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An average values global model for the switched reluctance machine

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Abstract

The main subject of this paper is to present a new simplified global model for the switched reluctance machine (SRM), useful, namely, for the command and control analysis of this kind of system. The concepts of power and energy are used in the global model construction, with the definition of global parameters and variables. This new global model presents an advantage over the classical detailed one, which is the reduction of the number of dynamic equations. The time-dependent global parameters disadvantage, which appears in the model construction process, is overcome with the consideration of average values for global variables and global parameters, allowing the representation of the machine non-detailed behavior. An average values global model is obtained. An 8/6 SRM with four phases, eight stator poles and six rotor poles is characterized and used for illustration of the system behavior, regarding its variables evolution. In the SRM command it is assumed that the m.m.f. is imposed, which means that the system current is controlled.

Keywords: Switched reluctance machine; Global model

1. Introduction

The switched reluctance machine (SRM) has been under particular attention of researchers in the last two decades. The technological progress, particularly in the field of electronics and informatics, stimulates the development of new and better solutions to improve its performance.

A consequence of this is the significant position that the SRM is assuming nowadays in the industrial and domestic markets [3], replacing successfully other types of electrical machines, mainly because of its good performance, robustness and low cost.

The working principle of the SRM is simple and well known [2,1]. Nevertheless, its dynamic modeling and the physical interpretation of its behavior are not always so simple. To this fact contribute the machine magnetic saliencies, both in stator and rotor, and consequently its non-linear characteristics.

The main subject of this paper is to obtain a new model for the SRM, useful for studying the system behavior particularly for the system command and control analysis. The purpose is not only to obtain a new global model like in [5], but a simplified global model, with the definition of non-time-dependent global parameters and global variables. Effectively, using the power and energy concepts, a model is constructed with global parameters and variables. The

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introduction of global variables, like a global current, a global voltage and the angular position of the global current, with the introduction of global parameters, like the global circuit resistance and the global circuit inductance, are integrated in a methodology in order to obtain an average values SRM model.

The new model is obtained in two steps. In the first one, the definition of global parameters and variables leads to a simpler SRM model when compared with the classical detailed one. Usually the number of electrical equations, used in the SRM models, is equal to the number of machine phases and with this approach only one electrical equation is considered, having a significant reduction in the number of dynamic equations. However, some difficulties are still present, in special, the structure of the global parameters that are time-dependent through their dependence on other variables. This is the motivation for the second step, when average values are considered and an average values global model for the SRM is obtained.

The new model is a significant simplification of the system dynamic representation and has the advantage over the detailed one of having, not only a reduced number of dynamic equations, but also non-time-dependent parameters, which is important regarding the SRM global behavior study and the control analysis.

The model for the SRM is obtained and tested in a typical situation, with some assumptions regarding the machine behavior, particularly the number of phases that are energized at a time. The usual command of this machine, characterized by the feed of one phase at a time, and the current in two phases during the commutation process, is taken into account. The results, obtained with the global model, are compared with the results obtained with the detailed model.

An 8/6 SRM with four phases, eight stator poles and six rotor poles is characterized and used to illustrate the system behavior.

2. SRM detailed model

The SRM dynamic model usually used is based on the circuits' theory and it is assumed that the machine has the same number of electrical circuits as the number of machine phases.

In this context, for each phase/circuit Eq. (1) can be written, where j denotes the phase/circuit, u_j the phase/circuit voltage, i_j the phase/circuit current, Ψ_j the phase/circuit coil linkage flux and R the phase/circuit resistance, which is assumed to be the same for every phase/circuit.

$$u_j = Ri_j + \frac{\mathrm{d}\psi_j}{\mathrm{d}t} \tag{1}$$

This means that for a SRM with n phases/circuits one needs to consider n equations as (1).

To have the complete detailed model it is necessary to consider the mechanical Eq. (2), where the dynamic torque balance is represented. In this equation, ω represents the machine angular speed, T_e the electromagnetic torque, T_L the load torque and J is the inertia momentum.

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = T_\mathrm{e} - T_\mathrm{L} \tag{2}$$

The electromagnetic torque is obtained by (3) where W_cm is the magnetic co-energy and θ is the rotor angular position.

$$T_\mathrm{e} = \frac{\partial W_\mathrm{cm}}{\partial \theta} \tag{3}$$

Finally, to complete the model, it is necessary to establish the functional relations (4) for each phase coil linkage flux and for the magnetic co-energy.

$$\psi_j(\theta, i_1, \cdots, i_n), \qquad W_\mathrm{cm}(\theta, i_1, \cdots, i_n) \tag{4}$$

3. SRM global model

The purpose of this approach is to obtain a simpler dynamic model for the SRM, which means, in a first step, to reduce the number of system equations.

The knowledge of the machine operational principles is useful, namely the fact that usually only one or two phases are energized at a time. With this assumption, the new system representation, in a first step, can be done without any loss of information.

The first goal is to replace the n circuits, related to the n phases of the machine by a lower number of circuits – global circuits. Attending to the operational characteristics of this machine, it will only be defined one global circuit, with a global current i defined by (5)

$$i = \sum_{j=1}^{n} i_j \tag{5}$$

The current in each phase i_j, can be written as in (6), where γ_j denotes the partition of the global current by the n circuits of the machine.

$$i_j = \gamma_j i \tag{6}$$

The global voltage of this circuit u can be defined using the electrical power supplied to the machine p, which is equal to the sum of the power supplied to each phase p_j as presented in (7),

$$p = \sum_{j=1}^{n} p_j = \sum_{j=1}^{n} u_j i_j = \left(\sum_{j=1}^{n} u_j \gamma_j \right) i = ui \tag{7}$$

In this way, the global voltage of the global circuit u is defined by (8)

$$u = \sum_{j=1}^{n} u_j \gamma_j \tag{8}$$

This voltage u can be seen as the voltage of the physical system power supply when only one phase is energized. Otherwise, the physical meaning of u may be seen as the necessary voltage in the global circuit to obtain the same existing m.m.f.

Using Eq. (1), the total electrical power p can be written as (9)

$$p = \sum_{j=1}^{n} u_j \gamma_j i = R \sum_{j=1}^{n} \gamma_j^2 i^2 + \sum_{j=1}^{n} \frac{\mathrm{d}\psi_j}{\mathrm{d}t} \gamma_j i \tag{9}$$

By analogy with the global voltage, it is possible to define the global linkage flux ψ , as presented in (10),

$$\psi = \sum_{j=1}^{n} \psi_j \gamma_j \tag{10}$$

Taking into account the definition of the phase linkage flux, expressed in (11) as

$$\psi_j = \frac{\partial W_{\mathrm{cm}}}{\partial i_j} \tag{11}$$

It can be obtained the time derivative of the global flux (12)

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \sum_{j=1}^{n} \frac{\mathrm{d}\psi_j}{\mathrm{d}t} \gamma_j + \frac{1}{i} \sum_{j=1}^{n} \frac{\partial W_{\mathrm{cm}}}{\partial \gamma_j} \frac{\mathrm{d}\gamma_j}{\mathrm{d}t} \tag{12}$$

With this result, the expression of the global power (9) can be re-written as (13)

$$p = R_{\mathrm{eq}} i^2 + i \frac{\mathrm{d}\psi}{\mathrm{d}t} - \sum_{j=1}^{n} \frac{\partial W_{\mathrm{cm}}}{\partial \gamma_j} \frac{\mathrm{d}\gamma_j}{\mathrm{d}t} \tag{13}$$

where R_{eq} is a global parameter, the global resistance of the global circuit, and is defined by (14)

$$R_{eq} = R\sum_{j=1}^{n}\gamma_j^2 \tag{14}$$

Dividing (13) by the global current i, the electrical equation of the global circuit (15) is obtained, where some global variables and parameters are presented.

$$u = R_{eq}i + \frac{d\psi}{dt} - \frac{1}{i}\sum_{j=1}^{n}\frac{\partial W_{cm}}{\partial \gamma_j}\frac{d\gamma_j}{dt} \tag{15}$$

To go forward with the global modeling it is useful to define a new global variable ρ which is an angular position that represents the angular position of the global current i which means, in a particular sense, the angular position of the energized coils. If more than one phase is excited, ρ can be seen as the "angular gravity centre" of the global current.

In these conditions, γ_j, with $j = 1\ldots n$, that represent the total current partition among the machine phases, are functions of the new variable ρ (16),

$$\gamma_j = \gamma_j(\rho) \tag{16}$$

The electrical equation of the global circuit is then re-written (17) using the previously defined global variables and parameters.

$$u = R_{eq}i + \frac{d\psi}{dt} - \frac{1}{i}\frac{\partial W_{cm}}{\partial \rho}\frac{d\rho}{dt} \tag{17}$$

The set of the system global variables is completed with the mechanicals variables θ and ω, respectively the rotor angular position and the machine angular speed.

With the defined new global variables the functional relations (4) are then written as (18)

$$\psi(i, \theta, \rho) \qquad W_{cm}(i, \theta, \rho) \tag{18}$$

The mechanical Eq. (2) maintains its form and the electromagnetic torque is also obtained by the same Eq. (3).

The developed model was obtained considering the SRM working principles and characteristics and presents a formal simplicity over the detailed one, because it has a lower number of dynamic equations. However, the global parameters are not constants, being time-dependent through their dependence on variables, which still represents a problem for the system analysis and simplification.

Besides the advantage of reducing the number of dynamic equations, this global model can also be seen as a way to find a more simplified model with assumptions regarding the system global parameters and variables behavior.

4. SRM average values global model

A more simplified model, which will represent the system global behavior, can be obtained considering average values of its global parameters and defining average values global variables. Of course, that with this simplified model the detailed evolution of the system global variables is no longer possible. It is just possible to obtain the important general and global description of the machine behavior.

Using (17), the average value of the system electric power is defined by (19),

$$\bar{p}_e = \langle ui \rangle = \left\langle R_{eq}i^2 \right\rangle + \left\langle i\frac{d\psi}{dt} \right\rangle - \left\langle \frac{\partial W_{cm}}{\partial \rho}\frac{d\rho}{dt} \right\rangle \tag{19}$$

where $\langle x\, y \rangle$ represents the $(x\, y)$ average value. Eq. (19) can be written as (20)

$$\bar{p}_e = \left\langle R_{eq} \right\rangle \bar{i}^2 + \bar{i}\frac{d\bar{\psi}}{dt} - \frac{\partial \bar{W}_{cm}}{\partial \bar{\rho}}\frac{d\bar{\rho}}{dt} + \Delta \tag{20}$$

where $\bar{x}$ represents a new variable that is the average value of the global variable x.

Considering the global current i, approximately constant and also an approximately constant machine speed ω in each electric period, it is possible to write the following electrical Eq. (21), where Δ was neglected in this situation. This equation is a part of the SRM average values global model.

$$\bar{u} = \langle R_{\mathrm{eq}} \rangle \bar{i} + \frac{\mathrm{d}\bar{\psi}}{\mathrm{d}t} - \frac{1}{\bar{i}} \frac{\partial \bar{W}_{\mathrm{cm}}}{\partial \bar{\rho}} \frac{\mathrm{d}\bar{\rho}}{\mathrm{d}t} \tag{21}$$

The mechanical equation will be represented by (22)

$$J\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}t} = \overline{T_{\mathrm{e}}} - T_{\mathrm{L}} \tag{22}$$

To better understand this model construction and development, it is assumed, from now on, the absence of magnetic saturation and the existence of null mutual inductances [4]. With these assumptions it is possible to establish the following functional relations:

$$\psi = \sum_{j=1}^{n} L_j(\theta)\gamma_j^2(\rho)\, i = L_{\mathrm{eq}} i \tag{23}$$

$$W_{\mathrm{cm}} = \frac{1}{2}\sum_{j=1}^{n} L_j(\theta)\gamma_j^2(\rho)\, i^2 = \frac{1}{2} L_{\mathrm{eq}} i^2 \tag{24}$$

where a new global parameter was introduced. L_{eq} is the inductance of the global circuit, defined by (25),

$$L_{\mathrm{eq}} = \sum_{j=1}^{n} L_j(\theta)\gamma_j^2(\rho) \tag{25}$$

With these relations and parameters it is possible to rewrite the previous electrical Eq. (21), as (26),

$$\bar{u} = \langle R_{\mathrm{eq}} \rangle \bar{i} + \langle L_{\mathrm{eq}} \rangle \frac{\mathrm{d}\bar{i}}{\mathrm{d}t} + \left\langle \frac{\partial L_{\mathrm{eq}}}{\partial \theta} \right\rangle \bar{\omega}\bar{i} + k \left\langle \frac{\partial L_{\mathrm{eq}}}{\partial \rho} \right\rangle \frac{\mathrm{d}\bar{\rho}}{\mathrm{d}t}\bar{i} \tag{26}$$

In the mechanical Eq. (22), the average value of the electromagnetic torque is defined by

$$\overline{T_{\mathrm{e}}} = \frac{1}{2} \left\langle \frac{\partial L_{\mathrm{eq}}}{\partial \theta} \right\rangle \bar{i}^2 \tag{27}$$

This new simplified model has the advantage over the classical detailed one of having, not only a reduced number of dynamic equations, but also non-time-dependent parameters, which is important for the system global dynamic analysis.

5. SRM characterization

To illustrate the system behavior and the performance of the new model, it is considered an 8/6 SRM with four phases, eight stator poles and six rotor poles, which cross-section is represented in Fig. 1.

The values of some constructive parameters of the machine, given by the manufacturer, are: $\beta_{\mathrm{s}} = \beta_{\mathrm{r}} = 22{,}5°$, $R = 1{,}3\,[\Omega]$, $L_{j\mathrm{min}} = 0.0119\,[\mathrm{H}]$, $L_{j\mathrm{max}} = 0.1112\,[\mathrm{H}]$; where β_{s} and β_{r} are the pole arc for stator and rotor, respectively.

Considering the absence of magnetic saturation, as previously referred (independence of L_j from i_j) and attending to the machine geometry, the evolution of the self-inductance of each of the four phases, can be drawn, as a function of the rotor angular position, θ, for the corresponding mechanical period, which is $\theta \in [-30°; 30°]$. This $L_j(\theta)$ evolution is represented in Fig. 2.

Another important aspect for the SRM characterization is the evolution of the γ_j functions (the total current partition among phases).

The γ_j functions depend on the global current position, ρ. To have a univocal definition of the ρ variable, and attending to the usual operation status of this kind of system, it is considered the existence of, at maximum, two contiguous phases simultaneously energized, during the commutation process. Outside the commutation process only

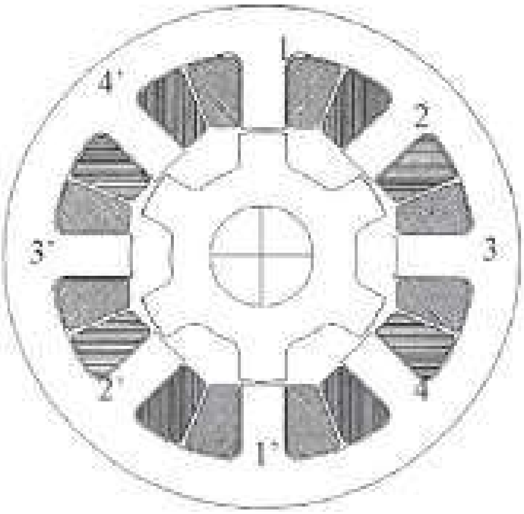

Fig. 1. Cross-section of the 8/6 SRM.

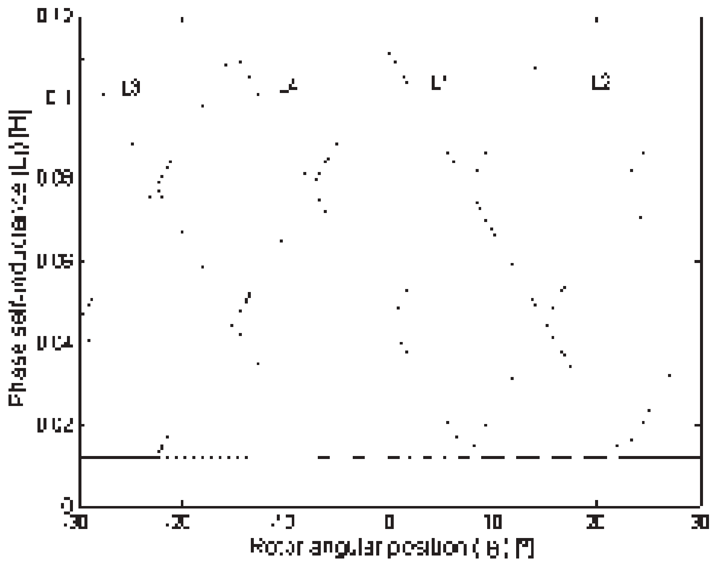

Fig. 2. Self-inductance evolution of each phase, function of θ.

one phase is excited at a time. Otherwise, if more than two phases were considered to be energized at the same time, it would be insufficient to have only one global current position variable.

With these considerations, a linear relation can be established between the γ_j functions and the ρ variable, resulting on the γ_j evolution presented in Fig. 3, for a mechanical angle period of $\rho\in[-90°; 90°]$.

With the definition and representation of the phase self-inductance ($L_j(\theta)$) and the total current partition per phase ($\gamma_j(\rho)$), it is possible to represent the evolution of the previously defined global parameters, R_{eq} (ρ) and L_{eq} (θ, ρ), which was done in [5].

6. Results

For studying and evaluating the new model performance it will be considered a particular, but representative, situation of having a controlled current, with the corresponding imposed m.m.f. It is then possible to define a command law where the command variable, the global current position ρ varies with the rotor angular position θ having a linear transition between phase currents in the commutation process.

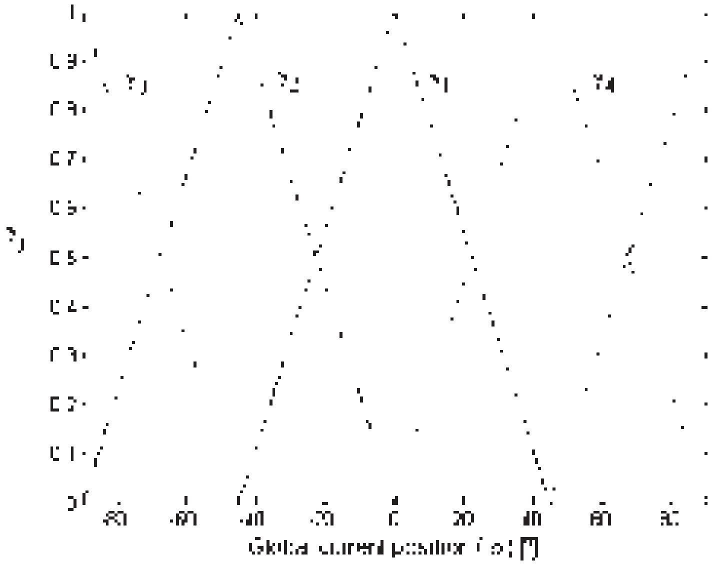

Fig. 3. Phase partition of the global current, function of ρ.

Table 1
Command law

θ-values	Phases ON	ρ-values
$-30^\circ \le \theta \le -22.5^\circ$	4	$\rho = 45^\circ$
$-22.5^\circ < \theta < -15^\circ$	4 and 1	$\rho = 45^\circ - 6(\theta + 22.5^\circ)$
$-15^\circ \le \theta \le -7.5^\circ$	1	$\rho = 0^\circ$
$-7.5^\circ < \theta < 0^\circ$	1 and 2	$\rho = -6(\theta + 7.5^\circ)$
$0^\circ \le \theta \le 7.5^\circ$	2	$\rho = -45^\circ$
$7.5^\circ < \theta < 15^\circ$	2 and 3	$\rho = -45^\circ - 6(\theta - 7.5^\circ)$
$15^\circ \le \theta \le 22.5^\circ$	3	$\rho = -90^\circ$ ($\equiv 90^\circ$)
$22.5^\circ < \theta < 30^\circ$	3 and 4	$\rho = 90^\circ - 6(\theta - 22.5^\circ)$

This commutation process occurs during the ascending evolution of the phase self-inductance coefficients (positive torque) and during the entire superposition interval for the angular rotor position and for contiguous phases.

With these considerations the system command law is expressed in Table 1.

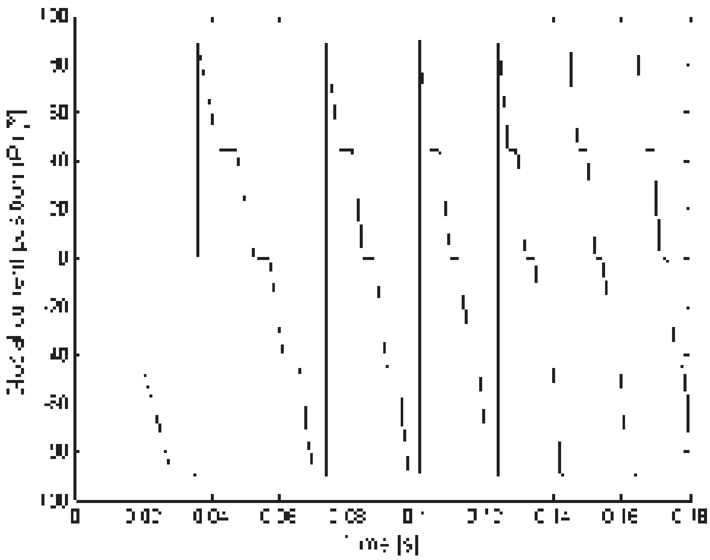

Fig. 4. Time evolution of the global current position, ρ, for one rotor revolution.

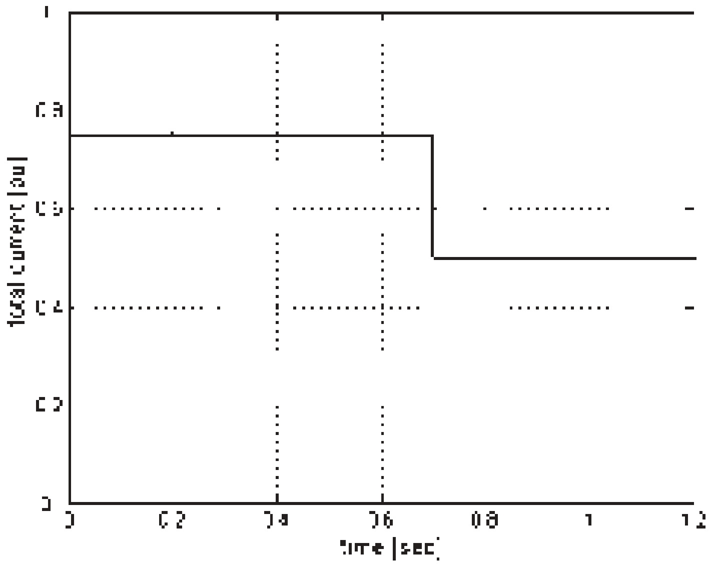

Fig. 5. Time evolution of the machine total current, i.

To have a better perception of this particular command law, a time evolution of the global current position – ρ – is represented in Fig. 4, for one rotor revolution and where the total current is kept constant and the machine speed is starting up.

Considering the SRM average values global model, an approximation of this command law should be performed by:

$$\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}t} = k_1\bar{\omega} \tag{28}$$

With this command law approximation, the electrical Eq. (26) can be transformed in (29):

$$\bar{u} = \langle R_{\mathrm{eq}} \rangle \bar{i} + \langle L_{\mathrm{eq}} \rangle \frac{\mathrm{d}\bar{i}}{\mathrm{d}t} + \left(\left\langle \frac{\partial L_{\mathrm{eq}}}{\partial \theta} \right\rangle + k_2 \left\langle \frac{\partial L_{\mathrm{eq}}}{\partial \rho} \right\rangle \right) \bar{\omega}\bar{i} \tag{29}$$

Having a controlled current, it is established a global current value time evolution like the one represented in Fig. 5. A quickly total current transition is initially imposed and later a similar quickly transition is imposed to the total current (0,7 s).

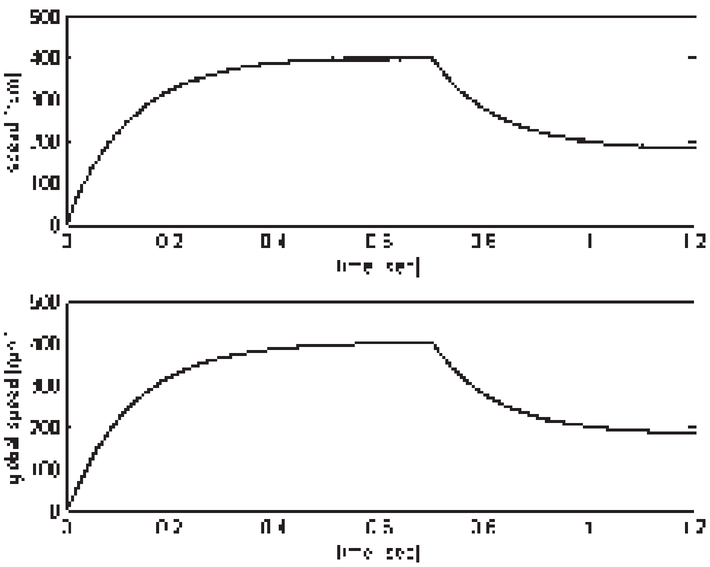

Fig. 6. Time evolution of the machine speed (detailed model (ω) and average values global model ($\bar{\omega}$)).

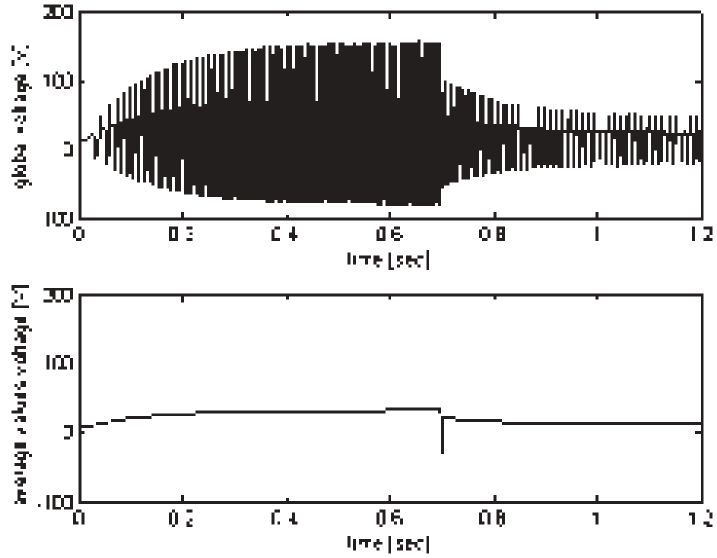

Fig. 7. Time evolution of the global voltage (global model (u) and average values global model ($\overline{u}$)).

With this total current time evolution, the evolution of the machine speed was obtained for the detailed model representation and for the new average values global model (Fig. 6). The results illustrate the capability of describing the global machine behavior, in this particular situation, of the new SRM average values global model.

Fig. 7 also illustrates the performance of the new model. The time evolution of the global voltage, previously defined, is represented in both situations. The upper evolution refers to the global model, initially obtained, which has the disadvantage of having time-dependent parameters (but does not have any loss of system information), and the lower evolution where the global voltage of the average values global model is represented. There is a good agreement regarding the comparison of the average value for the upper global voltage (u) with the values obtained for the lower global voltage ($\overline{u}$), with the average values global model.

A zoom over the time evolution of the machine electromagnetic torque is represented in Fig. 8 for the same essay. In solid line it is represented the evolution with the detailed model and in dashed line it is represented the evolution with the average values global model.

The purpose of this figure is to illustrate the SRM behavior. It can be seen that with the almost the time imposed constant global current evolution, particularly in the commutation process, the electromagnetic torque could not have a constant value (detailed model). To have a constant torque evolution, with no ripple, a non-constant global current and a different command law should be established.

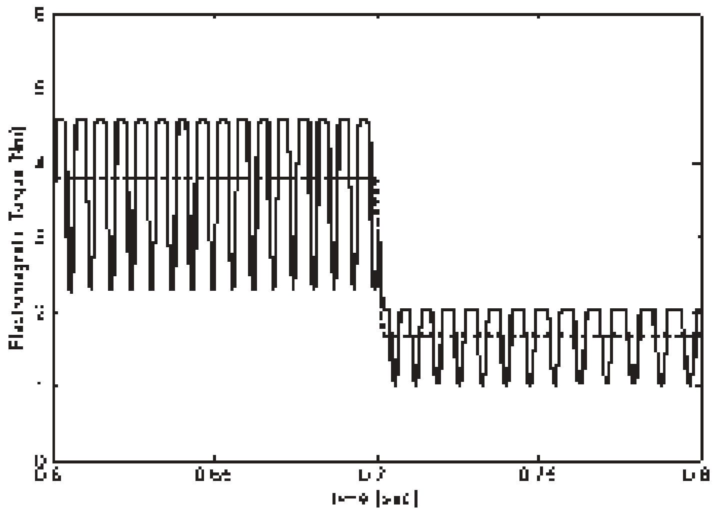

Fig. 8. Time evolution of the electromagnetic torque (detailed model (T_e – solid line) and average values global model ($\overline{T_e}$- dashed line)).

7. Conclusions

An average values global model for the SRM was obtained and tested in a typical situation. This model was developed having fewer dynamic equations than the ones usually considered to describe the SRM behavior. In fact, the number of electrical equations that is usually used in the SRM models is equal to the number of machine phases and with this new model only one electrical equation is used.

This is possible with some assumptions regarding the machine behavior, particularly the number of phases that are energized at a time.

The model was established in two steps. The purpose of the first one was to reduce the number of the system dynamic equations. The obtained global model was simpler than the detailed one and was based on the typical operating conditions of the SRM. No information was lost in this first step; however the presence of time-dependent global parameters was a problem. This problem was overcome in the second step, when a major simplification was introduced. It was considered the achievement of average values for the global parameters and new average values global variables were defined.

For testing the new simplified model a typical situation was considered. An imposed m.m.f. was the base for the obtained results, that validate, in this important situation, the average values global model.

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