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Singular configurations of planar parallel manipulators

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The paper deals with determining singular configurations for a group of 3 DOF planar parallel manipulators whose links form revolute and prismatic pairs. Relations for Jacobian matrix determinants for selected mechanisms are derived. The form of the equations differs depending on the structure of the inner part of the manipulators belonging to the Assur groups of the 3rd class. An analysis of the relations shows that singular configurations occur when Assur points coincide. Due to the approach proposed, singular configurations can be determined using classical kinematic analysis methods, without it being necessary to specify a Jacobian determinant and to define its zeroing conditions. Also a method of determining singular configurations from auxiliary mechanisms' characteristic point trajectory is proposed. The research results make it easier to define the conditions in which singular configurations occur.

Keywords: *singularity, planar parallel manipulators*

1. Introduction

Due to their advantages, such as the ability to carry considerable loads at high speed and with great precision, planar parallel manipulators can have numerous applications. They are characterized by a relatively small workspace which is partly inaccessible due to the occurrence of singular configurations. Once the system enters a singular configuration, it cannot be pulled out of it and may be damaged as a result. Therefore the determination of singular configurations is vitally important for planning the motion of actual manipulators and for their analysis and design.

There are many works, e.g., [1], [2], [3], which deal with the singular configurations of planar parallel manipulators. This paper presents the results of research aimed at deriving generalizations which would cover as large a group of 3 DOF planar parallel systems as possible. The knowledge gained may contribute to a wider use of the mechanisms.

2. Jacobian matrix determinant as indicator of singular configurations

A singular configuration is a peculiar position of the driven link (the effector) in which the manipulator cannot be effectively controlled [4]. Analyses of the motion of parallel manipulators indicate singular configurations, irrespective of the modelling method.

In order to describe the mechanism in absolute coordinates for a system with n movable links, $k = 3n$ linearly independent equations need to be formulated [5]. For a mechanism with mobility m the number of equations which follow from the equations of constraints (joint constraints $\mathbf{\Phi}^W$) is $k - m$. The other equations are derived from the formulation for the motion of the driving links (excitations $\mathbf{\Phi}^C$). Hence a multi-body mechanical system is described by the following system of equations:

$$\mathbf{\Phi}(\mathbf{q},t) = \begin{bmatrix} \mathbf{\Phi}^W(\mathbf{q}) \\ \mathbf{\Phi}^C(\mathbf{q},t) \end{bmatrix} = 0. \tag{1}$$

After differentiating Equation (1), taking into account that matrix $\mathbf{q}$ is time variable, the following equation of velocity is obtained:

$$\dot{\mathbf{q}} = -\mathbf{\Phi}_q^{-1}\mathbf{\Phi}_t. \tag{2}$$

The equation has a solution for:

$$\det(\mathbf{\Phi}_q) \neq 0. \tag{3}$$

The above notation of derivatives conforms to the Lagrange symbols, where the subscript t denotes a derivative over time and $\mathbf{\Phi}_q$ is a Jacobian of the constraints matrix $\mathbf{\Phi}$. The form of the matrix $\mathbf{\Phi}_q$ depends on the kind of problem (forward or inverse kinematics problem). A singular configuration occurs when the Jacobian matrix determinant is equal to zero.

Generally, three types of singularity can be distinguished in the workspace of planar parallel manipulators [1], [6]:
- forward kinematics problem singularity $\det(\mathbf{\Phi}_{q1}) = 0$,
- inverse kinematics problem singularity $\det(\mathbf{\Phi}_{q2}) = 0$,
- mixed singularity $\det(\mathbf{\Phi}_{q1}) = 0$ and $\det(\mathbf{\Phi}_{q2}) = 0$.

3. Analysis of singularities of manipulators with symmetrical legs with driving links at base

Three degrees of freedom planar parallel manipulators with one driving link in each of the three legs are considered. 384 systems, including 56 designs with driving links at the base, form a complete set of possible design solutions. Figure 1 shows symmetrical systems with driving links at the base.

Each of the manipulators considered consists of three kinematic chains connecting end-effector 7 to base 0. In each chain, there are driving links 1, 2, 3 located at the base and intermediate links 4, 5, 6 connected to the driving links and end-effector 7. A singular configuration analysis was performed using the relations for the Jacobian matrix determinant for the forward kinematics problem.

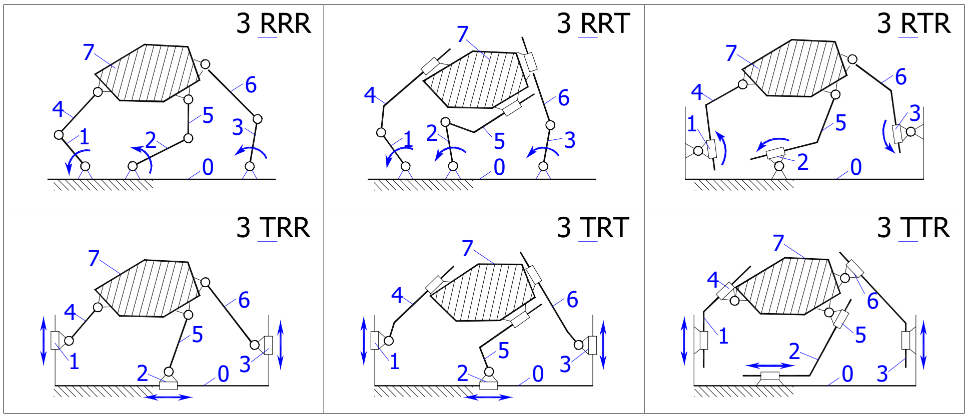


Fig. 1. Kinematic schemes of the mechanisms analysed

Appropriate equations of constraints were formulated for all the types of the systems shown in Figure 1. Then the vector of equations was differentiated whereby successive Jacobian matrices were obtained. Finally, relations for Jacobian determinants were derived. The relations are presented in the Table.

The manipulators presented in Table 1 are grouped according to the form of their determinants. One should note that the relations for determinants are identical in the systems whose inner parts, after the driving links and the base have been removed, are identical [7]. Table 1 also includes configurations which fulfil condition det $(\mathbf{\Phi}_q) = 0$. It becomes apparent that there is an analogy to Assur's classification of planar systems [8] – the inner systems are the 3rd class Assur groups. Hence the following general condition for the occurrence of manipulator singular configurations can be formulated:

a singular configuration for the considered systems occurs when the configuration of the inner group is characterized by the coincidence of Assur points [7].

A special case of a singular configuration (not shown in the table, but satisfying equation det $(\mathbf{\Phi}_q) = 0$) is the coincidence of the Assur points in infinity. This is manifested in the parallelism of appropriate straight lines.

4. Determination of singular configurations through kinematic analysis by graphical method

Conditions for the occurrence of singular configurations for the systems considered were defined by specifying the Jacobian determinant det $(\mathbf{\Phi}_q)$ and equating it to zero. But the observation that a singular configuration corresponds to the coincidence of the Assur points allows us to determine the singular configurations of all the systems and it is not necessary to specify the Jacobian determinant and to define conditions for equation det $(\mathbf{\Phi}_q) = 0$.

The conclusions drawn from the formulated condition of the occurrence of singular configurations allow us to determine singular configurations for manipulators with asymmetrical legs with driving links at the base and for manipulators with driving links located in the legs. By analogy to the configurations shown in Table 1 it is possible to determine the singular configurations of each manipulator belonging to the group under consideration. Figure 2 shows singular configurations for the 2$\underline{R}$TR1$\underline{R}$RR and $\underline{T}$RT$\underline{R}$TR$\underline{R}$RR systems with driving links at the base, determined from the positions of the Assur points.

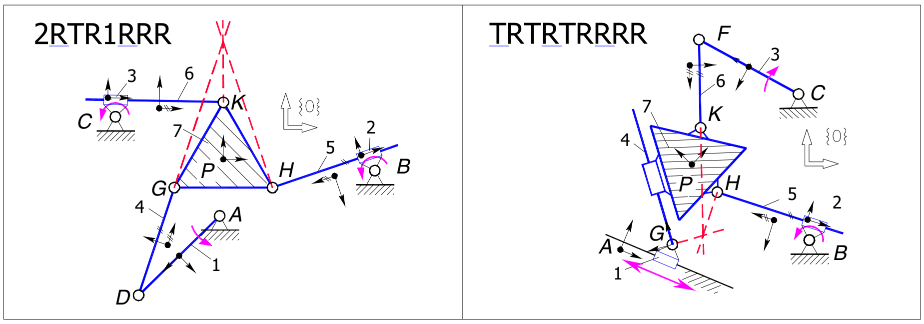


Fig. 2. Singular configuration of manipulators with asymmetrical legs

Hence singular configuration for a given manipulator can be determined if the conditions at which the coincidence of the Assur points occurs are known.

One of the possible ways of determining such conditions is a kinematic analysis performed using graphical methods. In the forward kinematics problem for each manipulator belonging to the group considered, the velocities of the three driving links are given, while the velocity of the end-effector point *P* is sought. In order to determine this velocity, first the velocity of the peculiar point *M* (an Assur point) belonging to the end-effector and then the velocities of the other end-effector points should be calculated.

In Table 1, we have:

l_i – the length of links $i = \{4, 5, 6\}$,

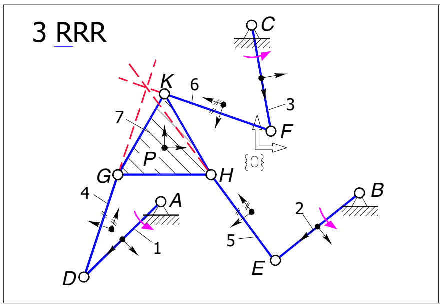
${}^7x_n$, ${}^7y_n$ – the coordinates of the point n in the system of effector 7, $n = \{G, H, K\}$,

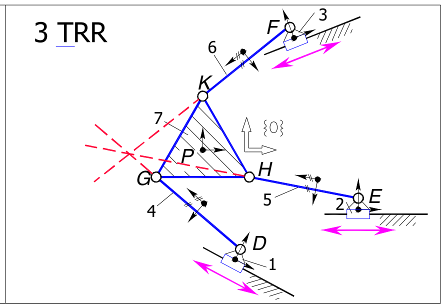
$\Theta_{j,k} = \Theta_j - \Theta_k$ – the difference in the angles of orientation of links $j, k = \{4, 5, 6, 7\}$,

The above observation perfectly fits the manipulator 3R$\underline{T}$R described in [9]. In order to solve the manipulator's kinematics, one must determine the velocity of the point *M* (Figure 3b) and then calculate the velocities of the other points of end-effector. The velocity of the point *M* is determined by solving the following system of vector equations:

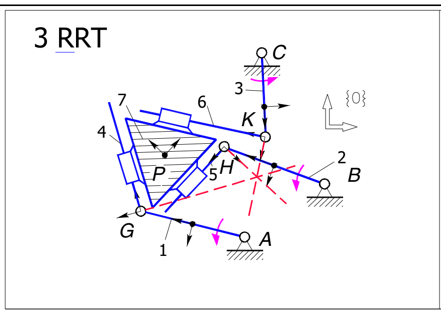
$$\begin{cases} \overline{\mathbf{v}_M} = \overline{\mathbf{v}_G} + \overline{\mathbf{v}_{MG}}, \\ \overline{\mathbf{v}_M} = \overline{\mathbf{v}_H} + \overline{\mathbf{v}_{MH}}, \end{cases} \tag{4}$$

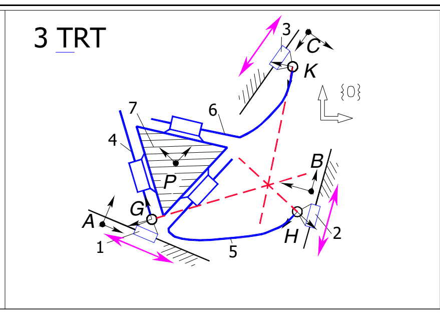
Table. Jacobian determinants



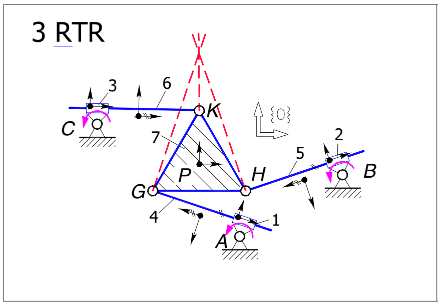


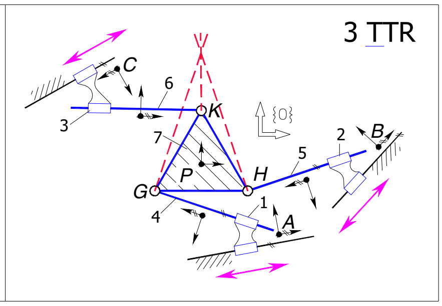
$$\det(\mathbf{\Phi}_q) = l_4\, l_5\, l_6 \left[\sin\Theta_{5,6}\left(-{}^7x_G \sin\Theta_{4,7} + {}^7y_G \cos\Theta_{4,7}\right) + \sin\Theta_{6,4}\left(-{}^7x_H \sin\Theta_{5,7} + {}^7y_H \cos\Theta_{5,7}\right) + \sin\Theta_{4,5}\left(-{}^7x_K \sin\Theta_{6,7} + {}^7y_K \cos\Theta_{6,7}\right)\right] \quad (5)$$





$$\det(\mathbf{\Phi}_q) = \sin\Theta_{5,6}\left(x_4 \cos\Theta_4 + y_4 \sin\Theta_4\right) + \sin\Theta_{6,4}\left(x_5 \cos\Theta_5 + y_5 \sin\Theta_5\right) + \sin\Theta_{4,5}\left(x_6 \cos\Theta_6 + y_6 \sin\Theta_6\right) \quad (6)$$





$$\det(\mathbf{\Phi}_q) = \sin\Theta_{5,6}\left[{}^7x_G \cos\Theta_{4,7} + {}^7y_G \sin\Theta_{4,7}\right] + \sin\Theta_{6,4}\left[{}^7x_H \cos\Theta_{5,7} + {}^7y_H \sin\Theta_{5,7}\right] + \sin\Theta_{4,5}\left[{}^7x_K \cos\Theta_{6,7} + {}^7y_K \sin\Theta_{6,7}\right] \quad (7)$$

where:

$$\begin{aligned} &\overline{\mathbf{v}_G} = \overline{\mathbf{v}_D} + \overline{\mathbf{v}_{GD}}, \qquad && \overline{\mathbf{v}_D} = \overline{\mathbf{v}_A} + \overline{\mathbf{v}_{DA}}, \qquad && \overline{\mathbf{v}_{DA}} = \overline{\mathbf{v}_{41}}, \\ &\overline{\mathbf{v}_H} = \overline{\mathbf{v}_E} + \overline{\mathbf{v}_{HE}}, && \overline{\mathbf{v}_E} = \overline{\mathbf{v}_B} + \overline{\mathbf{v}_{EB}}, && \overline{\mathbf{v}_{EB}} = \overline{\mathbf{v}_{52}}. \end{aligned} \tag{8}$$

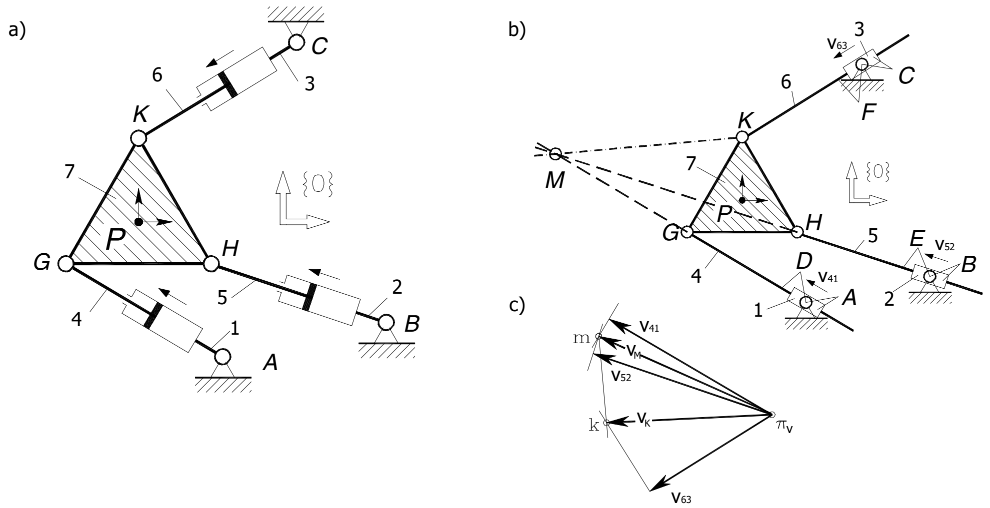


Fig. 3. Kinematic scheme of the manipulator 3R<u>T</u>R (a), equivalent mechanism (b), velocity diagram (c)

Hence after substituting Equations (4) and (8) we arrive at:

$$\begin{cases} \overline{\mathbf{v}_M} = \overline{\mathbf{v}_{41}} + \overline{\mathbf{v}_{GD} + \mathbf{v}_{MG}}, \\ \overline{\mathbf{v}_M} = \overline{\mathbf{v}_{52}} + \overline{\mathbf{v}_{HE} + \mathbf{v}_{MH}}. \end{cases} \tag{9}$$

In the system of Equations (9), vectors $\mathbf{v}_{41}$ and $\mathbf{v}_{52}$ and the direction of vector sums $\mathbf{v}_{GD} + \mathbf{v}_{MG}$ and $\mathbf{v}_{HE} + \mathbf{v}_{MH}$ are known, which allows the velocity of the point M (Figure 3c) to be determined. Then the velocity of the point K (Figure 3c) is obtained from this system of equations:

$$\begin{cases} \overline{\mathbf{v}_K} = \overline{\mathbf{v}_{63}} + \overline{\mathbf{v}_{KF}}, \\ \overline{\mathbf{v}_K} = \overline{\mathbf{v}_M} + \overline{\mathbf{v}_{KM}}. \end{cases} \tag{10}$$

Having solved the system of Equations (10), one can calculate the velocity of the point K and then the other end-effector velocities (including the velocity of the point P). When vectors $\mathbf{v}_{KF}$ and $\mathbf{v}_{KM}$ are parallel to each other, the velocity of none of the effector points can be determined. This happens in the case of the configurations shown in Figure 4 [9]. A singular configuration occurs when the normals to the velocity directions $\mathbf{v}_{GD}$, $\mathbf{v}_{HE}$, $\mathbf{v}_{KF}$ intersect at one point (Figure 4a) or are parallel (Figure 4b).

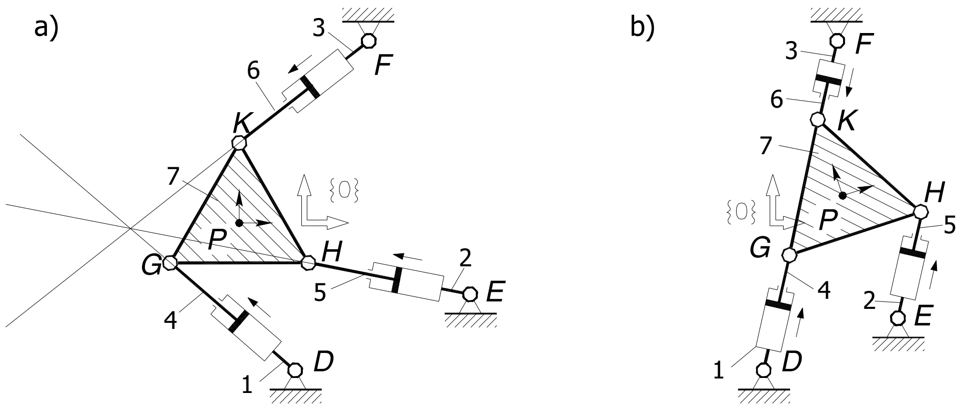


Fig. 4. Singular configurations of manipulator 3RTR

5. Singular configurations within workspace – auxiliary mechanisms

The determination of a singular form of configuration (on the basis of the determinant or Assur points) is the first step only. Then singular configurations against the background of the workspace must be determined. This is a complex problem since the determinant is nonlinearly dependent on the position parameters.

The singular configurations of manipulators with a known geometry are mostly determined through a numerical search of the entire workspace and by calculating the Jacobian determinant. But this is ineffective and laborious. The generalization about the conditions of the occurrence of singular configurations in the systems considered allows an effective and quick determination of the configurations. For this purpose auxiliary mechanisms based on the geometrical interpretation of the singular configurations should be introduced [10]. By determining all the possible positions of the auxiliary mechanisms it is possible to determine all the singular configurations of a manipulator – the trajectory of auxiliary mechanism point *P* marks out the manipulator's singular configurations.

Figure 5 shows auxiliary mechanisms for the manipulator 3TRR. The geometry of the base and that of links 1–7 corresponds to the manipulator's geometry, while links 8, 9, 10 are responsible for a peculiar configuration: the intersection of appropriate directions and parallelism for the cases shown in Figures 5a and 5b, respectively [10].

By analogy to the auxiliary mechanisms presented one can propose auxiliary mechanisms for the other manipulators.

The advantage of auxiliary mechanisms lies in the fact that singular configurations can be quickly and effectively determined against the background of the workspace.

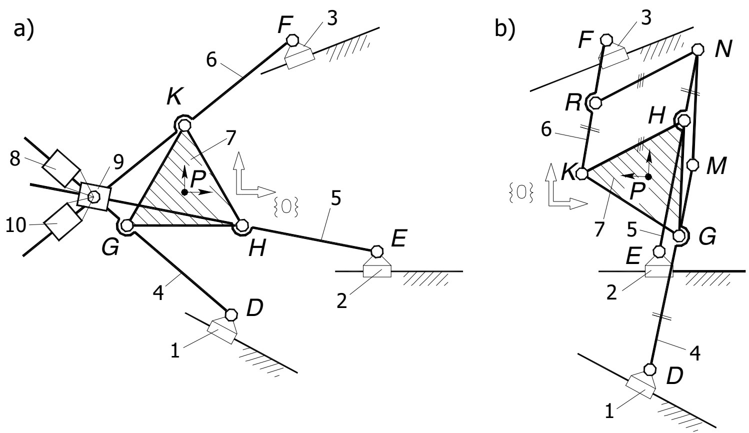


Fig. 5. Auxiliary mechanisms for determining singular configurations characterized by: a) intersection, b) parallelism of normals to relative velocities

A detailed analysis of the auxiliary mechanisms allows one to formulate further conclusions. The auxiliary mechanism shown in Figure 5a has two degrees of freedom. Consequently, the orientation of link 7 and one coordinate (x_P or y_P) of the point P can be defined arbitrarily. Even if the effector has a constant orientation, the mechanism is still capable of motion. The point P plots a trajectory which forms a continuous curve of singular positions for a given orientation of end-effector link 7. By successively changing the orientation of link 7 and each time determining the trajectory of the point P one obtains several curves which represent the singular configurations. In order to determine singular configurations, it is most convenient to present them in the system $XY\Theta_7$ since the locations of singular configurations change with the orientation of the effector. Hence it follows that the singular configurations for the whole workspace form a continuous surface in the system $XY\Theta_7$. This means that the manipulator's workspace delimited by the singularities of the inverse problem cannot be fully utilized without disassembly. Thus one can say that the available workspace of a given planar parallel manipulator is delimited by the singularities of both the forward and inverse kinematic problems.

The auxiliary mechanism shown in Figure 5b has one degree of freedom. As the orientation of the end-effector changes, the position of the point P changes in a specific way. Thus singular configurations characterized by parallelism of appropriate straight lines occur less often.

6. Conclusion

The results of research aimed at formulating generalizations covering possibly the largest group of 3 DOF planar parallel systems with driving links in each of the three legs have been presented.

Compact relations for the Jacobian matrix determinants for symmetrical systems with driving links at the base have been derived. The form of the equations depends on the structure of the system fragment which remains after the base and the driving links are removed. The system fragments form the so-called 3rd class Assur groups [6]. A detailed analysis of the expressions for the Jacobian determinants showed that singular configurations occur when the Assur points coincide.

The approach presented allows one to determine singular configurations by classical kinematic analysis methods without calculating the Jacobian matrix determinant and to define conditions in which equation det $(\mathbf{\Phi}_q) = 0$ is fulfilled. As a result, the conditions of the occurrence of singular configurations and the singular configurations themselves can be easily determined and interpreted.

The finding that the singular configurations of the forward kinematic problem occur only when the Assur points coincide served as the basis for defining auxiliary mechanisms which simplify the analysis of singular configurations.

The research results presented may contribute to a wider use of planar parallel manipulators. The demonstrated simple way of determining singular configurations and their nature will facilitate the design of such manipulators.

References

[1] Gosselin C., Angeles J.: *Singularity analysis of closed-loop kinematic chains*, IEEE Trans. On Robotics and Automation, 1990, 6(3).

[2] Gosselin C., Sefrioui J.: *Polynomial solutions for the direct kinematic problem of planar three-degree-of-freedom parallel manipulator*, International Conference on Advanced Robotics, Pisa, 1991.

[3] Knapczyk J., Morecki A.: *Podstawy robotyki. Teoria i elementy manipulatorów i robotów*, WNT, 1999.

[4] Daniali H.R.M., Zsombor-Murray P.J., Angeles J.: *Singularity analysis of a general class of planar parallel manipulators*. IEEE Int. Conf. On Robotics and Automation, Nagoya, May 25–27, 1995.

[5] Shabana A.A.: *Computational Dynamics*, John Wiley & Sons, Inc., 1995.

[6] Tsai L-W.: *Robot analysis. The mechanics of serial and parallel manipulators*, John Wiley & Sons, Inc., New York, 1999.

[7] Gronowicz A., Prucnal-Wiesztort M.: *Położenia osobliwe manipulatorów równoległych płaskich*, XIX Konferencja Naukowo-Dydaktyczna Teorii Maszyn i Mechanizmów, Kraków, 12–14 października, 2004.

[8] Miller S.: *Układy kinematyczne. Podstawy projektowania*, WNT, 1988.

[9] Wender Ph., Chablat D.: *Workspace and assembly modes in fully-parallel manipulators: a descriptive study. Advances in Robot Kinematics: Analysis and Control*, Kluwer Academic Publishers, 1998.

[10] Prucnal-Wiesztort M.: *Wyznaczanie położeń manipulatorów równoległych płaskich na tle strefy roboczej,* XIX Konferencja Naukowo-Dydaktyczna Teorii Maszyn i Mechanizmów, Kraków, 12–14 października, 2004.

Położenia osobliwe płaskich manipulatorów równoległych

W artykule podjęto problem wyznaczania położeń osobliwych grupy płaskich manipulatorów równoległych o trzech stopniach swobody, których człony tworzą pary obrotowe i postępowe. Przedstawiono wyrażenia opisujące wyznaczniki macierzy Jacobiego dla wybranych mechanizmów. Forma otrzymanych równań zależy od budowy strukturalnej wewnętrznej części manipulatorów tworzących tzw. grupy Assura III klasy. Analiza otrzymanych wyrażeń pokazała, że położenia osobliwe odpowiadają tym, w których punkty Assura pokrywają się. Zaprezentowane podejście umożliwia określenie konfiguracji osobliwych jedynie na podstawie klasycznych metod analizy kinematycznej, bez obliczania wartości wyznacznika macierzy Jacobiego i definiowania warunków zerowania się tego wyrażenia. Ponadto zaproponowano metodę określania położeń osobliwych na podstawie trajektorii punktu charakterystycznego mechanizmów pomocniczych. Przedstawione wyniki badań ułatwiają określanie warunków występowania położeń osobliwych.