematic chains which consist of both human (two links connected with a 1 DOF articulation such as the one shown in Fig. 3) and robot links. We limited our search to the space of kinematic chains with up to 8 links, since we expect that a kinematic chain consisting of more links would add excessive complexity and weight to the structure. Please note that the kinematic chains which are enumerated comprise the links corresponding to human segments (two since we are focusing on a single human joint), hence the number of robot links is $n_{robot} = n - 2$.

A. 4 links

From the schematization shown in Fig. 3 it can be derived that the simplest kinematic chain with the desired characteristics can be obtained by connecting links 3 and 4 through a revolute joint. In such a way we can obtain the 1 DOF four-bar mechanism shown in Fig. 4(a), which corresponds to the 2-regular graph of order 4.

B. 5 links

In this case the only suitable topology consists of all binary links, the 2 DOFs five-bar mechanism shown in Fig. 4, corresponding to the 2-regular graph of order 5.

C. 6 links

In the case of matrices of order 6, we found 3 possible topologies. One is the 3 DOFs six-bar mechanism corresponding to the 2-regular graph of order 6 (shown in Fig. 5(a)); the two others are 1 DOF kinematic chains (shown in Fig. 5(b,c). The corresponding graphs are shown in Fig. 5.

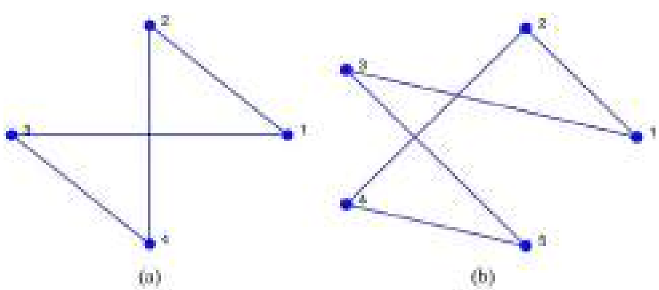


Fig. 4. Possible topologies for four (a) and five (b) links. These topologies are represented by planar 2-regular graphs, for which $DOFs = n - 3$.

D. 7 links

In the case of matrices of order 7, we find the 4 DOF seven-bar mechanism (2-regular graph of order 7 shown in Fig. 6 (a)), and also 4 topologies with 2 DOFs (Fig. 6 (b) to (e).

E. 8 links

The case of matrices of order 8 produces 24 valid independent topologies, 1 with 5 DOFs (the 2-regular graph of order 8), 7 with 3 DOFs and 16 with 1 DOF. Three examples of the obtained graphs are shown in Fig. 7, while the complete list of generated topologies is included in the additional material, which can be downloaded from the link `http://www.evryon.eu/atlas.html`.

The number of topologies found for each applicable combination of links number and DOFs is summarized in Table I.

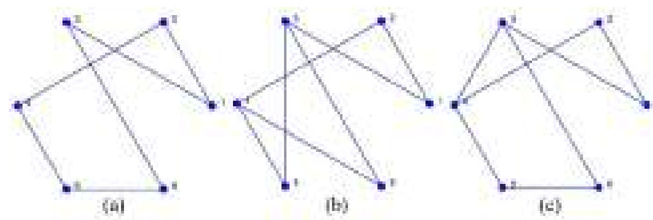


Fig. 5. Possible topologies for six links with 3 DOFs (a) and 1 DOFs (b) and (c).

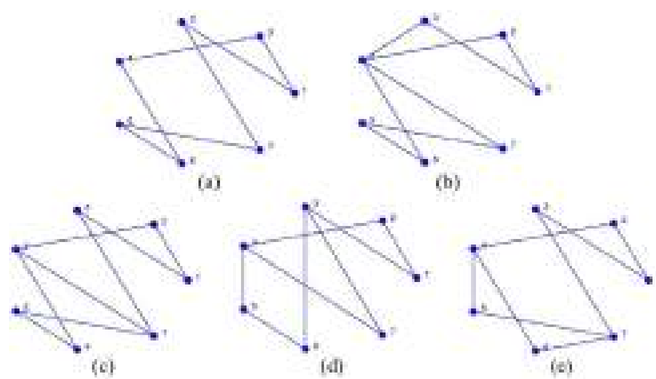


Fig. 6. Possible topologies for seven links with 4 DOFs (a) and 2 DOFs (b) to (e).

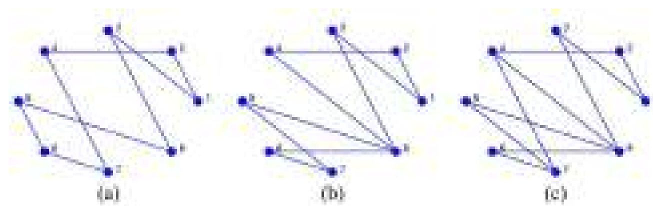


Fig. 7. Three examples of the obtainable topologies for eight links with 5 DOFs (a), 3 DOFs (b) and 1 DOF (c).

TABLE I
ENUMERATION OF KINEMATIC STRUCTURES

Number of links (human + robot)	DOFs	Independent topologies
4	1	1
5	2	1
6	1	2
6	3	1
7	4	1
7	2	4
8	5	1
8	3	7
8	1	16

V. CONCLUSIONS AND FUTURE WORK

Non-anthropomorphic kinematic solutions for robotic orthoses promise positive properties from both the standpoints of the dynamical interaction with the human body and of ergonomics. Despite of that, it is complex to synthesize an appropriate kinematic structure based only on engineering insight and intuition. In order to facilitate this task, a systematic approach can be useful, since it allows to obtain the number of independent kinematic solutions with desired kinematic properties, which generalize the set of solutions of the problem of kinematic synthesis of a non-anthropomorphic wearable robot with given requirements. In this paper a method for the systematic enumeration of all the topological solutions for a problem of kinematic synthesis of a robotic orthosis with given requirements is presented.

The method has been applied to the case of a 1 DOF human articulation, and has produced the atlas of topological solutions containing up to 6 robot links which are summarized by Table I. The same method can be extended to more complex design problems such as that of an exoskeleton for the lower or upper limbs without conceptual modifications but only modifying the conditions described in section IIIA.

The enumeration of all topological solutions is the first stage in a problem of kinematic synthesis. At this stage it is possible to have a list of the topologies of the kinematic chains with given properties, e.g. number of links and degrees of freedom. After this phase an optimization of the morphological properties of the structure (e.g. link lengths) can be performed for the definitive design of the kinematic structure of the robot, by considering the necessity of providing an adequate range of motion of the human joint. Other objectives, which are not analyzable only with a purely kinematic analysis (such as minimizing the exchange of tangential forces on human skin and optimizing the dynamical interaction with the human body), can be also taken into account for the optimization of the kinematic structure. These analyses will be conducted in future works.

VI. ACKNOWLEDGMENTS

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