AN ACTIVE CONTOUR MODEL FOR IMAGE SEGMENTATION: A VARIATIONAL PERSPECTIVE

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ABSTRACT

Image segmentation is a crucial step in computer vision, medical imaging and image processing. There has been recently an interest in a nonlinear partial differential equation based approach, motivated by a more systematic approach for image segmentation. In this paper, a novel active contour model expressed in terms of an energy functional is formulated in a calculus of variations framework. The key idea behind the proposed technique is to segment objects from the background of images that may have non-uniform brightness because of illumination. Simulation results showing a much improved performance of the proposed method in image segmentation are analyzed and illustrated.

1. INTRODUCTION

Image segmentation refers to the process of distinguishing objects from background. A variety of approaches have been developed for solving this problem. For gray scale images, one may classify the segmentation methods into four popular approaches namely, edge-based methods, threshold techniques, connectivity-preserving relaxation methods, and region-based techniques.

Over the last decade, variational methods and partial differential equations (PDE) based techniques [1, 2, 3, 4, 5, 6] have been introduced for a variety of purposes including but not limited to image denoising, curve evolution, mathematical morphology, and image segmentation. This last topic will be the focus of the present paper.

Recently, Chan and Vese [7] proposed an active contour model expressed in the variational and curve evolution framework to detect objects in an image. Its variational integral requires computing the image statistics (mean value) inside and outside the evolving implicit curve. This model, however, fails when the brightness of the image is not uniform, that is, the gray level distributions inside and outside the evolving curve are overlapping. Another active contour model combining boundary and region-based segmentation approaches is proposed in [8].

In this paper, we present a novel variational active contour model for image segmentation which mitigates the latter problem and demonstrates more robustness in the presence of contrast variability. The key idea behind the proposed technique is to locate the contours of objects at the optimal position in images that may have non-uniform brightness because of illumination.

In the next section we briefly outline the method of active contours without edges proposed in [7]. In Section 3, a novel variational model for image segmentation is proposed. This model takes into account the information about the boundary of the objects in the target image. In Section 4, we provide experimental results to show a much improved performance of the proposed technique in image segmentation. Finally, in Section 5 we give some conclusions.

2. PROBLEM STATEMENT

In the variational framework, an image I is considered as a real-valued function $I : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$, where Ω is a nonempty open and bounded set with Lipschitz boundary $\partial\Omega$ (usually Ω is a rectangle in $\mathbb{R}^2$). Throughout, $\boldsymbol{x} = (x_1, x_2)$ denotes a pixel location in Ω, $|\cdot|$ denotes the Euclidean norm, "∇" stands for the gradient, and "$\nabla\cdot$" denotes the divergence operator.

An active contour is an evolving implicit curve Γ defined as a zero-level set of a real-valued function $u : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$ that deforms within an image I according to a variational flow in order to detect or recover objects in the image background.

The active contour model proposed in [7] is based on the theory of level set method and curve evolution. The basic idea of this model is to minimize the following variational

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integral

$$\begin{aligned}\mathcal{F}_\epsilon(u) &= \alpha\int_\Omega \delta_\epsilon(u)|\nabla u|d\boldsymbol{x} + \beta\int_\Omega H_\epsilon(u)d\boldsymbol{x} \\ &\quad +\mu_1\int_\Omega |I-m_1|^2 H_\epsilon(u)d\boldsymbol{x} \\ &\quad +\mu_2\int_\Omega |I-m_2|^2(1-H_\epsilon(u))d\boldsymbol{x},\end{aligned}$$

where m_1 and m_2 are the mean values of the image I inside and outside the curve defined as a zero-level set of u respectively (see Fig. 1(a)), and $\alpha, \beta, \mu_1, \mu_2$ are the regularizing parameters to be estimated or chosen *a priori*. H_ϵ is the regularized Heaviside function defined as

$$H_\epsilon(z) = \frac{1}{2}\left(1+\frac{2}{\pi}\arctan\left(\frac{z}{\epsilon}\right)\right)$$

and δ_ϵ its derivative in the distributional sense (see Fig. 1(b)). Note that $\lim_{\epsilon\to 0} H_\epsilon = H$ (i.e. the standard Heaviside function).

The most important first-order necessary condition to be satisfied by any minimizer of the functional $\mathcal{F}_\epsilon$ is that its first variation $\delta\mathcal{F}_\epsilon(u;v)$ vanishes at u in the direction of v, that is

$$\delta\mathcal{F}_\epsilon(u;v) = \frac{d}{dh}\mathcal{F}_\epsilon(u+hv)\bigg|_{h=0} = 0, \qquad (1)$$

Using the fundamental lemma of the calculus of variations, relation (1) yields the Euler-Lagrange equation as a necessary condition to be satisfied by minimizers of $\mathcal{F}_\epsilon$

$$\begin{aligned}\delta_\epsilon(u)\Bigg(-\alpha\nabla\cdot\left(\frac{\nabla u}{|\nabla u|}\right)+\beta \\ +\mu_1(I-m_1)^2-\mu_2(I-m_2)^2\Bigg) = 0\end{aligned}$$

To solve this nonlinear equation, a variety of iterative methods may be applied such as the gradient descent flow expressed as a PDE given by

$$\begin{aligned}u_t &= \delta_\epsilon(u)\Bigg(\alpha\nabla\cdot\left(\frac{\nabla u}{|\nabla u|}\right)-\beta \\ &\quad -\mu_1(I-m_1)^2+\mu_2(I-m_2)^2\Bigg),\end{aligned}$$

where $u(\boldsymbol{x}, t=0) = u_0(\boldsymbol{x})$ is the initial implicit curve (contour) and we assume homogeneous Neumann boundary conditions. But in the case of images with non-uniform brightness, Chan-Vese method cannot locate the boundary at the proper position.

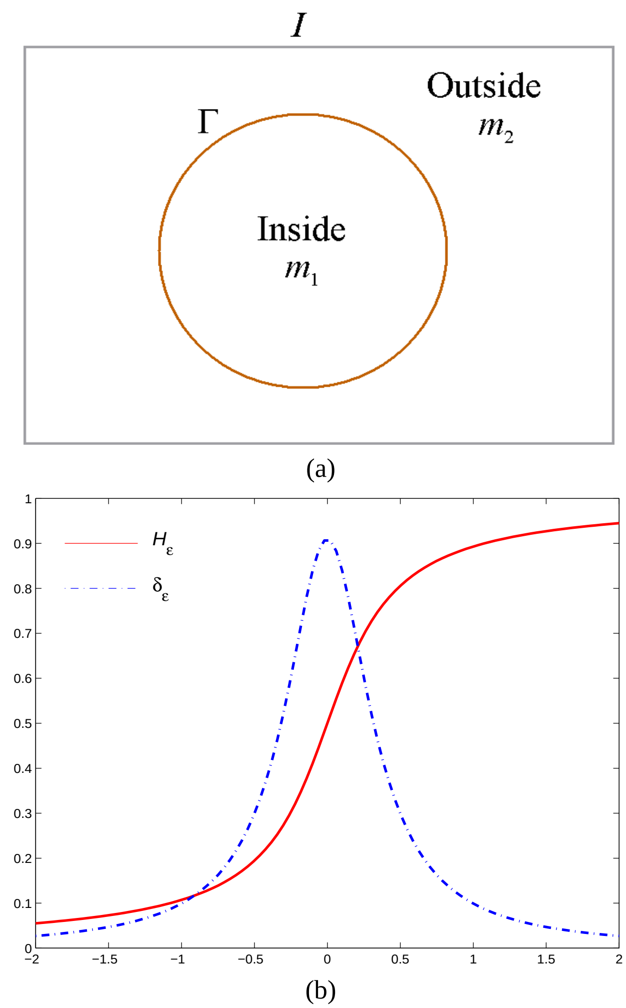

Fig. 1. (a) Model illustration. (b) Regularized Heaviside function H_ϵ and its distributional derivative δ_ϵ.

3. PROPOSED METHOD

Suppose that the original image $I \in C^2(\overline{\Omega})$ (for instance, we consider a regularization of the image I with the Gaussian mollifier, i.e. $G_\sigma \star I$, and σ is the Gaussian variance).

In evaluating the Chan-Vese energy functional, no information about the boundary of the objects in the image I was taken into account. An effective way to include such information is to use the Laplacian of the image I. Since $\Delta I \geq 0$ in the dark background area, and $\Delta I \leq 0$ in the bright object area near the contour, it follows that the difference of ΔI in the background and the object will be maximum when the contour is at the optimal position as depicted in Fig. 2. In other words, the following functional

$$\mathcal{D}_\epsilon(u) = \int_\Omega\Big(H_\epsilon(u)\Delta I-(1-H_\epsilon(u))\Delta I\Big)d\boldsymbol{x}$$

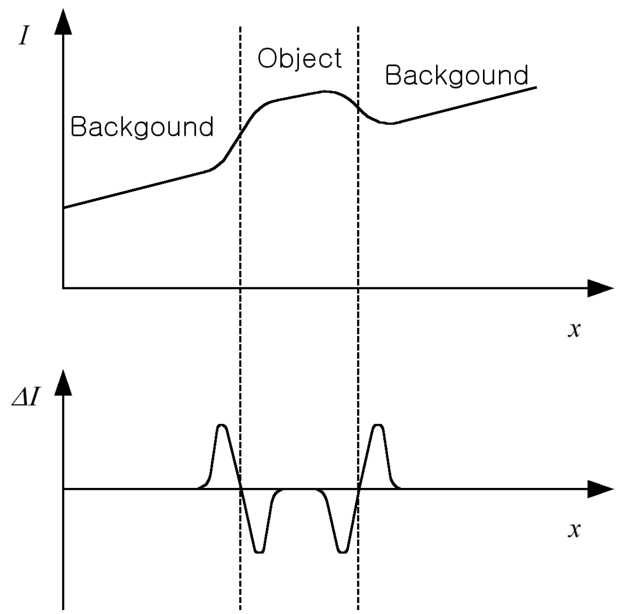

Fig. 2. The Laplace operator around the boundary between the object and the background.

$$= \int_\Omega (2H_\epsilon(u) - 1)\Delta I d\boldsymbol{x}$$

will be maximized when the boundary is located at the correct position. Hence we expect a significant improvement of Chan-Vese method and a more accurate segmentation algorithm if we minimize the variational integral $\mathcal{L}_\epsilon(u) = \mathcal{F}_\epsilon(u) - \eta\mathcal{D}_\epsilon(u)$, where η is a positive constant. It follows that

$$\begin{aligned} \mathcal{L}_\epsilon(u) &= \alpha \int_\Omega \delta_\epsilon(u)|\nabla u| d\boldsymbol{x} + \beta \int_\Omega H_\epsilon(u) d\boldsymbol{x} \\ &\quad + \mu_1 \int_\Omega |I - m_1|^2 H_\epsilon(u) d\boldsymbol{x} \\ &\quad + \mu_2 \int_\Omega |I - m_2|^2 (1 - H_\epsilon(u)) d\boldsymbol{x} \\ &\quad - \eta \int_\Omega (2H_\epsilon(u) - 1)\Delta I d\boldsymbol{x} \end{aligned}$$

Using the Euler-Lagrange variational principle, the minimizer of $\mathcal{L}_\epsilon$ can be interpreted as the steady state solution to the following gradient descent flow

$$\begin{aligned} u_t &= \delta_\epsilon(u) \left(\alpha \nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) - \beta - \mu_1 (I - m_1)^2 \right. \\ &\quad \left. + \mu_2 (I - m_2)^2 + 2\eta \Delta I \right) \end{aligned}$$

where $u(\boldsymbol{x}, t = 0) = u_0(\boldsymbol{x})$ is the initial implicit curve (contour) and we assume homogeneous Neumann boundary conditions.

4. SIMULATION RESULTS

This section presents simulation results where the proposed method is applied to segment objects in images with non-uniform as well as uniform brightness. In all the experiments, we choose the regularizing parameters in our model as follows: $\alpha = 0.01 \max(u(\boldsymbol{x}))^2$, $\beta = 1$, $\mu_1 = \mu_2 = 1$, $\eta = 500$.

In order to evaluate the performance of the proposed technique in image segmentation, Chan-Vese method and the proposed method are applied to various images. Fig. 3 is an example of uniform illumination case. Objects can be segmented by both methods. Fig. 4 is an example of a non-uniform brightness case. Fig. 4(c) is the result of Chan-Vese method, and it shows clearly that the objects cannot be segmented because the brightness of the image is not uniform. The proposed method shows an improved performance in Fig. 4(d).

The problem of non-uniform illumination frequently occurs in various industrial applications. Fig. 5 is one of the examples. Fig. 5(a) is an example of a vehicle license plate image taken in Korea. There is a growing demand for traffic data concerning traffic flow and automatic vehicle identification, especially from law enforcement to control for instance highway traffic, access to parking lots or finding stolen vehicles. In order to maximize the use of vision technology, the images of the traffic scene should be captured and analyzed in real time. The main goals of a vehicle licence plate recognition system are the segmentation and recognition of the numbers and characters. It is difficult, however, to segment the characters in vehicle license plates under harsh operating environments such as changing light intensities of day and night and differing weather conditions. In Fig. 5(a) the brightness of the background in the left side of the original image is almost the same as the brightness of the numerals in its right side. The results depicted in Fig. 5 show that our proposed technique outperforms the Chan-Vese model.

5. CONCLUSIONS

In this paper, we proposed a novel variational model expressed in the level-set theory framework to achieve image segmentation. We demonstrated the improved performance of the proposed technique in a variety of images. The key strength of our active contour model is that it robustly locates the contour of objects at the optimal position in case of non-uniform illumination as well as uniform illumination. Our proposed technique can be applied with robustness to a variety of industrial computer vision applications.

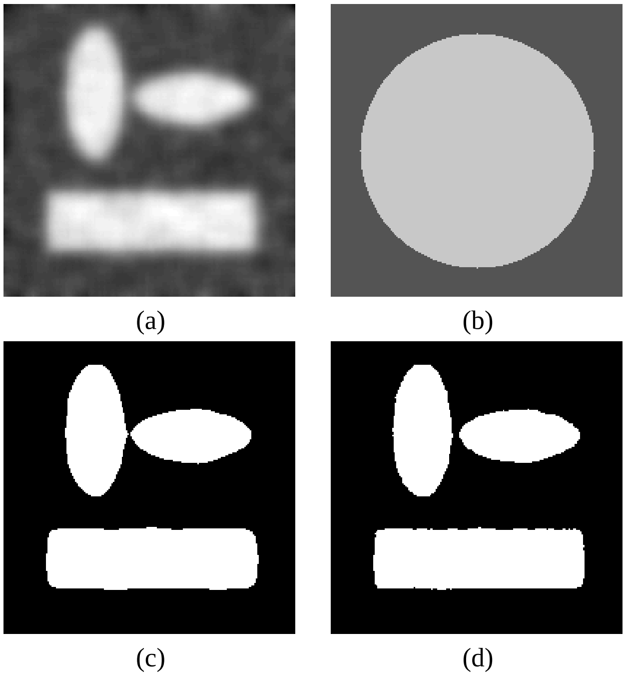

Fig. 3. Segmentation results (uniform brightness): (a) Original image, (b) Initial curve, (c) Chan-Vese method, (d) Proposed method.

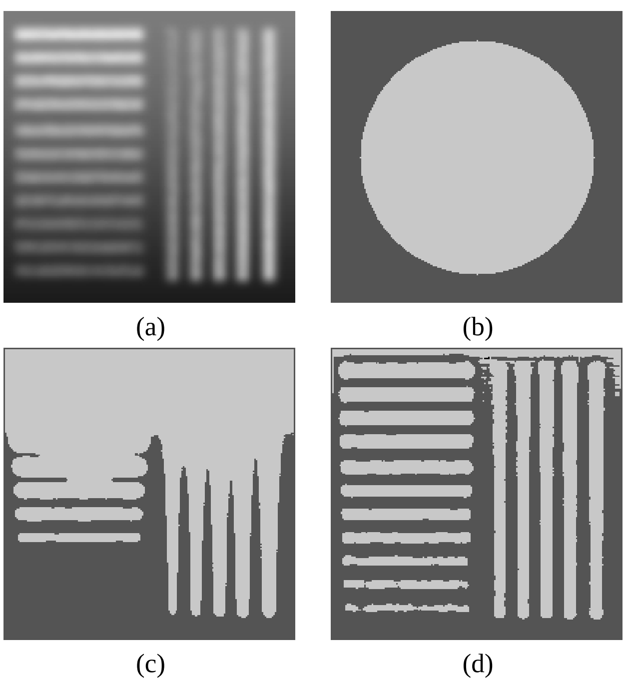

Fig. 4. Segmentation results (non-uniform brightness) : (a) Original image, (b) Initial curve, (c) Chan-Vese method, (d) Proposed method.

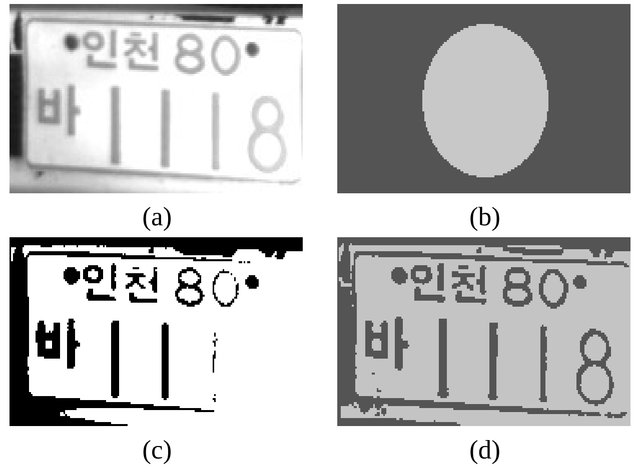

Fig. 5. Segmentation results (vehicle license plate with non-uniform illumination): (a) Original image, (b) Initial curve, (c) Chan-Vese method, (d) Proposed method.

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